Interest, Reserves and Prices*

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Abstract

We would like to propose a new framework for monetary policy analysis that encompasses, as a special case, the Neo-Wicksellian paradigm. A general form of an aggregate-demand equation reveals a role for liquidity, as well as less effective movements in future real rates with respect to current ones, in stimulating aggregate demand. The quantity of reserves and their interest rate both matter for determining inflation and economic activity.

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1 Introduction

The Neo-Wicksellian framework that is currently popular for monetary policy analysis presents two peculiar features (see Galì, 2008, and Woodford, 2003, for a general presentation of the model). The first is that the central bank can have full control of inflation and output through movements in the policy rate. As a consequence, once the policy rate is specified, there is no additional role for the composition and size of the central bank's balance sheet or, in general, for money aggregates to influence prices and economic activity. The second important feature is that the policy rate coincides with the nominal interest rate that consumers face when deciding on how much to save and consume. In this way, the policy rate directly influences spending and, therefore, aggregate demand.

We propose a framework that generalizes the standard paradigm by relaxing the two features underlined above. First, reserves are going to be an additional tool on top of the policy rate (interest rate on reserves) that the central bank needs to specify in order to control inflation and output. Second, the policy rate does not coincide anymore with the nominal rate relevant for the consumption/saving choice, which is now influenced by a novel monetary transmission mechanism.

The departure from the standard Neo-Wicksellian paradigm arises because of a proper modelling of the banking sector, which unveils that the two peculiar features underlined above are the result of special cases nested in the new framework.

First, we explain why the policy rate does not necessarily coincide with the nominal rate relevant for the consumption/saving choices. In our framework, the only agents holding central bank reserves are banks, which use them to collateralize deposits. In this way, the interest rate on deposit is going to depend on the interest rate the central bank sets on its reserves. Deposits are a saving instrument for households but they also provide liquidity services. The interest rate on deposits will therefore command a liquidity premium with respect to other illiquid securities that households may hold. However, it is exactly the interest rate on these illiquid securities that is the one that matters for their consumption/saving choices and for aggregate demand. As such, it will be linked to the policy rate only through the connection with the deposit rate and the liquidity premium. Via this new monetary-policy transmission mechanism, central bank reserves become an additional tool available for the monetary policymaker to control aggregate demand, since in general they can influence the liquidity premium through the quantity of deposits that they back. An additional implication of the new framework is that also government tax policy can influence liquidity premia and, through them, aggregate demand.

The standard Neo-Wicksellian paradigm is nested when reserves do not provide any non-pecuniary benefits either to banks or directly to households or when, even if they do provide such benefits, conditions are such that agents get zero marginal benefits from holding them, i.e. liquidity is fully satiated. This occurs either when taxation is high to back a sufficient supply of government liquidity or when, with the absence of this backing, reserves grow even to overshoot the equilibrium inflation rate.

The Neo-Wicksellian paradigm, instead, does not coincide with the so-called narrow-banking regime in which deposits are fully backed by reserves or with the case in which households directly hold deposits at the central bank – a situation that, in recent debates, has been labelled as 'central-bank digital currency'. In these cases, there is still a disconnection between the policy rate and the relevant rate for consumption/saving choices. However, reserves – which are still a tool that the central bank needs to specify when setting monetary policy – no longer influence inflation and economic activity. What matters for the liquidity premium is just the overall supply of government liquidity, which is controlled solely through taxation. Only when taxation is appropriately high to satiate liquidity, the Neo-Wicksellian framework results again as a special case.

Our framework provides a novel aggregate demand equation that generalizes the one used in standard New-Keynesian models. Output depends not only on the current and expected future real rates but also on the supply of liquidity in the economy. A higher supply of liquidity, which lowers liquidity premia, stimulates aggregate demand and therefore output. The connection between output and real rates also changes. It is no longer the case that an increase in future real rates lowers output by the same magnitude as movements in the current real rate do. Instead, in our general analysis, it happens that future real rates count less for short-run aggregate demand and output. The model is consistent with a reduced power of forward guidance in stimulating aggregate demand.

We use our framework to study how a standard monetary policy shock propagates in the economy in comparison with the New-Keynesian model and we investigate the effects of varying central bank reserves on output and inflation. We also analyze the deflationary impact of a liquidity shock, which originates from a contraction in the supply of liquidity.

The banking model is also enriched with a credit channel for which banks supply loans to the corporate sector. Loans are requested by intermediate-good producers to finance the purchase of capital and are subject to default depending on the firm's revenues. Banks, which are subject to a limited-liability constraint, raise equity to absorb losses on loans. The credit channel operates through the intermediaries' leverage, which in the end depends on the probability distribution of the default. We study how a credit crunch, due to a sudden fall in leverage, propagates to affect output and inflation.

This work is related to recent literature that has provided departures from the standard New-Keynesian model. Benigno and Nisticò (2017) already presents a model in which the central bank has two policy instruments: the interest rate on reserves and their quantity. In their model interest-bearing reserves provide liquidity services through a cash-in-advance constraint together with a privately issued asset. They use their framework to study how an exogenous reduction in the liquidity properties of private assets affects inflation and output, given different monetary policies. Their banking model is stylized and they do not provide a general framework that nests the standard Neo-Wicksellian paradigm, as we do here. More recently, Diba and Loisel (2020) have presented a New-Keynesian model with the central bank supplying interest-bearing reserves and in which the monetary policymaker also has two policy instruments, like here. In their framework, reserves enter the aggregate-supply equation, unlike in our model, because they reduce firms' borrowing costs. Their focus is mainly on equilibrium determinacy and on policies at the zero-lower bound. Diba and Loisel (2021) use this class of models to account for some features of the US economy in the aftermath of the Great Financial crisis: little inflation volatility, no deflation and subdued inflation after quantitative-easing policies.

Piazzesi, Rogers and Schneider (2021) also emphasize the disconnection between money-market rates and the interest rate relevant for consumption/saving choices. They present a banking model in which monetary policy operates either through a corridor or a floor system. We focus on the floor system by providing an enriched banking model that also features a credit channel. The two analyses differ also in the way they characterize the monetary/fiscal policy regime and, therefore, in the implications in terms of the determination of output and inflation.

Bigio and Sannikov (2021) integrate monetary policy analysis through a corridor system via a banking model that displays a liquidity and a credit channel. However, in their case, when the corridor around the policy rate shrinks to zero, the only policy instrument remains the interest rate on reserves while the quantity of reserves becomes irrelevant. In our model, instead, reserves are always a policy instrument and are relevant for inflation and economic activity even with a zero corridor system, provided they supply some non-pecuniary benefits.

Bigio and Sannikov (2021) and Piazzesi, Rogers and Schneider (2021) distinguish between a model in which reserves are scarce and the central bank conducts policy through a corridor system, or a model in which reserves are abundant and the central bank conducts policy through a "floor system".[1] Our model is one in which there is no use of central bank settlement balances as a means of clearing payments between banks. The corridor system shrinks to zero at the policy rate, i.e. the interest rate on reserves.[2] This is in line with what they label as the "floor system", which, however, works regardless the scarcity or abundance of reserves. Moreover, irrespective of the size of the reserves, the central bank always has two independent tools to specify policy, the interest rate on reserves and their quantity.[3] We show that the quantity of reserves can be relevant for determining inflation and economic activity, unless the reserves do not provide any non-pecuniary marginal benefits. If anything, the abundance of reserves may imply irrelevance of reserves for inflation or output but, in general, the abundance or scarcity of reserves are not relevant dimensions to discriminate between a corridor or a floor system.

Canzoneri et al. (2008) and Canzoneri, Cumby and Diba (2017) are early models in which there is a disconnection between the policy rate and the interest rate relevant for consumption/saving choices. Curdia and Woodford (2010, 2011) present models

[1] De Fiore, Hoerova and Uhlig (2018) also present a banking model in which there are frictions in the money market.

[2] Our analysis is in line with the discussion of Woodford (2001) on how the central bank can still control money-market interest rates when central bank settlement balances cease to be used to clear payments across banks.

[3] In fact, there are three independent policy tools including the specification of central bank remittances policy.

with borrowers and savers in which credit spreads arise because of intermediation activity. However, in their context, the policy rate is still the relevant factor for the savers' consumption/saving choices. The central bank's balance sheet is also an additional policy instrument when there are financial frictions, but it acts only on credit spreads.

Our work is also related to the literature on the "forward-guidance puzzle" as elaborated by Del Negro, Giannoni and Patterson (2013) in which the New-Keynesian model gives too much power to forward guidance in affecting current demand. Recent works such as Werning (2015) and McKay, Nakamura and Steinsson (2016) have tried to reconcile the puzzle by using incomplete market models. Our framework, instead, delivers a new AD equation in which forward guidance is less powerful even when markets are complete. A similar result is obtained in Diba and Loisel (2020).

Finally, our model, in the special case of a narrow-banking regime, is also related to the recent literature that has studied central bank digital currency by allowing households to directly hold deposits at the central bank (see the work of Niepelt, 2021, and again Piazzesi, Rogers and Schneider, 2021).

The present work starts with Section 2, providing the main intuition for why our framework departs from the standard Neo-Wicksellian paradigm. Section 3 presents a simple model assuming flexible prices, costless bank equity and an exogenous default on loans. Section 4 studies the equilibrium with some examples. Section 5 extends the model with costly equity, endogenous default and sticky prices. Section 6 studies the implications of this more general model in a log-linear approximation. Section 7 concludes the work.

2 Main mechanism and intuition

In this section, we highlight the main difference between our model and the standard Neo-Wicksellian paradigm. In the latter, the economy can be simply described by an AS-AD model in which the policy rate acts directly on the AD equation. Consider a standard Euler equation in a perfect-foresight model

$$U_c(C_t) = \frac{\beta(1+i_t)}{\Pi_{t+1}} U_c(C_{t+1}) \tag{1}$$

in which $U_c(\cdot)$ is the marginal utility of consumption, C_t; β is the rate of time preference, with $0 < \beta < 1$; i_t is the nominal interest rate at time t and Π_{t+1} is the gross inflation rate between time t and $t+1$. A key assumption in the baseline Neo-Wicksellian paradigm is that the policy rate controlled by the central bank is the same as the nominal rate influencing the AD equation. Upward movements in the policy rate cause a contraction in demand for the given future consumption and inflation rate. More generally, by setting the policy rate, the central bank can control the path of inflation and output.

The framework proposed in this work is still consistent with the Euler equation (1). However, there is no longer a direct link between the policy rate and the nominal interest rate in the Euler equation. The latter will identify, more properly, the risk-free rate on (private) illiquid securities. For the sake of simplicity, focusing on the perfect-foresight equilibrium, the first-order conditions of the household imply the Euler equation (1)

$$U_c(C_t) = \beta \frac{(1 + i_t^B)}{\Pi_{t+1}} U_c(C_{t+1}), \tag{2}$$

in which i_t^B is now the natural nominal rate of interest. Households also hold other types of risk-free debt, which instead provides liquidity services. This class of securities might include bank deposits and/or treasury debt. An asset-pricing condition links the interest rate on liquid securities, i_t^D, to the natural nominal rate of interest

$$1 + i_t^D = (1 - \mu_t)(1 + i_t^B) \tag{3}$$

where μ_t, with $\mu_t \geq 0$, is the liquidity premium

$$\mu_t = V_q \left(\frac{Q_t}{P_t} \right),$$

with $V_q(\cdot)$ being the marginal utility from liquid securities, and Q_t the amount of liquid securities held by households in their portfolio. It is assumed that $V_q(Q_t/P_t) = 0$ for values of Q_t/P_t above a satiation level $\bar{q}$, i.e. $Q_t/P_t \geq \bar{q}$.

The last step to understand the novelty of the monetary transmission mechanism in our framework is the banking model. Financial intermediaries supply deposits and raise equity to invest in central-bank reserves, private risk-free securities and loans to firms. The banking equilibrium implies that the deposit rate will be a weighted average of the policy rate and the natural nominal rate of interest

$$(1 + i_t^D) = (1 - \rho)(1 + i_t^B) + \rho(1 + i_t^R),$$

where i_t^R is the policy rate while ρ is the reserve/collateral requirement with $0 \leq \rho \leq 1$ and $D_t = \rho R_t$; R_t are central bank reserves. Moreover, the following inequality holds: $(1 + i_t^B) \geq (1 + i_t^R)$.

We can then combine the above three equations to obtain

$$(1 + i_t^B) = \frac{\rho}{\rho - V_q \left(\frac{1}{\rho} \frac{R_t}{P_t} \right)} (1 + i_t^R)$$

showing the novel relationship between the policy rate and the rate directly influencing consumption/saving choices. We have used here the shortcut that treasury debt is zero and $Q_t = D_t$. If the economy is not satiated with liquidity, i.e. $V_q(Q_t/P_t) > 0$, movements in the policy rate have amplifying effects on the natural nominal rate of interest, with everything else being equal. On the other hand, variations in central bank reserves do affect the natural nominal interest rate independently of the movements of the policy rate. An increase in reserves, ceteris paribus, lowers the liquidity services of deposits and through that the natural rate of interest, having therefore an expansionary effect on the economy.

Note that even with full reserve requirement ($\rho = 1$), like in a narrow banking regime, there will be a difference between the policy rate and the natural nominal rate of interest. In this respect, it is important to note that the narrow-banking regime is isomorphic to a model in which households directly hold deposits at the central bank. In this case, the asset-pricing equation (3) is replaced by

$$1 + i_t^R = (1 - \mu_t)(1 + i_t^B),$$

and therefore

$$(1 + i_t^B) = \frac{1}{1 - V_q\left(\frac{R_t}{P_t}\right)}(1 + i_t^R).$$

In our general framework, the Neo-Wicksellian model is nested either i) when there is full satiation of liquidity, in which case $(1 + i_t^B) = (1 + i_t^R) = (1 + i_t^D)$, or ii) when there are no securities available that provide liquidity services or iii) when, even if some securities provide liquidity services, reserves are not in this class, like in the case in which they do not provide any collateral benefits ($\rho = 0$).

