Managing Monetary Policy Normalization*

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Abstract

We propose a new framework for monetary policy analysis to study monetary policy normalization when exiting a liquidity trap. The optimal combination of reserves and interest rate policy requires an increase in liquidity (reserves) a few quarters after the policy rate is set at the effective lower bound. Removal of accommodation requires that quantitative tightening starts before the liftoff of the policy rate. Moreover, the withdrawal of liquidity takes place at a very slow pace relative to the normalization of the policy rate.

1 Introduction

The global financial crisis of 2008-09 and the recent pandemic shock have pushed major Central Banks to reduce their official policy rates to historically low levels embracing unconventional monetary policies, including quantitative easing (QE), with purchases of government debt and, in some instances, private-sector financial assets, to the end of providing monetary accommodation and meeting their policy objectives. In engaging in QE operations, Central Banks finance the purchases of government bonds by issuing bank reserves, thus increasing both the central banks' assets and liabilities. With economic activity recovering and inflation peaking up, Central Banks have started removing policy accommodation. Reducing the pace of asset purchases (tapering) is the starting point in this process. Normalizing monetary policy consists in lifting the policy rate and reducing the size of Central Banks' balance sheets (quantitative tightening, QT): this process entails strategic choices in terms of the timing, the pace and the sequence of policy interventions that potentially could be shaped by country-specific institutional features.

In particular, how should reserves be managed at the effective lower bound in conjunction with interest rate policies? What is the optimal size of Central Banks' balance sheets (reserves)? How does fiscal policy and in particular the fiscal capacity determine the supply of reserves and its adjustment during the normalization process?

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In the reference framework for monetary policy analysis (the Neo-Wicksellian framework, see Galì, 2008, and Woodford, 2003, for a general presentation of the model), the central bank can have full control of inflation and output through movements in the policy rate: there is no role for reserves and the size of Central Banks' balance sheets is irrelevant. Once the policy rate is specified, it is a sufficient policy tool to control aggregate demand, to influence prices and economic activity. This key feature is a by-product of the fact that in that environment the policy rate coincides with the nominal interest rate that consumers face when deciding on how much to save and consume.

To answer to the aforementioned policy questions, we propose a new framework for monetary policy analysis that, while encompassing as a special case the standard approach, provides a novel way to characterize the interaction between nominal interest rates and balance-sheet policies. First, reserves are going to be an additional tool on top of the policy rate (e.g. interest rate on reserves) that the central bank can use to control inflation and output. Second, the policy rate does not coincide with the nominal rate relevant for the households' consumption/saving choice, which is now influenced by a novel monetary transmission mechanism based on what we label the "liquidity channel".

In order to capture the salient aspects of recent Central Banks' policy actions, we need to consider an explicit role for the banking system (as the only holders of Central Banks' reserves), a role for the fiscal authority (as the issuer of liquid government bonds) and a role for selected assets (deposit and government bonds) as provider of liquidity services to the private sectors.

Within this rich context, we first explain why the policy rate does not necessarily coincide with the nominal rate relevant for the consumption/saving choices. The gap between these interest rates creates a role for reserve policy (QE or QT) in affecting aggregate demand. In our framework, the only agents holding central bank reserves are banks, which use them to collateralize deposits. In this way, the interest rate on deposit is going to depend on the interest rate the central bank sets on its reserves. Since deposits provide liquidity services, the interest rate on deposits command a liquidity premium with respect to other illiquid securities that households may hold. However, it is exactly the interest rate on these illiquid securities that is the one that matters for their consumption/saving choices and for aggregate demand. As such, it will be linked to the policy rate only through the connection with the deposit rate and the liquidity premium. Via this new monetary-policy transmission mechanism, central bank reserves become an additional tool available for the monetary policymaker to control aggregate demand, since in general they can influence the liquidity premium through the quantity of deposits that they back. An additional implication of the new framework is that also fiscal policy can influence liquidity premia and, through them, aggregate demand. In this general framework, inflation and output determination is achieved through a joint interaction between monetary and fiscal policy, unlike in the conventional framework.