Our model also delivers a novel AD equation that has important differences with respect to the standard one used in New-Keynesian models. In a log-linear approximation it takes the following form:

$$\hat{Y}_t = -\sigma v_\rho E_t \sum_{T=t}^{\infty} v_\rho^{T-t}(\hat{\imath}_T^R - (\pi_{T+1} - \pi)) + \epsilon_q(1 - v_\rho)E_t \sum_{T=t}^{\infty} v_\rho^{T-t}\hat{q}_T,$$

in which $\hat{Y}_t$ is the deviation of output from the steady state, $\hat{\imath}_t^R$ is the deviation of the policy rate from the steady state, π_t is the log of the inflation rate and π its target; $\hat{q}_t$ is the overall supply of liquidity, σ the intertemporal elasticity of substitution in consumption and ϵ_q the inverse of the elasticity of liquidity demand. The parameter v_ρ is in the range $0 < v_\rho \leq 1$. The New-Keynesian AD equation is nested when $v_\rho = 1$. In the more general case in which $v_\rho < 1$, there are two important differences. First, liquidity ($\hat{q}$) matters for aggregate demand. Higher supply of liquidity has an expansionary effect on aggregate demand. Second, the effects on demand of the variations in the current real rate are dampened by the factor $v_\rho < 1$ while movements in future real rates matter even less. The AD equation can then be consistent with less power of forward guidance in influencing aggregate demand. In the New-Keynesian model, liquidity does not affect demand, and current and future real rates have all the same impact, thereby enhancing the importance of forward guidance.

Our model is also enriched by a credit channel since firms are financed by intermediaries to purchase capital. Unless this credit channel is shut down, the AS equation is more elaborated than that of the standard New-Keynesian model since the equilibrium in the market of capital goods and loans matters for the determination of inflation and output.

3 Model

We present the model in blocks starting from the banking sector, then households, firms and, finally, the government, which includes the treasury and the central bank.

3.1 Banking model

At a generic time t there is a potentially infinite number of intermediaries that can start intermediation activity without any entry cost. Each intermediary lives for two periods. Intermediaries entering at time t face the following budget constraint:

$$L_t + R_t + A_t = D_t + N_t \tag{4}$$

in which L_t are loans, which are supplied to firms at the interest rate i_t^L, R_t are the holdings of central bank reserves which are remunerated at the rate i_t^R, A_t are the holdings of short-term private debt that carries an interest rate i_t^B. Intermediaries can finance their assets by issuing deposits D_t, at the interest rate i_t^D, and by raising equity N_t.

They are subject to a reserve/collateral requirement of the form $R_t \geq \rho D_t \geq 0$ with $0 \leq \rho \leq 1$. The two extremes of the interval characterize two interesting cases. When $\rho = 1$, intermediaries need to back all deposits by reserves, like in a narrow banking system. When $\rho = 0$, there is no reserve/collateral requirement, but reserves should be non-negative, $R_t \geq 0$.

Intermediaries can also invest in cash, which is going to be dominated by reserves. The economy is therefore cashless in equilibrium but not without cash as a store of value. The possibility that reserves can be transformed into cash implies the existence of a zero-lower bound on the interest rate on reserves, $i_t^R \geq 0$.

Note that A_t can be negative, meaning – in this case – borrowing, as it will be shown in the household's problem $i_t^B \geq i_t^D$. Therefore, deposits are a better way of financing intermediaries' assets. The reason for such inequality is that A_t represents a form of private indebtedness that is risk-free but not liquid, whereas deposit provides liquidity services and therefore receives a liquidity premium. However, given the demand for deposits and the supply of equity, intermediaries might rely on a more costly way of financing, if needed. Note also that i_t^B is the natural nominal rate of interest, previously defined, since it is the rate that directly influences the consumption/saving choice of households, as the next section is going to show.

Intermediary profits, Ψ_{t+1}, at time $t+1$ are given by

$$\Psi_{t+1} = (1 + i_t^L)(1 - \phi_{t+1})L_t + (1 + i_t^B)A_t + (1 + i_t^R)R_t - (1 + i_t^D)D_t. \tag{5}$$

Loans are subject to an exogenous and random default rate ϕ_{t+1} with $\phi_{t+1} \in [0, \phi_{\max}]$ in which $\phi_{\max}$ is the maximum default rate, with $\phi_{\max} < 1$. The default rate on loans is the only source of randomness in intermediary profits. Later, we are going to endogeneize default on the basis of firm's solvency. Intermediaries are subject to a limited-liability constraint, for their profits should be non-negative in all contingencies. This constraint can be written as

$$\Psi^{\min} = (1 + i_t^L)(1 - \phi^{\max})L_t + (1 + i_t^B)A_t + (1 + i_t^R)R_t - (1 + i_t^D)D_t \geq 0. \tag{6}$$

Finally, they maximize expected rents, which are equal to the expected discounted value of profits minus the value of equity,

$$\begin{aligned} \mathcal{R}_t &= E_t\left\{M_{t+1}\Psi_{t+1}\right\} - N_t \qquad (7) \\ &= E_t\left\{M_{t+1}\left[(1 + i_t^L)(1 - \phi_{t+1})L_t + (1 + i_t^B)B_t + (1 + i_t^R)R_t - (1 + i_t^D)D_t\right]\right\} - N_t. \end{aligned}$$

In the first line of (7), we have discounted profits with the stochastic discount factor M_{t+1}, which is the same as that of the consumers since they own intermediaries. In the second line we have substituted, in the equation, profits with (5).

Intermediaries choose L_t, A_t, R_t, D_t, N_t to maximize (7) under the budget constraint (4), the limited-liability constraint (6) and the collateral constraint $R_t \geq \rho D_t \geq 0$. The first-order conditions of the problem are

$$E_t \left\{ M_{t+1}(1+i_t^L)(1-\phi_{t+1}) \right\} - \lambda + \psi(1+i_t^L)(1-\phi^{\max}) = 0, \tag{8}$$

$$E_t \left\{ M_{t+1}(1+i_t^B) \right\} - \lambda + \psi(1+i_t^B) = 0, \tag{9}$$

with respect to L_t and A_t, in which λ and ψ are the Lagrange multipliers attached to the constraints (4) and (6), with $\psi \geq 0$.

The first-order conditions with respect to R_t and D_t are

$$E_t \left\{ M_{t+1}(1+i_t^R) \right\} - \lambda + \psi(1+i_t^R) + \upsilon = 0, \tag{10}$$

and

$$E_t \left\{ M_{t+1}(1+i_t^D) \right\} - \lambda + \psi(1+i_t^D) + \rho\upsilon = 0, \tag{11}$$

respectively, in which υ, with $\upsilon \geq 0$, is the Lagrange multiplier attached to the constraint $R_t \geq \rho D_t$. Finally, the first-order condition with respect to N_t is

$$\lambda = 1. \tag{12}$$

We can now derive the implications of the optimality conditions by combining them. First, when anticipating a result of the household's problem, note that $E_t \left\{ M_{t+1}(1+i_t^B) \right\} = 1$. Using it and the first-order condition (12) in (9), we obtain that $\psi = 0$. The first important implication is that the limited-liability constraint (6) is not binding because intermediaries can always raise enough equity to absorb the maximum loss on loans in a way as to make profits non-negative in each state. Later, we are going to extend the banking model to a case in which raising equity is costly, thus implying a binding limited-liability constraint. At this stage, a frictionless market for equity is better for simplifying the analysis and conveying the results in a clearer way.

Using $\lambda = 1$ and $\psi = 0$ in (8) we can obtain

$$E_t \left\{ M_{t+1}(1+i_t^L)(1-\phi_{t+1}) \right\} = 1 \tag{13}$$

which can be manipulated to get the spread between the lending rate and the natural nominal rate of interest as

$$\frac{(1+i_t^L)}{(1+i_t^B)} = \frac{1}{(1-\tilde{E}_t\phi_{t+1})}, \tag{14}$$

in which $\tilde{E}_t(\cdot)$ is the expectation operator under the risk-neutral measure and, therefore, $\tilde{E}_t\phi_{t+1} = E_t \left\{ M_{t+1}\phi_{t+1} \right\} / E_t M_{t+1}$. Note that the denominator on the right-hand side of equation (14) is below the unitary value. There is then a positive spread between the lending rate and the risk-free rate on illiquid bonds, a spread that is a function of the expected default rate on loans.

Finally, (10) implies that the value of the non-negative Lagrange multiplier υ is

$$\upsilon = 1 - \frac{(1+i_t^R)}{(1+i_t^B)},$$

and, therefore, $i_t^B \geq i_t^R \geq 0$. The reserve requirement is going to be binding in equilibrium whenever holding reserves is costly, i.e. the return is less than that on illiquid bonds, $(1+i_t^B) > (1+i_t^R)$. The spread between the deposit and bond rate can be obtained by using (12), $\psi = 0$ and the above equality for υ in (11):

$$(1+i_t^D) = \rho(1+i_t^R) + (1-\rho)\max[(1+i_t^B),(1+i_t^R)]. \tag{15}$$

Some interesting cases can be discussed. Consider first a binding reserve-requirement constraint, i.e. $R_t = \rho D_t$ and $\upsilon > 0$, equation (15) implies that

$$(1+i_t^D) = \rho(1+i_t^R) + (1-\rho)(1+i_t^B).$$

The deposit rate at which intermediaries are willing to supply deposit is a weighted average of the policy rate and the natural nominal rate of interest, with a weight given by the parameter ρ.

In a narrow banking regime, when $\rho = 1$, the deposit rate coincides with the policy rate, $i_t^D = i_t^R$, but in general $i_t^B > i_t^D = i_t^R$. At the other extreme, when $\rho = 0$ and reserves no longer provide non-pecuniary benefits, it follows that $i_t^D = i_t^B$ whereas the first-order condition (10) implies that $i_t^B > i_t^R$ when $\upsilon > 0$, i.e. $R_t = 0$, and $i_t^B = i_t^R$ when $\upsilon = 0$, i.e. $R_t > 0$. Since reserves are controlled by the central bank, they can be supplied in a positive amount, which is going to be held by banks at zero premium with respect to other default-free securities, i.e. $i_t^B = i_t^R$. Therefore, all interest rates are equalized, $i_t^B = i_t^R = i_t^D$, and the Neo-Wicksellian regime is nested.

Consider now a non-binding collateral constraint, i.e. $R_t \geq \rho D_t$ and $\upsilon = 0$. In this case, equations (10) and (11) imply $i_t^B = i_t^R = i_t^D$ and the Neo-Wicksellian regime is nested again.

To conclude the banking problem, the level of equity raised by intermediaries is such as to make the limited-liability constraint not binding.[4] Therefore, using (4) in (6), we obtain the following inequality:

$$\begin{aligned} N_t &\geq \left[1 - \frac{(1+i_t^L)}{(1+i_t^B)}(1-\phi^{\max})\right] L_t + \frac{i_t^D - i_t^B}{1+i_t^B} D_t + \frac{i_t^B - i_t^R}{1+i_t^B} R_t \\ N_t &\geq \left[1 - \frac{(1-\phi^{\max})}{(1-\tilde{E}_t\phi_{t+1})}\right] L_t \end{aligned}$$

in which we have used (14) and (**??**) in moving from the first to the second line.

[4]Note that intermediaries' rents are going to be zero in equilibrium.

3.2 Households

Consider a representative consumer maximizing the following intertemporal utility:

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[U(C_t)+V\left(\frac{D_t+B_t^h}{P_t}\right)\right]\right\}$$

in which β with $0<\beta<1$ is the intertemporal discount factor in preferences, E_{t_0} is the conditional-expectation operator at time t_0, $U(\cdot)$ and $V(\cdot)$ are increasing and concave functions of their respective arguments, with $V(\cdot)$ reaching a satiation point at a generic value $\bar{q}$; C is a consumption good and D are deposits from which agents get non-pecuniary benefits (liquidity services), B_t^h are the holdings of the treasury debt, which is perfectly substitutable with deposits in providing liquidity services. The household is subject to the following flow budget constraint:

$$P_tC_t+(D_t+B_t^h)+(1+i_{t-1}^B)B_t+N_t\leq P_t^KK+\Phi_t+\Psi_t+(1+i_{t-1}^D)(D_{t-1}+B_{t-1}^h)+B_{t-1}-T_t$$

in which P is the price of the consumption good, which is produced by firms, while P^K is the price of the endowment of capital. As will be detailed in the next section, firms transform capital into consumption goods. The household can invest its savings in two securities: D, deposit, and B^h, treasury notes, which are a perfect substitute for providing liquidity services and both pay an interest rate i^D. It can borrow or lend through private risk-free bonds, B, which pay an interest rate i^B. All securities are risk free, but deposit and treasury's notes provide liquidity services, whereas private bonds are illiquid.[5]

There is a subtle justification for why we are assuming that some risk-free securities, such as deposits and treasury debt, provide liquidity services while other securities do not. The key observation is that central bank reserves are always free of risk and are repaid, since the central bank issues those liabilities without being subject to a solvency constraint.[6] In attributing a liquidity role to treasury debt, we are implicitly assuming that the central bank always backs the treasury by extending the special risk-free properties of its liabilities to the treasury debt. Deposits are issued by intermediaries and backed, although partially, by reserves. For this guarantee, we also attribute a liquidity role to deposits. On the contrary, private debt needs to satisfy a solvency condition to be risk free, which makes it different from central bank reserves and from all the other securities that are implicitly backed by the central bank.

Households can also finance intermediaries with equity, N. On the right-hand side of the budget constraint, it has an endowment of physical capital K, and receives nominal profits from firms, Φ, and intermediaries, Ψ; T are lump-sum taxes levied by the government. Later, we are going to extend the model to allow for an endogenous supply of capital. The consumption/saving choices are subject to an appropriate borrowing limit.

[5] Note that in the household's budget constraint B denotes debt.

[6] See the discussion in Benigno (2020) and Woodford (2000).

The following Euler equation characterizes the choice with respect to the illiquid bonds:

$$E_t\{M_{t+1}\} = \frac{1}{1+i_t^B}, \tag{16}$$

so that the expected value of the stochastic discount factor is equal to the price of the illiquid bonds – the inverse of the gross nominal interest rate. As noted, we label this interest rate as the natural nominal rate of interest since it is the one that directly affects the saving-consumption choices. The nominal stochastic discount factor is given by $M_{t+1} = \beta(U_c(C_{t+1})/P_{t+1})/(U_c(C_t)/P_t)$. The optimal choice with respect to the liquid securities, D_t and B_t^h, implies that

$$1 = \mu_t + (1+i_t^D)E_t\{M_{t+1}\}, \tag{17}$$

in which μ_t is the liquidity premia given by

$$\mu_t = \frac{V_q\left(\frac{D_t+B_t^h}{P_t}\right)}{U_c(C_t)},$$

with $V_q(\cdot)$ the derivative of the function $V(\cdot)$. Note that we can combine (16) and (17) to obtain

$$(1+i_t^D) = (1-\mu_t)(1+i_t^B)$$

saying that the interest rate on deposit is not higher than that on illiquid bonds. Only when the economy is satiated with liquidity the two rates are the same. The optimal choice of equity implies that its value is equal to the discounted value of intermediary profits:

$$N_t = E_t\{M_{t+1}\Psi_{t+1}\}.$$

Finally, the intertemporal budget constraint of the consumer holds with equality at all times.

3.3 Firms

We now turn to the firms' problem. They borrow from intermediaries to finance the purchase of capital which is used to produce goods in the next period. The production function is given by $Y_{t+1} = Z_{t+1}K_t^\gamma$ with $0 < \gamma < 1$ and in which Z is a random disturbance. Firms live for two periods and in an overlapping way. A firm starting its activity at time t borrows an amount $L_t = P_t^K K_t$. A fraction $(1-\phi_{t+1})$ of its debt is randomly defaulted at time $t+1$, without costs. Therefore, at time $t+1$ firms' profits are given by

$$\begin{aligned}\Phi_{t+1} &= P_{t+1}Y_{t+1} - (1+i_t^L)(1-\phi_{t+1})L_t \\ &= P_{t+1}Z_{t+1}K_t^\gamma - (1+i_t^L)(1-\phi_{t+1})P_t^K K_t.\end{aligned}$$

Firms choose the amount of capital to maximize the expected discounted value of profits:

$$E_t\{M_{t+1}\Phi_{t+1}\} = E_t\left\{M_{t+1}\left(P_{t+1}Z_{t+1}K_t^\gamma - (1+i_t^L)(1-\phi_{t+1})P_t^K K_t\right)\right\}.$$

The optimal choice implies

$$\gamma E_t \left\{ M_{t+1} P_{t+1} Z_{t+1} K_t^{\lambda-1} \right\} = (1 + i_t^L) E_t \left\{ M_{t+1}(1 - \phi_{t+1}) P_t^K \right\},$$

which can be solved to obtain the optimal demand for capital

$$\begin{aligned} K_t &= \left[\frac{P_t^K}{P_t} \frac{(1 + i_t^L) E_t \left\{ M_{t+1}(1 - \phi_{t+1}) \right\}}{\gamma E_t \left\{ \tilde{M}_{t+1} Z_{t+1} \right\}} \right]^{\frac{1}{\gamma-1}} \\ &= \left[\frac{P_t^K}{P_t} \frac{1}{\gamma E_t \left\{ \tilde{M}_{t+1} Z_{t+1} \right\}} \right]^{\frac{1}{\gamma-1}} \end{aligned} \tag{18}$$

in which $\tilde{M}_{t+1}$ is the real stochastic discount factor $\tilde{M}_{t+1} \equiv \beta U_c(C_{t+1})/U_c(C_t)$. In going from the first to the second line, we have used (13).

3.4 Government

The government is composed of the treasury and the central bank. The treasury's budget constraint is

$$B_t^g = (1 + i_{t-1}^D) B_{t-1}^g - T_t - T_t^c \tag{19}$$

showing that the short-term debt issued by the treasury B^g carries the nominal interest rate i^D, as already discussed; the treasury can pay debt via taxes, T, and use remittances, T^c, received from the central bank. The central bank has the following budget constraint:

$$B_t^c - R_t = (1 + i_{t-1}^D) B_{t-1}^c - (1 + i_{t-1}^R) R_{t-1} - T_t^c \tag{20}$$

since it can issue interest-bearing reserves R at the rate i^R and it holds treasury notes as assets, denoted by B^c. Note that the central bank is not subject to any solvency condition since its liabilities define what a currency is. As mentioned, we are assuming that the central bank backs the treasury, which is, therefore, not subject to a solvency condition, too.

4 Equilibrium

We will now discuss the equilibrium of the model. Asset-market equilibrium requires that all bonds issued by the treasury are held by the central bank and the households, therefore $B_t^g = B_t^c + B_t^h$; the debt issued by the private sector is held by intermediaries $A_t = B_t$; the markets for loans and deposits are in equilibrium, as well as the market of central bank reserves. The demand for capital is equal to the fixed supply K and households hold all intermediary equity. As a consequence of the asset market equilibrium, the goods market equilibrium implies that

$$Y_t = Z_t K^\gamma = C_t,$$

output is equal to consumption. Using demand for capital (18) and the equilibrium in the capital market $K_t = K$, we obtain that the relative price of capital is

$$\frac{P_t^K}{P_t} = \gamma E_t \left\{ \beta \frac{U_c(Y_{t+1})}{U_c(Y_t)} Z_{t+1} \right\} K^{\gamma - 1}.$$

Having determined the real variables, we can now look at prices and interest rates.

Recall the spread in the banking equilibrium between the lending and the natural nominal rate of interest:

$$\frac{(1+i_t^L)}{(1+i_t^B)} = \frac{1}{(1-\tilde{E}_t \phi_{t+1})}, \tag{21}$$

while the relationship between the deposit rate, policy rate and nominal natural rate of interest is:

$$(1+i_t^D) = \rho(1+i_t^R) + (1-\rho)\max((1+i_t^B), (1+i_t^R)), \tag{22}$$

with $i_t^B \geq i_t^R \geq 0$.

On the other hand, the household's demand for securities implies that the spread between deposits and the natural nominal rate of interest should satisfy

$$\frac{(1+i_t^D)}{(1+i_t^B)} = \left(1 - \frac{V_q\left(\frac{D_t + B_t^h}{P_t}\right)}{U_c(Y_t)}\right), \tag{23}$$

with

$$\frac{1}{1+i_t^B} = E_t \left\{ \beta \frac{U_c(Y_{t+1})}{U_c(Y_t)} \frac{P_t}{P_{t+1}} \right\}. \tag{24}$$

The last equation relevant for the determination of the nominal variables is the intertemporal budget constraint of the government, which holds in equilibrium as the mirror image of the intertemporal budget constraint of the private sector

$$\frac{(1+i_{t-1}^R)R_{t-1} + (1+i_{t-1}^D)B_{t-1}^h}{P_t} = E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c(Y_T)}{U_c(Y_t)} \left[\frac{T_T}{P_T} + \frac{i_t^B - i_t^D}{1+i_t^B} \frac{B_t^h}{P_t} + \frac{i_t^B - i_t^R}{1+i_t^B} \frac{R_t}{P_t} \right] \right\}. \tag{25}$$

The outstanding overall real liabilities of the whole government (treasury and central bank), with respect to the private sector, should be equal, at each point in time, to the present-discounted value of real taxes and of the seigniorage revenues the government gets by selling liabilities, either reserves or treasury notes, at an interest rate lower than the natural nominal rate of interest.

The equilibrium conditions (21) to (25) determine the paths of the variables $(i_t^L, i_t^D, i_t^B, P_t)$, i.e. the market interest rates and the price level, given the paths of the policy variables: the interest rate on reserves and their quantity, i_t^R and R_t, set by the central bank, the tax rates and the supply of treasury debt, T_t and B_t^g, set by the treasury. Note that deposits and reserves are linked through the reserve requirement in the case it binds, $D_t = R_t/\rho$. The key feature of our framework is that, unless there is satiation of liquidity in the economy, the path of deposits and the treasury debt are relevant to determine the natural nominal interest rate i_t^B. With satiation of liquidity $V_q(\cdot) = 0$, then $i_t^B = i_t^D = i_t^R$ and the policy rate moves one-to-one with the natural nominal rate of interest and the deposit rate.

4.1 Examples

To understand the determination of prices and interest rates, we now make the simplifying assumptions that the economy is deterministic and that output is constant at $Y_t = Y$. First, we discuss how monetary and fiscal policies are set. The central bank chooses the interest rate on reserves and their quantity, i.e. i_t^R and R_t. In particular, it follows a constant interest rate policy, $i_t^R = i^R$, whereas reserves can vary over time at the constant rate r, $R_t = R_{t-1}(1+r)$, with $r > -1$. Consider the flow budget constraint of the central bank:

$$B_t^c - R_t = (1 + i_{t-1}^D)B_{t-1}^c - (1 + i_{t-1}^R)R_{t-1} - T_t^c. \tag{26}$$

First, note that i_t^D is not under direct control of the central bank. If i_t^R and R_t are set, there is still one degree of freedom to specify the central bank's policy. We assume that the central bank rebates its profits to the treasury, i.e. $T_t^c = i_{t-1}^D B_t^c - i_{t-1}^R R_t$. It follows in (26) that $B_t^c - R_t$ is constant over time and, therefore, that the central bank's purchases of treasury bonds follow the law of motion $B_t^c = B_{t-1}^c + rR_{t-1}$. On the contrary, had policy been specified in terms of an exogenous path for B_t^c, R_t would have been endogenously determined by (26).

The treasury sets the following tax policy

$$\frac{T_t}{P_t} = (1-\beta)\tau - (i_{t-1}^B - i_{t-1}^D)\frac{B_{t-1}^h}{P_t} - (i_{t-1}^B - i_{t-1}^R)\frac{R_{t-1}}{P_t} \tag{27}$$

rebating to households the revenues the government gets by issuing liabilities at a lower rate than the natural rate of interest, in which τ is a positive real tax. Given the tax policy (27) and the remittances policy of the central bank, the supply of treasury debt, B_t^g, is determined by

$$B_t^g = (1 + i_{t-1}^D)B_{t-1}^g - T_t - T_t^c. \tag{28}$$

Specifying the tax policy as in (27) is worthwhile since it can determine the price level at time t_0. Insert (27) into (25) to obtain

$$\frac{(1 + i_{t_0-1}^B)(R_{t_0-1} + B_{t_0-1}^h)}{P_{t_0}} = \tau, \tag{29}$$

which shows that the price level P_{t_0} is determined by τ, given initial conditions R_{t_0-1}, $B_{t_0-1}^h$ and $i_{t_0-1}^B$. Moreover, note that by aggregating the budget constraints of treasury (28) and central bank (26), the consolidated government budget constraint can be written as

$$B_t^h + R_t = (1 + i_{t-1}^D)B_{t-1}^h + (1 + i_{t-1}^R)R_{t-1} - T_t.$$

By inserting the tax rule (27), we get

$$\frac{B_t^h + R_t}{P_t} = (1 + i_{t-1}^B)\frac{B_{t-1}^h + R_{t-1}}{P_t} - (1-\beta)\tau$$

and therefore, using (29), that

$$\frac{B_t^h + R_t}{P_t} = \beta\tau \tag{30}$$

at all times $t \geq t_0$. The real value of government liabilities is proportional to the parameter τ of the tax rule.

Finally, equations (22) to (24) determine interest rates and inflation. Recall them and exploit the simplifying assumption stated in this section to write

$$(1 + i_t^D) = \rho(1 + i_t^R) + (1 - \rho)\max((1 + i_t^B), (1 + i_t^R)). \tag{31}$$

$$\frac{(1 + i_t^D)}{(1 + i_t^B)} = \left(1 - \frac{V_q\left(\frac{D_t + B_t^h}{P_t}\right)}{U_c(Y)}\right), \tag{32}$$

$$1 + i_t^B = \frac{1}{\beta}\frac{P_{t+1}}{P_t}, \tag{33}$$

with the further restriction that $i_t^B \geq i_t^R \geq 0$. Note that, given that the price level at time t_0 is determined by the tax policy, the natural nominal rate of interest determines the inflation rate through equilibrium condition (33). However, this rate is not directly controlled by the central bank. To determine the inflation path, one needs to understand how the policy rate passes into the natural nominal rate of interest to affect the inflation rate. We proceed by studying the problem by means of three distinct cases: $\rho = 0, \rho = 1$ and $0 < \rho < 1$.

4.1.1 Case I: $\rho = 0$

We start with the simple case in which there is no reserve/collateral requirement. In Section 3.1, we have already shown that all money-market rates are equalized, $i_t^B = i_t^R = i_t^D$. Reserves are supplied in a positive amount, even negligible, and held by banks.[7] By moving the policy rate, the central bank can control in a direct way the rate relevant for the consumption/saving choices of households. The Neo-Wicksellian paradigm emerges. There are no liquidity premia and deposits will be supplied and demanded to satiate liquidity in the economy.

Concerning deposits, we have assumed that they provide liquidity to households on the grounds that they have some backing from reserves. However, when $\rho = 0$ there is no such backing. There are two possible interpretations. One could be to assume that intermediaries have special powers to transform illiquid risk-free securities into liquid deposits, since in any case $i_t^B = i_t^D$. Alternatively, we could assume that deposits do not provide any liquidity services while the treasury bonds do, since they are backed by the central bank. In this case, it is still true that $i_t^B = i_t^R = i_t^D$ but, the interest rate on the treasury debt could be lower than these rates if liquidity is not satiated. This latter result is interesting since it shows that the existence of securities carrying a convenience yield is not per se a sufficient condition for breaking the link

[7]See also Woodford (2000) on how the central bank can control money-market rates with a negligible supply of reserves.

between the policy rate and the natural nominal rate of interest. What matters is the transmission mechanism of the policy rate through the banking sector.