The standard Neo-Wicksellian paradigm is nested when reserves do not provide any non-pecuniary benefits either to banks or directly to households or when, even if they do provide such benefits, conditions are such that agents get zero marginal benefits from holding them, i.e. liquidity is fully satiated. This occurs when taxation is high to back a sufficient supply of government liquidity.

Our framework provides a novel aggregate demand equation that generalizes the one used in standard New-Keynesian models. Output depends not only on the current and expected future real rates but also on the supply of liquidity in the economy. A higher supply of liquidity, which lowers liquidity premia, stimulates aggregate demand and therefore output. The connection between output and real rates also changes. It is no longer the case that an increase in future real rates lowers output by the same magnitude as movements in the current real rate do. Instead, in our general analysis, it happens that future real rates count less for short-run aggregate demand and output. The model is consistent with a reduced power of forward guidance in stimulating aggregate demand.

We use our framework to study how to optimally manage interest rate and reserve policies in an economy that faces a liquidity trap lasting for a finite duration of time. Given the connection between liquidity and fiscal policy, this becomes a joint monetary and fiscal policy problem.

The optimal combination of reserves and interest rate policy, requires to increase liquidity (reserves) few quarters after the policy rate is at the effective lower bound. Our analysis implies also that the removal of accommodation requires quantitative tightening to start before the liftoff of the policy rates. Moreover the withdrawal of liquidity takes place at a very slow pace relatively to the normalization of the policy rate. Intuitively an active management of reserves help in reducing the output losses by its effect on the liquidity premium and ultimately on aggregate demand.

In general, our results can also be read de-coupled along inflation-versus-output objectives: i) to stabilize inflation at the target the increase in government liquidity should be moderate, peak at the end of the trap, withdrawn just before the liftoff the policy rate and reabsorbed only very slowly; the policy rate should stay at the zero-lower bound longer than the duration of the shock; ii) to stabilize output, liquidity should increase more, peak during the middle of the liquidity trap and completely be reabsorbed as policy rates normalize; the liftoff of the policy rate from zero interest-rate policies may happen at the time the shock disappears. A higher weight on inflation stabilization, as often the case in the welfare function of this class of models, implies i) to dominate in the optimal policy problem.

This work is related to the literature that has studied the optimal interest rate policy in a liquidity trap, e.g. Eggertsson and Woodford (2003, 2004) and Werning (2011). In these works, reserves are irrelevant to determine optimal policy during a liquidity trap. When taxes are distortionary, as in Eggertsson and Woodford (2004), the path of public debt matters, instead, but just as a way to smooth the impact of distortionary taxes rather than as an instrument to stimulate aggregate demand, through liquidity, like here.

Angeletos, Collard and Dellas (2022) is an important related work, which has studied an optimal taxation problem when public debt provides liquidity services, though with a different modelling strategy than here, showing that there is a departure from the standard tax smoothing result. They also study the optimal supply of liquidity. Our framework has a broader focus by also considering the interaction with monetary policy in a liquidity trap.

Our analysis is also related to recent literature that has provided departures from the standard New-Keynesian model. Benigno and Nisticò (2017) already present a model in which the central bank has two policy instruments: the interest rate on reserves and their quantity. In their model reserves provide liquidity services through a cash-in-advance constraint together with a privately issued asset. They use their framework to study how an exogenous reduction in the liquidity properties of private assets affects inflation and output, given different monetary policies. Their banking model is stylized and they do not provide a general framework that nests the standard Neo-Wicksellian paradigm, as we do here. More recently, Diba and Loisel (2020, 2021) have also built a New-Keynesian model with the central bank having two policy instruments, like here. In their framework, intermediaries demand reserves to reduce the costs associated with the supply of loans, which are demanded by firms because of a working-capital constraint. Reserves enter directly into the aggregate supply equation. Instead, in the framework proposed here, reserves are held to collateralize deposits and the money-market channel in our model could also be disjoint from the loans market. Diba and Loisel (2020) finds that the equilibrium can be determinate even with interest-rate pegging because reserves are also an instrument of policy. Diba and Loisel (2021) focuses on the quantitative properties of policies at the zero lower bound, showing that their framework can be consistent with no significant deflation and little inflation volatility.