4.1.2 Case II: "Narrow" banking or central bank digital currency, $\rho = 1$

We now focus on the special case of full backing of deposits by reserves, i.e. $\rho = 1$. This framework is isomorphic to one in which households directly hold accounts at the central bank, something that in recent proposals has been labeled 'central bank digital currency'.

Start by observing that

$$\frac{D_t + B_t^h}{P_t} \leq \frac{R_t + B_t^h}{P_t} = \beta\tau \tag{34}$$

where we have used $R_t \geq D_t$ to derive the first inequality and (30) for the final equality. When $\rho = 1$, equation (31) says that deposits and policy rates are equal, $i_t^D = i^R$. We study first under which conditions the Neo-Wicksellian equilibrium arises, i.e. $i_t^B = i^R$. Using equation (33), the inflation rate is determined at the constant $\Pi = P_{t+1}/P_t = \beta(1 + i^R)$. When $i_t^D = i^R$, liquidity is fully satiated. Therefore, the equilibrium level of deposits should satisfy

$$\frac{D_t + B_t^h}{P_t} \geq \bar{q}.$$

Using (34), we obtain the restriction on τ for such an equilibrium to exist, $\tau \geq \bar{q}/\beta$. Taxes should be set at a sufficiently high level to satiate liquidity.

When $\tau < \bar{q}/\beta$ there is a disconnection between the policy rate and the natural nominal rate of interest, thus departing from the Neo-Wicksellian analysis. Using equations (32), (33) and (34), we obtain that

$$1 + i^B = \frac{1 + i^R}{1 - \frac{V_q(\beta\tau)}{U_c(Y)}},$$

$$\Pi = \frac{\beta(1 + i^R)}{1 - \frac{V_q(\beta\tau)}{U_c(Y)}}.$$

There is now a spread between the natural nominal rate of interest and the policy rate. This spread decreases with τ. A similar negative relationship also arises between the equilibrium inflation rate and the tax rate. The inflation rate is going to be higher than under the Neo-Wicksellian regime, which is nested when liquidity is fully satiated by a sufficiently high τ.

The final remark is on the irrelevance of reserves for the equilibrium inflation rate and the natural nominal rate of interest, irrespective of whether the policy rate coincides or not with the latter. For a given τ, this section has shown that the size of the central bank's balance sheet or reserves does not matter, although reserves remain an additional instrument of monetary policy.

4.1.3 Case III: $0 < \rho < 1$

We now relax the assumption $\rho = 1$. This generalization brings novel policy implications: the size of the central bank's balance sheet is relevant for determining the inflation rate in addition to the standard specification of the policy rate, unlike the Neo-Wicksellian analysis. The Neo-Wicksellian equilibrium can be achieved not only by setting a sufficiently high tax rate but also with an appropriately high growth rate of reserves. However, this growth rate should be sufficiently high and definitely higher than the inflation rate in the Neo-Wicksellian equilibrium.

Start by considering

$$\frac{D_t + B_t^h}{P_t} \leq \frac{(\rho^{-1} - 1)R_t + R_t + B_t^h}{P_t} = (\rho^{-1} - 1)\frac{R_t}{P_t} + \beta\tau$$

where the first inequality uses $R_t \geq \rho D_t$ while the last equality uses (30). Consider the most interesting case in which taxes are not high enough to satiate liquidity, $\beta\tau < \bar{q}$, an assumption that we are going to make throughout the section. Reserves are going to matter for the type of equilibrium. We start by discussing an equilibrium in which liquidity is fully satiated starting with the initial period t_0. This depends on the growth rate of reserves r to satisfy a certain lower bound.

Consider the right-hand side of the above equation at time t_0

$$(\rho^{-1} - 1)\frac{R_{t_0}}{P_{t_0}} + \beta\tau = \tau(\rho^{-1} - 1)(1 + r)X_{t_0-1} + \beta\tau \tag{35}$$

in which we have substituted in $R_{t_0} = (1 + r)R_{t_0-1}$ and the price level at time t_0 with (29). Furthermore, we have defined $X_{t_0-1} \equiv R_{t_0-1}/[(1 + i_{t_0-1}^B)(R_{t_0-1} + B_{t_0-1}^h)]$. Liquidity at time t_0 is satiated whenever the above equation is not lower than the satiation level $\bar{q}$. This requires the growth rate of reserves to not be lower than $(\bar{q} - \beta\tau)/(\tau(\rho^{-1} - 1)X_{t_0-1})$. However, for liquidity to be satiated at all times, the real value of reserves should not decrease over time. Since in this equilibrium inflation is at the Neo-Wicksellian level, $\Pi^L = \beta(1 + i^R)$, $1 + r$ should not be lower than this value. The following inequality summarizes the conditions required on $1 + r$:

$$1 + r \geq \max\left(\Pi^L, \frac{\bar{q} - \beta\tau}{\tau(\rho^{-1} - 1)X_{t_0-1}}\right). \tag{36}$$

The interesting policy implication of the above result is that the growth rate of reserves, r, could also be quite high and well above the inflation rate in the equilibrium, which is given by Π^L. This statement is in general true: whenever τ is low, ρ is larger and X_{t_0-1} is small.

We now characterize equilibria in which the economy starts at a value of liquidity below the satiation level. In this case, $1 + r < (\bar{q} - \beta\tau)/(\tau(\rho^{-1} - 1)X_{t_0-1})$. Combine (31) – (33) to obtain

$$\Pi_{t+1} = \frac{\rho\beta(1 + i^R)U_c(Y)}{\rho U_c(Y) - V_q\left((\rho^{-1} - 1)\frac{R_t}{P_t} + \beta\tau\right)}. \tag{37}$$

Note that Π_{t_0+1} is in the interval (Π^L, Π^H) in which Π^L has been defined above and Π^H is defined by

$$\Pi^H \equiv \frac{\rho\beta(1+i^R)U_c(Y)}{\rho U_c(Y) - V_q\,(\beta\tau)}.$$

In particular, the inflation rate at t_0+1 is given by

$$\Pi_{t_0+1} = \frac{\rho\beta(1+i^R)U_c(Y)}{\rho U_c(Y) - V_q\,(\tau(\rho^{-1}-1)(1+r)X_{t_0-1} + \beta\tau)}.$$

Starting from a rate Π_{t_0+1} there are three possible paths for the inflation rate: 1) convergence to Π^L; 2) convergence to Π^H; 3) constant at Π_{t_0+1}. The three cases depend on $1+r$ to be higher, lower or equal than Π_{t_0+1}, respectively. The reason is that real reserves will be respectively increasing, decreasing or staying constant in the three equilibria. It is easy to start from the third equilibrium in which the inflation rate remains constant at Π_{t_0+1}, real reserves are constant too and $1+r = \Pi_{t_0+1}$, which is valid for an $\bar{r}$ that solves

$$(1+\bar{r}) = \frac{\rho\beta(1+i^R)U_c(Y)}{\rho U_c(Y) - V_q\,(\tau(\rho^{-1}-1)(1+\bar{r})X_{t_0-1} + \beta\tau)}. \tag{38}$$

Note that indeed there is a unique $1+\bar{r}$ in the interval (Π^L, Π^H) considering the restrictions $1+r < (\bar{q}-\beta\tau)/(\tau(\rho^{-1}-1)X_{t_0-1})$ and $R_t > 0$ for each t. When $\Pi_{t_0+1} = 1+\bar{r}$, the inflation rate remains constant at all times. The other two cases follow consequentially. When $(\bar{q}-\beta\tau)/(\tau(\rho^{-1}-1)X_{t_0-1}) - 1 > r > \bar{r}$, inflation converges to Π^L in a finite period of time; when $r < \bar{r}$, it converges to Π^H.

The latter result also has interesting policy implications. If the growth rate of reserves is not sufficiently high, inflation converges to a high level. In this equilibrium, the policy rate and the natural nominal rate of interest are disentangled.

5 Extensions

In this section, we extend the benchmark model presented in the previous analysis along three dimensions: 1) banks will face a cost of raising equity which is going to make the limited-liability constraint binding and, therefore, leverage will matter for determining lending spreads; 2) there will be rigidity in prices; 3) the default rate in the corporate sector is going to be endogeneized as a function of the firms' balance sheet and macroeconomic conditions. In extensions 2) and 3), the model economy is going to have two layers of production for a final and an intermediate good. Intermediate-good firms are the ones that borrow from intermediaries to finance the purchase of capital, which is now going to be endogenous since it is produced by final-good producers.

5.1 Cost of raising equity

We assume that intermediaries face a cost of raising equity. In particular, the cost, per unit of equity, is going to be an increasing function of leverage captured by the function $f(\delta_t)$ with $\delta_t = L_t/N_t$. We show derivations for a general functional form $f(\delta_t)$ but, in most of the analysis, we focus on the simple case in which $f(\delta_t) = f$, for a constant positive f. At time t, the intermediaries' budget constraint is

$$L_t + A_t + R_t = (1 - f(\delta_t))N_t + D_t, \tag{39}$$

showing that the cost of raising equity reduces the resources available for investment proportionally to the amount of equity. With this modification, intermediaries choose L_t, A_t, R_t, D_t, N_t to maximize (7) under the budget constraint (39), the limited-liability constraint (6) and the reserve requirement $R_t \geq \rho D_t$. The only first-order conditions that change in comparison with Section 3.1 are those with respect to L_t and N_t. They are now

$$E_t\left\{M_{t+1}(1+i_t^L)(1-\phi_{t+1})\right\} - \lambda(1+f'(\delta_t)) + \psi(1+i_t^L)(1-\phi^{\max}) = 0, \tag{40}$$

and

$$\lambda = \frac{1}{1-g(\delta_t)}, \tag{41}$$

respectively in which $g(\delta_t) = f(\delta_t) - f'(\delta_t)\delta_t \geq 0$. There is an important difference in comparison with the benchmark case of costless equity: using λ given by (41) in equation (9) implies now that the limited-liability constraint is binding and the multiplier ψ is given by

$$\psi(1+i_t^B) = \frac{g(\delta_t)}{1-g(\delta_t)}.$$

The benchmark analysis of Section 3.1 is nested when $f(\delta_t) = 0$ in which case $g(\delta_t) = 0$ and $\psi = 0$. The other important result is that, although λ is different with respect to that of Section 3.1, the equilibrium condition which interlinks deposits, reserves and the natural rate of interest is the same as (15). What instead changes is the spread between the lending rate and the natural nominal rate of interest, as it can be seen by combining the above three first-order conditions. It is now given by

$$\frac{(1+i_t^L)}{(1+i_t^B)} = \frac{1+f'(\delta_t)}{(1-g(\delta_t))(1-\tilde{E}_t\phi_{t+1}) + g(\delta_t)(1-\phi^{\max})},$$

which is a non-decreasing function of leverage.

Finally, the leverage ratio is determined by the binding limited-liability constraint. Using (39) and (15), we obtain

$$\frac{\delta_t}{(1-f(\delta_t))} = \frac{1}{\left[1 - \frac{(1+i_t^L)}{(1+i_t^B)}(1-\phi^{\max})\right]}. \tag{42}$$

To get an idea, we can focus on the simple case in which $f(\delta_t)$ is just equal to a constant $f > 0$. The spread between lending and the nominal natural rate of interest is given by

$$\frac{(1+i_t^L)}{(1+i_t^B)} = \frac{1}{(1-f)(1-\tilde{E}_t\phi_{t+1}) + f(1-\phi^{\max})},$$

which weighs with $1-f$ and f the expected and maximum default rate, respectively. The leverage ratio is instead determined by

$$\delta_t = \frac{1 - \tilde{E}_t \phi_{t+1}}{(\phi^{\max} - \tilde{E}_t \phi_{t+1})} - f. \tag{43}$$

Leverage is lower as the cost of equity increases. It rises with the expected default rate for a given $\phi^{\max}$. But for a given spread $(\phi^{\max} - \tilde{E}_t \phi_{t+1})$ it decreases with the expected default rate. In any case, it is always decreasing with respect to the maximum default rate.

5.2 Sticky prices

We add sticky prices by modelling an additional layer of production. There is an intermediate-good sector, which uses capital by borrowing from intermediaries, and a final-good sector, which uses the intermediate good to produce the consumption and capital goods. The final-good sector is subject to price rigidity. To clarify exposition, we relabel output and prices in the intermediate sector as Y^I and P^I and the productivity shock as Z^I. Now, Y, P and Z denote the respective variables in the final-good sector. Firms in this sector, of a unitary mass on the segment $[0,1]$, use the intermediate input to produce their good, combining it with labor according to the production function $Y_t(i) = Z_t(H_t(i))^{1-\theta}(Y_t^I(i))^{\theta}$ for a generic $i \in [0,1]$, in which $H_t(i)$ and $Y_t^I(i)$ are, respectively, the quantity of labor and of the intermediate input used by the generic firm i.

To model labor supply, we assume that households obtain disutility from supplying labor through the convex function $G(H_t)$ in which H_t is the aggregate supply of labor exerted at the nominal wage W. The optimal supply of labor equates the marginal rate of substitution between consumption and labor to the real wage

$$\frac{G_h(H_t)}{U_c(C_t)} = \frac{W_t}{P_t}.$$

Returning to the firm's problem, real marginal costs are given by

$$mc_t = \frac{1}{Z_t}\left(\frac{1}{1-\theta}\frac{W_t}{P_t}\right)^{1-\theta}\left(\frac{1}{\theta}\frac{P_t^I}{P_t}\right)^{\theta},$$

which are the same across all firms. Furthermore, note that cost minimization implies that

$$\frac{1}{1-\theta}\frac{W_t}{P_t}H_t(i) = \frac{1}{\theta}\frac{P_t^I}{P_t}Y_t^I(i)$$

which can be used to write the real marginal cost as

$$mc_t = \frac{1}{Z_t}\left(\frac{1}{1-\theta}\frac{W_t}{P_t}\right)(h_t)^{\theta}$$

in which $h_t = H_t(i)/Y_t^I(i)$ is also constant across firms.

Each firm faces a demand function of the form $Y_t(i) = (P_t(i)/P_t)^{-\varpi} Y_t$ in which $P_t(i)$ is the price of the good i and ϖ is the elasticity of substitution among the variety of goods produced with $\varpi > 1$. Prices are sticky following the Calvo model in which a fraction $1-\alpha$ is allowed to change its prices maximizing the expected present discounted value of its profits. Firms that are not adjusting prices index them to the target Π. The described framework delivers an Aggregate-Supply equation of the form

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\varpi-1}}{1-\alpha}\right)^{\frac{1}{\varpi-1}} = \frac{F_t}{J_t}, \tag{44}$$

in which F_t and J_t are given by

$$F_t = U_c(C_t)Y_t + \alpha\beta E_t\left\{\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\varpi-1} F_{t+1}\right\},$$

$$J_t = \frac{\varpi}{\varpi-1}U_c(C_t)\frac{Y_t}{Z_t}\left(\frac{1}{1-\theta}\frac{W_t}{P_t}\right)(h_t)^{\theta} + \alpha\beta E_t\left\{\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\varpi} J_{t+1}\right\}.$$

5.3 Endogenous default

We now turn to endogeneizing the default rate on the loans, which are requested by the intermediate-good firm to finance the purchase of capital. Recall the relabeling of output and prices in the sector as Y^I and P^I, respectively. We also modify the supply of capital and we make it endogenous. The goods produced by the final-good producer are requested by the consumer in the form of consumption and capital goods. Consumers sell the capital goods to the intermediate-good producer which starts its activity at time t borrowing $L_t = P_t K_t$. Profits at time $t+1$ are given by

$$\Phi_{t+1} = P^I_{t+1}Y^I_{t+1} - (1+i^L_t)(1-\phi_{t+1})P_t K_t.$$

To endogeneize default, consider that revenues cannot fall below the loan value, otherwise loans have to be seized for the portion that exceeds revenues. The default rate is given by

$$\phi_{t+1} = \max\left(1 - \frac{P^I_{t+1}Y^I_{t+1}}{L_t}, 0\right),$$

in which the maximum default rate is $\phi^{\max} = \max \phi_{t+1}$. To provide an alternative interpretation of the default rate, define ϑ_{t+1} as the revenue-to-loan ratio, i.e. $\vartheta_{t+1} = P^I_{t+1}Y^I_{t+1}/L_t$. Therefore, the default rate is nothing more than the payoff of a put option written on ϑ_{t+1} with a strike price 1 and premium $\mathcal{P}_t$. Therefore the expected rate of default, under the neutral measure is simply given by the option premium, i.e. $\tilde{E}_t\phi_{t+1} = \mathcal{P}_t$. The intermediate-good producer maximizes the expected discounted value of profits $E_t(M_{t+1}\Phi_{t+1})$, given technology $Y^I_t = Z^I_t K^{\gamma}_{t-1}$. The optimal choice of capital implies the following demand for capital

$$K_t = \left[\frac{(1+i^L_t)E_t\left\{M_{t+1}(1-\phi_{t+1})\right\}}{\gamma E_t\left\{\tilde{M}_{t+1}Z^I_{t+1}\frac{P^I_{t+1}}{P_{t+1}}\right\}}\right]^{\frac{1}{\gamma-1}}.$$

Note however that with costly equity, $(1+i_t^L)E_t\left\{M_{t+1}(1-\phi_{t+1})\right\}$ is no longer equal to the unitary value but is given by (40). Therefore

$$K_t = \left[\frac{\frac{(1+f'(\delta_t))}{1-g(\delta_t)} - \frac{g(\delta_t)}{1-g(\delta_t)}\frac{(1+i_t^L)}{(1+i_t^B)}(1-\phi_{\max})}{\gamma E_t\left\{\tilde{M}_{t+1}Z_{t+1}^I\frac{P_{t+1}^I}{P_{t+1}}\right\}}\right]^{\frac{1}{\gamma-1}}$$

which can be simplified to

$$K_t = \left[\frac{1+\frac{f(\delta_t)}{\delta_t}}{\gamma E_t\left\{\tilde{M}_{t+1}Z_{t+1}^I\frac{P_{t+1}^I}{P_{t+1}}\right\}}\right]^{\frac{1}{\gamma-1}} \tag{45}$$

having used (42).

Finally, recall that from the final-good firm's problem

$$\frac{P_t^I}{P_t} = \frac{\theta}{1-\theta}\frac{W_t}{P_t}h_t$$

therefore using it in (45) implies

$$K_t = \left[\frac{1+\frac{f(\delta_t)}{\delta_t}}{\gamma\frac{\theta}{1-\theta}E_t\left\{\tilde{M}_{t+1}Z_{t+1}^I\frac{G_h(H_{t+1})}{U_c(C_{t+1})}h_{t+1}\right\}}\right]^{\frac{1}{\gamma-1}}.$$

In the special case in which the cost function is constant, $f(\delta_t) = f$, the demand for capital is an increasing function of the leverage in the banking sector.

6 Stabilization policies

We study the transmission mechanism of monetary policy and shocks in the general framework outlined in the previous section. We proceed step by step first analyzing the liquidity channel and then adding the credit channel.

6.1 Liquidity channel

In order to focus on the liquidity channel, we make the simplifying assumption that $\theta = 0$, saying that the technology to produce the final goods only uses labor as input and not the intermediate good. Therefore, the demand for the intermediate good is zero, as well as its supply and the demand for loans. In this model, intermediaries issue deposits to invest in reserves and private bonds. With the simplification $\theta = 0$, the model is consistent with the standard New-Keynesian AS equation

$$\pi_t - \pi = \kappa\left(\hat{Y}_t - \tilde{Y}_t\right) + \beta E_t(\pi_{t+1} - \pi), \tag{46}$$

for a positive parameter κ; $\pi_t \equiv \ln P_t/P_{t-1}$ and $\pi \equiv \ln \Pi$. Inflation deviations from the target depend positively on the output gap $\hat{Y}_t - \tilde{Y}_t$ and on the one-period ahead inflation expectations; $\hat{Y}_t$ is the output deviation with respect to the steady state, while $\tilde{Y}_t$ is the flexible-price level of output. In general, variables with a hat denote log-deviations of the variable with respect to the steady state.

The aggregate demand equation, instead, differs from that of the standard New-Keynesian model. Although in both models the consumption Euler equation links output to the real rate, here the relevant nominal rate is the natural nominal rate of interest and not the policy rate. In a log-linear approximation, we obtain

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t^B - E_t(\pi_{t+1} - \pi)), \tag{47}$$

in which σ is the intertemporal elasticity of substitution in consumption. The banking model determines the following relationship between the relevant spreads

$$\upsilon(\hat{\imath}_t^B - \hat{\imath}_t^D) = (\rho + \upsilon - 1)(\hat{\imath}_t^B - \hat{\imath}_t^R). \tag{48}$$

Since intermediaries issue deposits to finance private risk-free bonds and to satisfy the reserve requirement, then the liquidity premium earned on deposits, which is on the left-hand side of the above equation, should compensate for the cost of holding reserves. Note that the parameter υ is given by the ratio, in the steady state, between the deposit and the natural nominal rate of interest, i.e. $\upsilon \equiv (1 + i^D)/(1 + i^B)$. This ratio is unitary in a steady state with full satiation of liquidity. In the more general case in which liquidity is not satiated, equilibrium in the liquidity market matters for the determination of spreads and therefore for output and inflation. In the liquidity market, the overall liquidity is positively related with output and the liquidity premium

$$\epsilon_q(1 - \upsilon)\hat{q}_t = (1 - \upsilon)\sigma^{-1}\hat{Y}_t - \upsilon(\hat{\imath}_t^B - \hat{\imath}_t^D). \tag{49}$$

In equation (49), $\hat{q}_t$ is the overall supply of liquidity expressed in log-deviation with respect to the steady state with $q_t \equiv [(\rho^{-1} - 1)R_t + R_t + B_t^h]/P_t$. The parameter ϵ_q is given by the inverse of the elasticity of the function $V()$ evaluated at the steady state, $\epsilon_q \equiv -V_{qq}(q)q/V_q(q)$, which is zero when there is full satiation of liquidity. The last variable relevant for understanding the new AD equation is the decomposition of the overall supply of liquidity between the one that can be controlled by the central bank, through the supply of interest-bearing reserves ($\hat{q}_t^R$), and the one controlled by the treasury ($\hat{q}_t^G$) through taxes:

$$\hat{q}_t = (1 - z)\hat{q}_t^R + z\hat{q}_t^G, \tag{50}$$

in which $\hat{q}_t^R$ and $\hat{q}_t^G$ are the supplies of reserves and net-government liabilities, respectively, expressed in real terms and in deviation with respect to the steady state; $q_t^R \equiv R_t/P_t$ and $q_t^G \equiv (R_t + B_t^h)/P_t$ while the parameter z, detailed in the Appendix, satisfies the bound $0 < z \leq 1$.

Some interesting cases are nested in the above framework. Consider first the case in which liquidity is fully satiated, which is captured by the parameters ϵ_q and υ going to zero and one, respectively. The interest rate on deposits, that on reserves and the natural nominal rate of interest are all equalized in and out of the steady state. The AD equation collapses to that of the benchmark New-Keynesian model

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi)) \tag{51}$$

in which the policy rate directly affects the real rate relevant for the consumption/saving choices.

The second interesting case is when $v < 1$ and $\rho = 1$. The last assumption says that reserves are fully backed by deposits as in the narrow-banking regime, which also corresponds to a case in which the central bank reserves are held directly by the households. In equation (50), the parameter z goes to one and the relevant liquidity aggregate is just government liquidity, $\hat{q}_t = \hat{q}_t^G$. When $\rho = 1$, equation (48) implies that the interest rates on reserves and deposits are equal. We can then combine (47) and (49) to obtain the AD equation

$$\hat{Y}_t = vE_t \hat{Y}_{t+1} - v\sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi)) + (1 - v)\sigma\epsilon_q \hat{q}_t^G. \tag{52}$$

There are some important differences between (52) and (51). First, note that the coefficient in front of the term $E_t\hat{Y}_{t+1}$ is less than the unitary value when $v < 1$. Similarly, the coefficient in front of the real interest rate is also smaller than in (51). An increase in the policy rate, with everything else being equal, impacts less output than in the standard New-Keynesian framework. The last important difference in equation (52) is that an increase in government liquidity has a positive impact on aggregate demand. To appreciate the difference with respect to the standard AD equation, solve equation (52) forward

$$\hat{Y}_t = -v\sigma E_t \sum_{T=t}^{\infty} v^{T-t}(\hat{\imath}_T^R - (\pi_{T+1} - \pi)) + (1 - v)\sigma\epsilon_q E_t \sum_{T=t}^{\infty} v^{T-t}\hat{q}_T^G. \tag{53}$$

Not only the current real rate has less impact on output, for given intertemporal elasticity of substitution in consumption σ, but also movements in the expected future rates influence current output less and with a decaying weight. Similarly for the supply of government liquidity. Note that a reduction in the supply of government liquidity has a contractionary effect on output.

Another interesting result of the new framework is the condition for the determinacy of equilibrium under an interest-rate rule. Assume a rule of the type $\hat{\imath}_t^R = \phi_\pi(\pi_t - \pi)$, the condition for determinacy is

$$\phi_\pi > 1 - \frac{(1 - v)(1 - \beta)}{\kappa\sigma v}.$$

The parameter capturing the reaction of the interest rate to inflation, ϕ_π, can also be less than the unitary value to get a unique equilibrium, in contrast with the result of the standard New-Keynesian model.

Consider now the more general case in which $v < 1$ and $\rho < 1$. Combining (47) to (50), we obtain

$$\hat{Y}_t = v_\rho E_t \hat{Y}_{t+1} - v_\rho\sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi)) + (1 - v_\rho)\sigma\epsilon_q \hat{q}_t, \tag{54}$$

in which $v_\rho \equiv (\rho + v - 1)/\rho$. The coefficient v_ρ coincides with v when $\rho = 1$ and it is a decreasing function of ρ.[8] Another novelty of equation (54) with respect to (52) is the distinct role of reserves in affecting output, since $\hat{q}_t$ matters now including both $\hat{q}_t^R$ and $\hat{q}_t^G$. Note that, given an appropriate tax policy detailed in the Appendix, the real value of government liabilities follow the process

$$\hat{q}_t^G = \hat{q}_{t-1}^G + \xi_t \tag{55}$$

for some stochastic disturbance ξ_t while

$$\hat{q}_t^R = \hat{q}_{t-1}^R + \hat{R}_t - (\pi_t - \pi), \tag{56}$$

in which $\hat{R}_t$ denotes the deviations of the central bank reserves from their steady-state growth rate.

Equations (46), (50), (54), (55) and (56) characterize the equilibrium for inflation π_t and output $\hat{Y}_t$ given the exogenous disturbances $\tilde{Y}_t$ ξ_t and considering that the central bank controls interest rates and the quantity of reserves, $\hat{\imath}_t^R$ and $\hat{R}_t$.

6.1.1 Shock to the policy rate

We run some experiments using the calibration detailed in the Appendix. In the first, we consider an unexpected upward movement in the policy rate assuming a simple interest-rate policy of the form

$$\hat{\imath}_t^R = \phi_\pi(\pi_t - \pi) + \phi_y \hat{Y}_t + \varepsilon_t$$

in which the interest rate on reserves reacts to the inflation deviations from the target, through a coefficient $\phi_\pi > 0$, and to the output deviations from the steady state, through a coefficient $\phi_y > 0$. The shock ε_t is assumed to follow an autoregressive process of order one with coefficient 0.95. The 25-basis-point shock increases the policy rate, with everything else being equal, by 1% at annual rates. Figure 1 compares the impulse responses of output, inflation, interest rates and money-market spreads between the benchmark New-Keynesian model and our model. The parameter ρ capturing the reserve-to-deposit ratio takes different values: $0.1, 0.3, 0.6$ and 1.