Piazzesi, Rogers and Schneider (2021) also emphasize the disconnection between money-market rates and the interest rate relevant for consumption/saving choices. They present a banking model in which monetary policy operates either through a corridor or a floor system. The main objective of their work is to compare the pass-through of the policy rate to other money market rates across the two systems. They show that equilibrium can be determinate even if the policy rate does not follow a Taylor rule. Arce et al. (2020) also focus on the relationship between the size of the balance sheet and the interbank rate. We instead focus on the floor system or alternatively on a corridor system that shrinks to zero. However, we are interested in the transmission mechanism of policy to inflation and output, emphasizing the specification of the monetary/fiscal policy regime, which in our framework is critical to determine prices and inflation. We study the interaction between fiscal policy, interest rates and reserves to determine equilibrium, which is novel with respect to the above works. None of the above papers has analyzed how reserves and interest rates should be set optimally during a liquidity trap episode.

Bigio and Sannikov (2021) integrate monetary policy analysis through a corridor system via a banking model that displays a liquidity and a credit channel. However, in their case, when the corridor around the policy rate shrinks to zero, the only policy instrument remains the interest rate on reserves while the quantity of reserves becomes irrelevant. In our model, instead, reserves are always a policy instrument and are relevant for inflation and economic activity even with a zero corridor system, provided they supply some non-pecuniary benefits.

Bigio and Sannikov (2021) and Piazzesi, Rogers and Schneider (2021) distinguish between a model in which reserves are scarce and the central bank conducts policy through a corridor system, or a model in which reserves are abundant and the central bank conducts policy through a "floor system".[1] Our model is one in which there is no use of central bank settlement balances as a means of clearing payments between banks. The corridor system shrinks to zero at the policy rate, i.e. the interest rate on reserves.[2] This is in line with what they label as the "floor system", which, however, works regardless the scarcity or abundance of reserves. Moreover, irrespective of the size of the reserves, the central bank always has two independent tools to specify policy, the interest rate on reserves and their quantity.[3] We show that the quantity of reserves can be relevant for determining inflation and economic activity, unless reserves do not provide any non-pecuniary marginal benefits. If anything, the abundance of reserves may imply irrelevance of reserves for inflation or output but, in general, the abundance or scarcity of reserves are not relevant dimensions to discriminate between a corridor or a floor system.

Canzoneri et al. (2008) and Canzoneri, Cumby and Diba (2017) are early models in which there is a disconnection between the policy rate and the interest rate relevant for consumption/saving choices. Curdia and Woodford (2010, 2011) present models with borrowers and savers in which credit spreads arise because of intermediation activity. However, in their context, the policy rate is still the relevant factor for the savers' consumption/saving choices. The central bank's balance sheet is also an additional policy instrument when there are financial frictions, but it acts only on credit spreads and not on the liquidity channel, like here.

Our work is also related to the literature on the "forward-guidance puzzle" as elaborated by Del Negro, Giannoni and Patterson (2013) in which the New-Keynesian model gives too much power to forward guidance in affecting current demand. Recent works such as Werning (2015) and McKay, Nakamura and Steinsson (2016) have tried to reconcile the puzzle by using incomplete market models. Our framework, instead, delivers a new Aggregate-Demand equation in which forward guidance is less powerful even when markets are complete. A similar result is obtained in Diba and Loisel (2020).

Finally, our model, in the special case of a narrow-banking regime, is also related to the recent literature that has studied central bank digital currency by allowing households to directly hold deposits at the central bank (see the work of Niepelt, 2021, and again Piazzesi, Rogers and Schneider, 2021).