Recall that when $\rho = 1$, the model collapses to the narrow-banking regime. Consider first the responses in the New-Keynesian model, the line "NK model". A contractionary monetary-policy shock reduces both output and inflation. The policy rate also falls given the reaction to output and inflation built into the policy rule. However, what drives the contraction in the economy is the increase in the real rate, computed in the Figure using the policy rate as $\hat{r}_t = \hat{\imath}_t^R - E_t(\pi_{t+1} - \pi)$. In the New-Keynesian model $\hat{\imath}_t^B = \hat{\imath}_t^D = \hat{\imath}_t^R$, and therefore all spreads are zero. Consider now the narrow-banking regime in which $\rho = 1$, and in which deposit and policy rates coincide, $\hat{\imath}_t^D = \hat{\imath}_t^R$. A spread still arises between the natural nominal rate of interest and the policy rate, i.e. $\hat{\imath}_t^B - \hat{\imath}_t^R$. Given that the parameter z in (50) is equal to the unitary value, $\hat{q}_t$ remains unchanged because it is only a function of the overall government

[8] Note that $\rho + v - 1 > 0$.

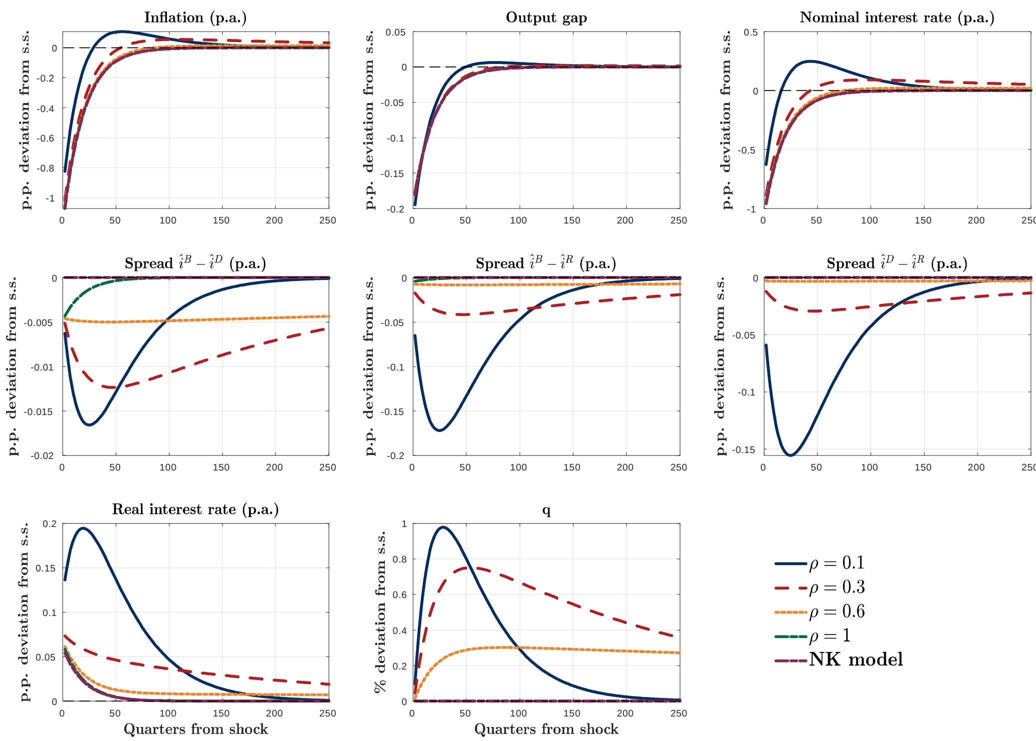

Figure 1: Impulse responses following a 25-basis-point monetary-policy shock. Variables are expressed in percentage point deviations from steady state (p.p. deviation from s.s.) or in percentage deviations from steady state (% deviation from s.s.); p.a. denotes percent per annum. The reserve-to-deposit ratio, ρ, is set to $\rho = 0.1, 0.3, 0.6, 1$. The line "NK model" describes the impulse response in the benchmark New-Keynesian model.

liabilities, $\hat{q}_t = \hat{q}_t^G$, while the spread $\hat{\imath}_t^B - \hat{\imath}_t^R$ is proportional to output $\hat{Y}_t$ through equation (49). Since output falls, the demand for liquidity falls. For an unchanged supply of liquidity, the liquidity premium should decrease to equilibrate the market, therefore $\hat{\imath}_t^B - \hat{\imath}_t^R$ goes down. Since the parameter v in (53) is calibrated close to the unitary value, the responses of output, inflation and real rate are otherwise indistinguishable from those of the benchmark New-Keynesian model. Differences arise when $\rho < 1$, the lower the value of ρ. In these cases, the real interest rate increases to a greater extent but with a not much impact on output and inflation, since the parameter v_ρ in (54) decreases with ρ, dampening the effect of the real rate on output. However, for a value of ρ equal to 0.1, the initial fall in output is larger than in the New-Keynesian model while inflation falls less. The main difference is in the recovery of output and inflation, which is now faster and up to the point of reaching positive values. What is the driver of this faster recovery? When $\rho < 1$, aggregate liquidity changes because of variations in the real value of reserves, $\hat{q}_t^R$. While the central bank maintains the nominal value of reserves constant, the fall in inflation raises its real value, increasing the overall supply of liquidity. This effect has a positive impact on output and inflation. The movements of $\hat{q}_t$ together with those of output also explain the larger fall in the spread $\hat{\imath}_t^B - \hat{\imath}_t^D$ through equation (49). The larger supply of liquidity needs to be accommodated by a fall in the liquidity premium.

6.1.2 A permanent increase in reserves

We now consider a policy of permanently increasing the central bank reserves. This policy has no effect at all in the standard New-Keynesian model, but it becomes a policy tool in the banking model when $\rho < 1$. In Figure 2, we consider a 0.25% permanent increase in the central bank reserves which produces a proportional long-run increase in the inflation rate by 0.25% at quarterly rates, 1% at annual rates. The Figure also plots the case $\rho = 1$ in which there is no effect at all of any independent variation of the central bank's reserves. In the other three cases, with $\rho < 1$, the shock is inflationary and through the AS equation also has long-run effects on output. The Figure shows a difference in the short-run response and in the speed of adjustment to the new equilibrium across the different calibrations of the parameter ρ. With lower values of ρ, inflation jumps more on impact and adjusts faster. On the contrary, output drops more while also adjusting faster. When ρ is low, $(1 - v_\rho)$ and $(1 - z)$ are larger in equations (49) and (50), respectively. The increase in reserves has, therefore, a larger impact on output through equation (54) for low values of ρ. However, the higher short-run increase in the real rate is responsible for the initial larger drop in output at low values of ρ. Since inflation jumps more when ρ is low, benefitting of the higher future output, the overall liquidity stabilizes faster to a new higher value. Liquidity premia also fall in a permanent way because of the permanent increase in the overall supply of liquidity.

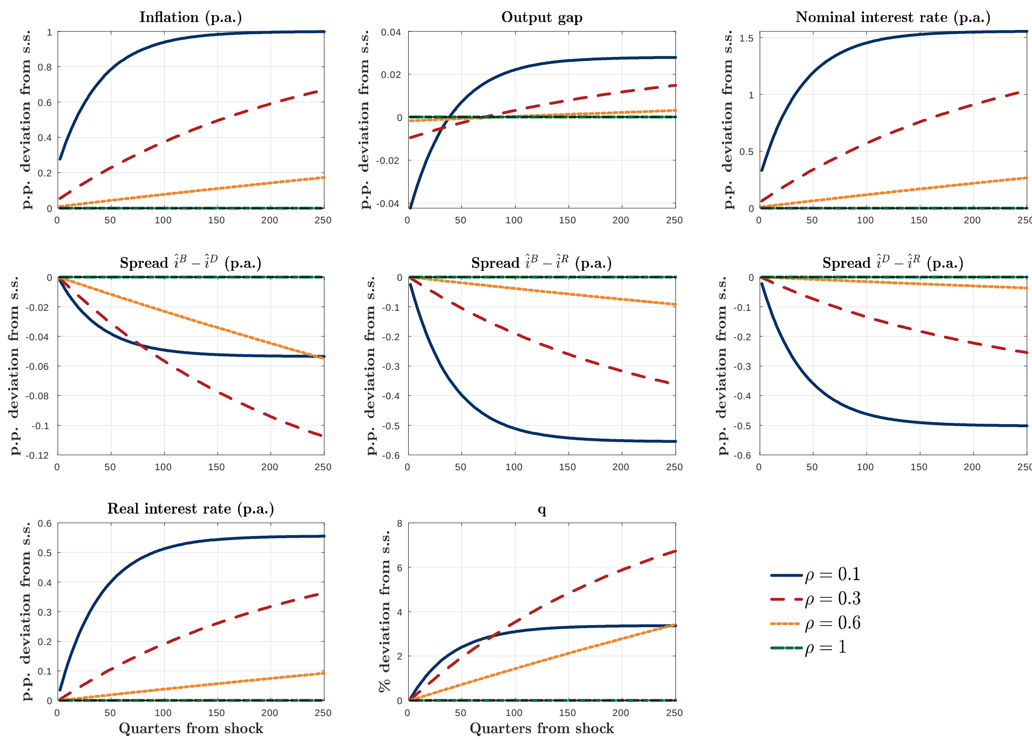

Figure 2: Impulse responses following a 0.25% increase in the central bank reserves. Variables are expressed in percentage point deviations from steady state (p.p. deviation from s.s.) or in percentage deviations from steady state (% deviation from s.s.); p.a. denotes percent per annum. The reserve-to-deposit ratio, ρ, is set to $\rho = 0.1, 0.3, 0.6, 1$.

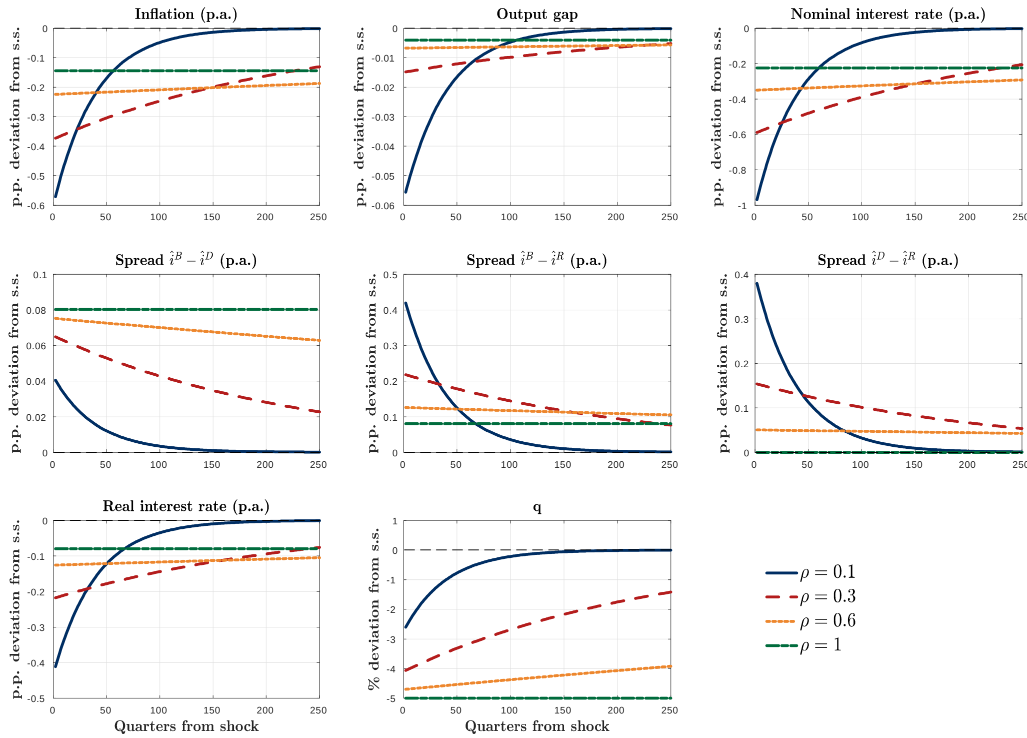

Figure 3: Impulse responses following a 5% drop in the supply of liquidity q_t^G. Variables are expressed in percentage point deviations from steady state (p.p. deviation from s.s.) or in percentage deviations from steady state (% deviation from s.s.); p.a. denotes percent per annum. The reserve-to-deposit ratio, ρ, is set to $\rho = 0.1, 0.3, 0.6, 1$.

6.1.3 A liquidity shock

In this last experiment, we mimic a liquidity crisis by assuming a negative shock ξ, which hits equation (55). This causes, first, a reduction in the real stock of government liabilities. We could also interpret it as a shock that affects the utility that agents obtain from liquidity, creating on impact an excessive demand for liquid assets. The two interpretations are isomorphic in the log-linear approximation of the model. The shock, again, does not have any effect in the standard New-Keynesian model. In Figure 3 we plot the responses of the variables of interest in the cases in which ρ takes values 0.1, $0.3, 0.6$ and 1. Let us start from the narrow-banking regime, $\rho = 1$. A permanent drop in q_t^G produces a permanent drop in q_t since $z = 1$ in (50). Looking at the equilibrium in the liquidity market via equation (49), a fall in the supply of liquidity requires a lower demand through a fall in output or/and a rise in the liquidity premium.

As the Figure shows, the economy immediately adjusts to the new equilibrium with a lower output and an increase in the liquidity premium. The lower output produces a deflation. Despite the deflation, the real rate falls because of the larger drop in the policy rate caused by the reaction to output and inflation. The lower supply of liquidity, however, dominates the fall in the real rate in the AD equation (53), implying a lower output level. When $\rho < 1$, the striking difference in comparison with case $\rho = 1$ is that the economy returns to the initial steady state. The adjustment is faster for low values of ρ. When $\rho < 1$, the overall supply of liquidity in the economy depends also on the central bank reserves, the lower ρ is. The initial deflation increases the real value of reserves and therefore the overall liquidity, pushing up output and inflation back to the steady state. When ρ is low, the negative short-run effect of the shock on output is larger since the coefficient $1 - \upsilon_\rho$ in (54) is larger. However, it is the larger drop in prices that again triggers a faster adjustment. Liquidity premia also rise more in the short run when ρ is low, consistently with (49), but adjust faster.

6.2 Credit channel

We now generalize the model to the case in which $\theta > 0$. Final-good producers now use labor and intermediate inputs in the production. Since an intermediate-good producer needs to finance fixed capital with loans, this activates a credit channel for which leverage in the banking sector matters for determining the spread on loans and then the amount of capital held by firms.

When $\theta > 0$, the AD block of the model is still consistent with equation (54), with the difference that it refers to consumption rather than output. Therefore we have

$$\hat{C}_t = \upsilon_\rho E_t \hat{C}_{t+1} - \upsilon_\rho \sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi)) + (1 - \upsilon_\rho)\sigma\epsilon_q \hat{q}_t. \tag{57}$$

The AS equation substantially changes from the New-Keynesian model and is now represented by

$$\pi_t - \pi = k\left[\sigma^{-1}\hat{C}_t + (\varphi - 1)\hat{Y}_t - \varphi(\hat{Z}_t + \theta\hat{Z}_t^I + \gamma\theta\hat{K}_{t-1})\right] + \beta E_t(\pi_{t+1} - \pi), \tag{58}$$

for some parameters k, φ given in the Appendix. The previous-period capital stock matters in the real marginal costs of firms and then affects the AS equation. The optimal choice of capital implies in a first-order approximation that

$$E_t\left\{\varphi(\hat{Y}_{t+1} - \hat{Z}_{t+1} - \theta\hat{Z}_{t+1}^I)\right\} = (1 + \gamma\varphi\theta)\,\hat{K}_t - s_\delta\hat{\delta}_t - \sigma^{-1}\hat{C}_t, \tag{59}$$

for some parameter s_δ. The capital stock depends positively on the expectation of final-good output and on leverage. Factors reducing intermediaries' leverage, like an increase in the default rate, reduce the stock of capital. The model is closed by the equilibrium in the goods market which implies, in a first-order approximation, that

$$\hat{Y}_t = s_c\hat{C}_t + s_k\hat{K}_t - s_k s_\delta\hat{\delta}_t, \tag{60}$$

for some parameters s_c, s_k detailed in the Appendix. The above set of four equations together with (50), (55) and (56) characterizes the equilibrium for $\hat{C}_t$, π_t, $\hat{K}_t$ and $\hat{Y}_t$ given $\hat{\delta}_t$, given the exogenous disturbances $\hat{Z}_t$, $\hat{Z}_t^I$, ξ_t and considering that the central bank controls the interest rate and the quantity of reserves, $\hat{\imath}_t^R$ and $\hat{R}_t$.

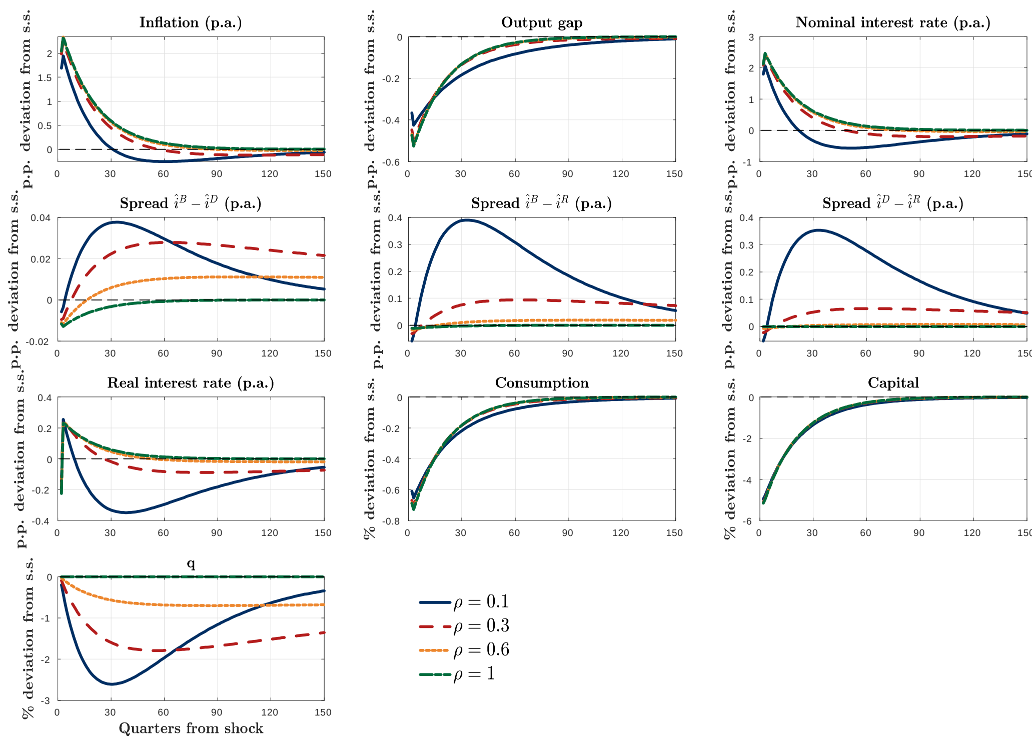

Figure 4: Impulse responses following a permanent movement in the maximum default rate, $\phi^{\max}$, from 25% to 50%. Variables are expressed in percentage point deviations from steady state (p.p. deviation from s.s.) or in percentage deviations from steady state (% deviation from s.s.); p.a. denotes percent per annum. The reserve-to-deposit ratio, ρ, is set to $\rho = 0.1, 0.3, 0.6, 1$.

In the experiment shown in Figure 4, we consider a shock that moves upward the maximum default rate $\phi^{\max}$, which through equation (43) lowers leverage and reduces loan supply. In particular, we consider a shock that moves on impact the maximum default probability $\phi^{\max}$ from 25% to 50%.[9] The shock then has a persistence of 0.95.

Figure 4 shows the impulse responses of the variables of interest. Capital, consumption and output all drop and then recover. Through the AS equation the lower stock of capital acts as a positive cost-push shock raising inflation. When $\rho < 1$ the higher inflation lowers the real value of reserves and therefore the overall supply of liquidity shrinks. Spreads in money markets increase in these cases.

7 Conclusion

We have proposed a new framework for monetary policy analysis that encompasses, as a special case, the Neo-Wicksellian paradigm. The nominal interest rate relevant for saving/consumption decisions can only be controlled by the central bank's simultaneous targeting of the interest rate on reserves and their quantity. The Neo-Wicksellian model is nested when liquidity is fully satiated.

Our framework provides a novel aggregate demand equation that generalizes the one used in standard New-Keynesian models. Output depends not only on the current and expected future real rate but also on the supply of liquidity in the economy. It happens that future real rates count less for short-run aggregate demand and output. The model is consistent with the reduced power of forward guidance in stimulating aggregate demand.

The model has been kept as simple as possible for tractability. It requires thorough extension in order to provide realistic quantitative analysis. We have also abstracted from analyzing optimal policy, which could result in important differences with respect to standard analysis.

[9] Since we are analyzing the model through a log-linear approximation and δ_t depends on higher-order terms, we consider that movements in the variable δ_t are as if they were exogenous in the first-order approximation.

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A Appendix

We describe in a compact way the general model which embeds the extensions presented in Section 5.

Starting from the household problem, we have the following equilibrium conditions derived from the optimal consumption/saving decisions:

$$E_t\left\{\beta\frac{U_c(C_{t+1})}{U_c(C_t)}\frac{P_t}{P_{t+1}}\right\}=\frac{1}{1+i_t^B} \tag{A.1}$$

$$1=\frac{V_q\left(\frac{D_t+B_t^h}{P_t}\right)}{U_c(C_t)}+\frac{1+i_t^D}{1+i_t^B}. \tag{A.2}$$

From the intermediary sector we obtain that the spread between the lending rate and the natural nominal rate of interest is

$$\frac{(1+i_t^L)}{(1+i_t^B)}=\frac{1+f'(\delta_t)}{(1-g(\delta_t))(1-\tilde{E}_t\phi_{t+1})+g(\delta_t)(1-\phi_{\max})}, \tag{A.3}$$

while that between the deposit rate, the policy rate and the natural nominal rate of interest is

$$(1+i_t^D)=\rho(1+i_t^R)+(1-\rho)\max((1+i_t^B),(1+i_t^R)), \tag{A.4}$$

with $i_t^R\geq 0$. Given the spread on loans, the leverage ratio is determined by

$$\frac{\delta_t}{(1-f(\delta_t))}=\frac{1}{\left[1-\frac{(1+i_t^L)}{(1+i_t^B)}(1-\phi_{\max})\right]}. \tag{A.5}$$

Turning to the production aspect, we have the following AS equation derived from the problem of the final-good producer:

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\varpi-1}}{1-\alpha}\right)^{\frac{1}{\varpi-1}}=\frac{F_t}{J_t}, \tag{A.6}$$

in which F_t and J_t are given by

$$F_t=U_c(C_t)Y_t+\alpha\beta E_t\left\{\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\varpi-1}F_{t+1}\right\}, \tag{A.7}$$

$$J_t=U_c(C_t)\frac{\varpi}{(1-\theta)(1-\varpi)}\frac{Y_t}{Z_t}\left(\frac{G_h(H_t)}{U_c(C_t)}\right)(h_t)^{\theta}+\alpha\beta E_t\left\{\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\varpi}J_{t+1}\right\}. \tag{A.8}$$

In the sector producing intermediate goods, we instead obtain that the demand for capital is

$$K_t^{\gamma-1}=\frac{1+\frac{f(\delta_t)}{\delta_t}}{\frac{\gamma\beta\theta}{1-\theta}E_t\left\{Z_{t+1}^I\frac{G_h(H_{t+1})}{U_c(C_t)}h_{t+1}\right\}}. \tag{A.9}$$

The model is closed with the goods market equilibrium. For the final good we have:

$$Y_t = C_t + K_t + f(\delta_t)\frac{N_t}{P_t}.$$

Noting that

$$N_t = \frac{L_t}{\delta_t} = \frac{P_t K_t}{\delta_t}$$

we can write

$$Y_t = C_t + K_t\left(1 + \frac{f(\delta_t)}{\delta_t}\right). \tag{A.10}$$

Moreover, equilibrium in the market of intermediate goods implies that

$$Y_t^I = \frac{\Delta_t}{Z_t} h_t^{\theta-1} Y_t \tag{A.11}$$

while that in the labor market

$$H_t = \frac{\Delta_t}{Z_t} h_t^{\theta} Y_t. \tag{A.12}$$

Moreover,

$$Y_t^I = Z_t^I K_{t-1}^{\gamma}. \tag{A.13}$$

The index of price dispersion Δ_t follows the law of motion:

$$\Delta_t \equiv \alpha \Delta_{t-1}\left(\frac{\Pi_t}{\Pi}\right)^{\varpi} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\varpi-1}}{1-\alpha}\right)^{\frac{\varpi}{\varpi-1}}. \tag{A.14}$$

Note that in the endogenous-default model the expected rate of default and the maximum default rate are given by

$$\tilde{E}_t \phi_{t+1} = \tilde{E}_t\left\{\max\left(1 - \frac{\theta}{1-\theta}\frac{G_h(H_{t+1})}{U_c(C_{t+1})}\frac{H_{t+1}}{K_t}\Pi_{t+1}, 0\right)\right\}, \tag{A.15}$$

$$\phi_t^{\max} = \max\left(1 - \frac{\theta}{1-\theta}\frac{G_h(H_{t+1})}{U_c(C_{t+1})}\frac{H_{t+1}}{K_t}\Pi_{t+1}\right). \tag{A.16}$$

Finally, the consolidated budget constraint of the government implies that

$$\frac{B_t^h + R_t}{P_t} = \frac{(1+i_{t-1}^D)}{\Pi_t}\frac{B_{t-1}^h}{P_{t-1}} + \frac{(1+i_{t-1}^R)}{\Pi_t}\frac{R_{t-1}}{P_{t-1}} - \frac{T_t}{P_t}. \tag{A.17}$$

The above set of seventeen equilibrium conditions together with $D_t \geq \rho^{-1}R_t$ involve the following twenty-one variables, $C_t, i_t^B, i_t^D, i_t^R, i_t^L, D_t/P_t, B_t^h/P_t, \delta_t,\ \Pi_t,\ K_t,\ F_t,\ J_t,$ $H_t,\ Y_t, Y_t^I,\ h_t,\ \Delta_t,\ \tilde{E}_t\phi_{t+1},\ \phi_t^{\max},\ R_t/P_t,\ T_t$. Finally, the transversality condition holds with respect to the overall government liabilities as a mirror image of the transversality condition of households. In the analysis in Section 6, we assume a tax policy such that the real value of government liabilities follows the process

$$\frac{B_t^h + R_t}{P_t} = \frac{B_{t-1}^h + R_{t-1}}{P_{t-1}} + \xi_t$$

for some stochastic process ξ_t. The tax policy is given by

$$\frac{T_t}{P_t} = \left[\frac{(1+i_{t-1}^D)}{\Pi_t} - 1\right]\frac{B_{t-1}^h}{P_{t-1}} + \left[\frac{(1+i_{t-1}^R)}{\Pi_t} - 1\right]\frac{R_{t-1}}{P_{t-1}} - \xi_t. \tag{A.18}$$

A.1 Steady state

We consider a steady state in which the interest rate on reserves is constant at $i_t^R = i^R$ and the real value of reserves and government liabilities are also constant. First, note that in the steady state

$$1 + i^B = \frac{\Pi}{\beta}$$

$$\frac{1 + i^D}{1 + i^B} = 1 - \frac{V_q(q)}{U_c(C)}$$

$$1 + i^D = \rho(1 + i^R) + (1 - \rho)\max\left(1 + i^B, 1 + i^R\right),$$

in which $q = (D + B^h)/P$. Note that the above three equations determine the spread between the deposit rate and the natural nominal rate of interest at

$$\frac{1 + i^D}{1 + i^B} = v = 1 - \frac{V_q(q)}{U_c(C)}$$

given q while C is determined in the equilibrium in the goods market. Therefore, we can also write

$$\frac{1 + i^B}{1 + i^R} = \frac{\rho}{\rho + v - 1}$$

and

$$\Pi = \frac{\beta\rho}{\rho + v - 1}(1 + i^R).$$

The following five equations determine the steady-state values of H, C, Y, K, Y^I given δ where we have normalized $Z = Z^I = 1$:

$$\frac{\varpi}{(1-\theta)(1-\varpi)}\left(\frac{G_h(H)}{U_c(C)}\right)\left(\frac{H}{Y^I}\right)^{\theta} = 1,$$

$$K^{\gamma-1} = \frac{1 + \frac{f(\delta)}{\delta}}{\frac{\gamma\beta\theta}{1-\theta}\frac{G_h(H)}{U_c(C)}\frac{H}{Y^I}},$$

$$Y = C + K\left(1 + \frac{f(\delta)}{\delta}\right),$$

$$Y = H^{1-\theta}K^{\gamma\theta},$$

$$Y^I = K^{\gamma}.$$

Combining them we get that output, consumption and the capital stock solve the following three equations:

$$Y = C + \beta\gamma\theta Y,$$

$$K = \frac{\beta\gamma\theta Y}{\left(1 + \frac{f(\delta)}{\delta}\right)},$$

$$\frac{\varpi}{(1-\theta)(1-\varpi)}\left(\frac{G_h\left(\left(\frac{Y}{K^{\gamma\theta}}\right)^{\frac{1}{1-\theta}}\right)}{U_c(C)}\right)\left(\frac{Y}{K^{\gamma}}\right)^{\frac{\theta}{1-\theta}}=1,$$

for given δ. Then, H and Y^I are determined by $Y = H^{1-\theta}K^{\gamma\theta}$ and $Y^I = K^{\gamma}$. Note that in the steady-state analysis we have kept δ as given. Recall that δ_t is determined by equations (A.3) and (A.5) and depends only on $\tilde{E}_t\phi_{t+1}$ and $\phi_{\max}$ which are both zero in a non-stochastic steady state. Here, instead, we are making the assumption that only these two variables, $\tilde{E}_t\phi_{t+1}$ and $\phi_{\max}$, are random, as if the economy were stochastic. They then determine δ.

A.2 Approximation of equilibrium conditions

Considering first the AD demand side of the model, we have the following first-order approximations of the equilibrium conditions (A.1), (A.2) and (A.4)

$$E_t\hat{C}_{t+1} = \hat{C}_t + \sigma(\hat{\imath}_t^B - E_t(\pi_{t+1} - \pi)) \tag{A.19}$$

$$\epsilon_q(1-v)\hat{q}_t = (1-v)\sigma^{-1}\hat{C}_t - v(\hat{\imath}_t^B - \hat{\imath}_t^D) \tag{A.20}$$

$$v\hat{\imath}_t^D = (\rho + v - 1)\hat{\imath}_t^R + (1-\rho)\hat{\imath}_t^B \tag{A.21}$$

in which we have defined variables with hat as the log-deviations of the respective variables with respect to the steady state; $\pi_t \equiv \ln P_{t+1}/P_t$, $\pi \equiv \ln \Pi$, $\epsilon_q \equiv -V_{qq}(q)q/V_q(q)$, $\sigma \equiv -U_c(C)C/U_{cc}(C)$. Note that we have defined $q_t = [(\rho^{-1}-1)R_t + R_t + B_t^h]/P_t$ and moreover

$$\hat{q}_t = z\hat{q}_t^G + (1-z)\hat{q}_t^R, \tag{A.22}$$

in which $\hat{q}_t^R$ and $\hat{q}_t^G$ are the log-deviations of the respective variables $q_t^R \equiv R_t/P_t$, $q_t^G \equiv (B_t^h + R_t)/P_t$ and in which $z \equiv q^G/q = q^G/(q^G + (\rho^{-1}-1)q^R)$.

We now turn to the approximation of the AS equation, given by (A.6) to (A.8). We obtain

$$\pi_t - \pi = k\left(\sigma^{-1}\hat{C}_t + \frac{\theta+\eta}{1-\theta}\hat{Y}_t - \frac{1+\eta}{1-\theta}\hat{Z}_t - \theta\frac{1+\eta}{1-\theta}\hat{Y}_t^I\right) + \beta E_t(\pi_{t+1} - \pi), \tag{A.23}$$

with

$$k \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$$

and in which we have used (A.11), (A.12) and note that Δ_t is zero in a first-order approximation. We take a log-linear approximation of equations (A.10) and (A.13):

$$\hat{Y}_t = s_c\hat{C}_t + s_k\hat{K}_t - s_k s_\delta\hat{\delta}_t \tag{A.24}$$

$$\hat{Y}_t^I = \hat{Z}_t^I + \gamma\hat{K}_{t-1} \tag{A.25}$$

in which

$$s_c \equiv \frac{\bar{C}}{\bar{Y}} = 1 - \beta\gamma\theta \qquad s_k \equiv \beta\gamma\theta \qquad s_\delta \equiv \frac{g(\bar{\delta})}{\bar{\delta} + f(\bar{\delta})}.$$

We can substitute (A.23) with (A.24) to obtain

$$\pi_t - \pi = k\left[\sigma^{-1}\hat{C}_t + (\varphi - 1)\hat{Y}_t - \varphi(\hat{Z}_t + \theta\hat{Z}_t^I + \gamma\theta\hat{K}_{t-1})\right] + \beta E_t(\pi_{t+1} - \pi), \quad \text{(A.26)}$$

in which we have defined

$$\varphi \equiv \frac{1+\eta}{1-\theta}.$$

Consider now a first-order approximation of (A.10) to obtain

$$E_t\left\{\hat{Z}_{t+1}^I + \hat{h}_{t+1} + \eta\hat{H}_{t+1} + \sigma^{-1}\hat{C}_t\right\} = (1-\gamma)\hat{K}_t - s_\delta\hat{\delta}_t. \quad \text{(A.27)}$$

We can use a first-order approximation of (A.11) and (A.12) to obtain, respectively, that

$$\hat{H}_{t+1} = \hat{Y}_{t+1} - \hat{Z}_{t+1} + \theta\hat{h}_{t+1},$$

$$\hat{h}_{t+1} = \frac{1}{1-\theta}(\hat{Y}_{t+1} - \hat{Z}_{t+1} - \hat{Y}_{t+1}^I) = \frac{1}{1-\theta}(\hat{Y}_{t+1} - \hat{Z}_{t+1} - \hat{Z}_{t+1}^I - \gamma\hat{K}_t).$$

We can then write (A.27) as

$$E_t\left\{\varphi(\hat{Y}_{t+1} - \hat{Z}_{t+1}) - \varphi\theta\hat{Z}_{t+1}^I\right\} = (1+\gamma\varphi\theta)\,\hat{K}_t - s_\delta\hat{\delta}_t - \sigma^{-1}\hat{C}_t. \quad \text{(A.28)}$$

Equations (A.19), (A.20), (A.21), (A.22), (A.24), (A.26), (A.28) are seven equations in the following ten stochastic sequences $\left\{\pi_t, \hat{C}_t, \hat{Y}_t, \hat{K}_t, \hat{\imath}_t^B, \hat{\imath}_t^R, \hat{\imath}_t^D, \hat{q}_t, \hat{q}_t^R, \hat{q}_t^G\right\}$, given the stochastic process $\left\{\hat{Z}_t, \hat{Z}_t^I, \hat{\delta}_t\right\}$ and initial condition $\hat{K}_{t_0-1}$. There are three degrees of freedom to specify policy. The central bank sets the interest rate on reserves and their quantity $\hat{R}_t$. Since $q_t^R = R_t/P_t$, we have that

$$\hat{q}_t^R = \hat{q}_{t-1}^R + \hat{R}_t - (\pi_t - \pi).$$

Given the tax policy specified in (A.18), the total supply of government liabilities, in a first-order approximation, follows

$$\hat{q}_t^G = \hat{q}_{t-1}^G + \xi_t.$$

Finally, note that in a first-order approximation, assuming that $f(\delta_t) = f$, $\hat{\delta}_t$ is given by

$$\hat{\delta}_t = \frac{\delta + f - 1}{\delta}\frac{1}{\bar{\phi}^{\max} - \bar{\phi}}\tilde{\phi}_t^a - \frac{\delta + f}{\delta}\frac{1}{\bar{\phi}^{\max} - \bar{\phi}}\tilde{\phi}_t^{\max}$$

in which $\bar{\phi}^{\max}$ and $\bar{\phi}$ are the stochastic steady-state values of $\phi_t^{\max}$ and $\tilde{E}_t\phi_{t+1}$, respectively, while $\tilde{\phi}_t^a \equiv \tilde{E}_t\phi_{t+1} - \bar{\phi}$ and $\tilde{\phi}_t^{\max} \equiv \phi_t^{\max} - \phi^{\max}$.

A.3 Calibration

We calibrate the model parameters as in the following table:

Table 1: Calibration of parameters

$\beta = 0.99$	$\upsilon = 0.996$
$\sigma = 0.66$	$\frac{q^R}{q^G} = 0.097$
$\eta = 0.2$	$\delta = 4.67$
$\epsilon_q = 1$	$f = 0.05$
$\gamma = 1$	$\phi_\pi = 1.5$
$\theta = 0.25$	$\phi_y = 0.5$
$\alpha = 0.80$	$\bar{\phi} = 0.05$

The rate of time preference is set to $\beta = 0.99$; the intertemporal elasticity of substitution in consumption σ is set to 0.66; the inverse of the Frisch elasticity of labor supply is set to $\eta = 0.2$; the inverse of the elasticity of the utility function with respect to liquidity is also set to the unitary value $\epsilon_q = 1$; the share of labor in the final-good production is set to 3/4 and therefore $\theta = 0.25$; the share of capital in the production of the intermediate good is set to the unitary value, $\gamma = 1$; the fraction of firms with fixed prices is set at $\alpha = 0.8$, implying a duration of price rigidities of five quarters. The steady-state ratio between the nominal natural rate of interest and the deposit rate $(1 + i^D)/(1 + i^B) = \upsilon$ is set to 0.996 and it is calibrated by taking the average over the last 5 years (March 2016 - March 2021) of the ratio between the 3-month Gross National Rate on Non-Jumbo Deposits (less than \$100,000) (code CD3NRNJ in FRED Database) and the 90-Day AA Asset-backed Commercial Paper Interest Rate (code RIFSPPAAAD90NB in FRED Database); the ratio between the total value of central bank reserves and the overall government liabilities including reserves is obtained by taking the average over the last available 5 years (March 2015 - March 2020) of the ratio between the Reserves of Depository Institutions, Nonborrowed (code NONBORRES in FRED Database) and the sum of the same reserves plus the Federal Government Total Liabilities (code FGTLBLQ027S in FRED Database), therefore $q^R/q^G = 0.097$. The loan-to-equity ratio is calibrated using the average over the last available 5 years (July 2015 - July 2020) of the ratio between the Loans and Leases in Bank Credit of all commercial banks (code LOANSNSA in FRED Database) and the Total Equity Capital of all commercial banks (code QBPBSTLKTEQK in FRED Database), therefore $\delta = 4.67$; the parameter f is set to 0.05 implying a 5%

cost per unit of equity. Finally, the policy parameters are set in a standard way at $\phi_\pi = 1.5$ and $\phi_y = 0.5$. The parameter ρ is instead set to take values 0.1, 0.3, 0.6 and 1.

Given the above primitive parameters, we can obtain the following composite parameters:

Table 2: Other parameters

$v_\rho = \frac{\rho+v-1}{\rho}$	$z = \frac{1}{1+(\rho^{-1}-1)\frac{q^R}{q^G}}$
$k = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$	$s_c = 1 - \beta\gamma\theta$
$s_k = \beta\gamma\theta$	$s_\delta = \frac{f}{\delta+f}$
$\varphi = \frac{1+\eta}{1-\theta}.$	$\bar{\phi}^{\max} = \bar{\phi} + \frac{1-\bar{\phi}}{\delta+f}$

Note that in Section 6.1, the benchmark New-Keynesian model is obtained by setting $\theta = 0$, $v = 1$ and $\epsilon_q = 0$.

In the impulse responses of Section 6.1, we have considered the following properties of the stochastic disturbances. Given the interest-rate rule

$$\hat{\imath}_t^R = \phi_\pi(\pi_t - \pi) + \phi_y \hat{Y}_t + \varepsilon_t$$

ε_t has been assumed to follow an AR(1) process with an auto-regressive coefficient equal to 0.95; the shock ξ_t in

$$\hat{q}_t^G = \hat{q}_{t-1}^G + \xi_t$$

has been assumed to be a white-noise process. Moreover, reserves have been assumed to follow the process

$$\hat{R}_t = \hat{R}_{t-1} + u_t$$

with u_t modelled as a white-noise process.

Finally, in Section 6.2, we have modelled a shock to $\tilde{\phi}_t^{\max}$ following an AR(1) process with an auto-regressive coefficient equal to 0.95. Note the following relationship between $\hat{\delta}_t$ and $\tilde{\phi}_t^{\max}$

$$\hat{\delta}_t = \frac{\delta + f - 1}{\delta} \frac{1}{\bar{\phi}^{\max} - \bar{\phi}} \tilde{\phi}_t^a - \frac{\delta + f}{\delta} \frac{1}{\bar{\phi}^{\max} - \bar{\phi}} \tilde{\phi}_t^{\max}.$$

Moreover, note that in the steady state

$$\delta + f = \frac{1 - \bar{\phi}}{\bar{\phi}^{\max} - \bar{\phi}}.$$

We fix the average default rate $\bar{\phi}$ at 5% on a quarterly basis and we retrieve $\bar{\phi}^{\max}$ using the above equation.