The present work starts with Section 2, providing the main intuition for why our framework departs from the standard Neo-Wicksellian paradigm. Section 3 presents a simple endowment economy with a banking sector in a flexible-price environment. Section 4 studies the equilibrium with some examples. Section 5 extends the model with price rigidities. Section 6 studies the implications of this more general model in a log-linear approximation. Section 7 studies the optimal policy in a liquidity trap and Section 8 concludes the work.

[1] De Fiore, Hoerova and Uhlig (2018) also present a banking model in which there are frictions in the money market.

[2] Our analysis is in line with the discussion of Woodford (2001) on how the central bank can control money-market interest rates when central bank settlement balances cease to be used to clear payments across banks.

[3] In fact, there are three independent policy tools including the specification of central bank remittances policy.

2 Reserve Effectiveness: the "Liquidity Channel"

In this section, we highlight the main difference between our model and the standard Neo-Wicksellian paradigm. In the latter, the economy can be simply described by an AS-AD model in which the policy rate acts directly on the AD equation. Consider a standard Euler equation in a perfect-foresight model

$$U_c(C_t) = \beta \frac{(1+i_t)}{\Pi_{t+1}} U_c(C_{t+1}), \tag{1}$$

in which $U_c(\cdot)$ is the marginal utility of consumption, C_t; β is the rate of time preference, with $0 < \beta < 1$; i_t is the nominal interest rate at time t and Π_{t+1} is the gross inflation rate between time t and $t+1$. A key assumption in the baseline Neo-Wicksellian paradigm is that the policy rate controlled by the central bank is the same as the nominal rate influencing the AD equation. Upward movements in the policy rate cause a contraction in demand for the given future consumption and inflation rate. More generally, by setting the policy rate, the central bank can control the path of inflation and output.

Our framework builds upon the same Euler equation (1) but with no direct link between the policy rate and the nominal interest rate. The latter identifies the risk-free rate on (private) illiquid securities: we refer to it as the “natural nominal interest rate”, i^B. For the sake of simplicity, focusing on the perfect-foresight equilibrium, the household’s Euler equation becomes

$$U_c(C_t) = \beta \frac{(1+i_t^B)}{\Pi_{t+1}} U_c(C_{t+1}). \tag{2}$$

In our novel framework, we allow households to hold other types of risk-free assets, which provides also liquidity services. This class of securities might include bank deposits and/or treasury debt. Portfolio optimization determines the links between the interest rate on liquid securities, i^D, and the natural nominal rate of interest, i^B :

$$1 + i_t^D = (1 - \mu_t)(1 + i_t^B), \tag{3}$$

where μ_t, with $\mu_t \geq 0$, is the liquidity premium

$$\mu_t = V_q\left(\frac{Q_t}{P_t}\right),$$

with $V_q(\cdot)$ being the marginal utility from holding liquid securities, and Q_t the amount of liquid securities held by households in their portfolio. It is assumed that $V_q(Q_t/P_t) = 0$ for values of Q_t/P_t above a satiation level $\bar{q} > 0$, i.e. $Q_t/P_t \geq \bar{q}$.

The last step to understand the novelty of the monetary transmission mechanism in our framework relies on the explicit modelling of the banking sector. Financial intermediaries supply deposits and raise equity to invest in central-bank reserves and privately-issued bonds. The banking equilibrium implies that the deposit rate will be a weighted average of the policy rate (the interest rate on reserves, i^R) and the natural nominal rate of interest, i^B:

$$(1 + i_t^D) = (1 - \rho)(1 + i_t^B) + \rho(1 + i_t^R),$$

where ρ is the reserve/collateral requirement with $0 \leq \rho \leq 1$ and $R_t = \rho D_t$; D denotes households' deposits and R central-bank reserves. Moreover, the following inequality holds: $(1+i_t^B) \geq (1+i_t^R)$.

We can then combine the above three equations to obtain

$$(1 + i_t^B) = \frac{\rho}{\rho - V_q\left(\frac{1}{\rho}\frac{R_t}{P_t}\right)}(1 + i_t^R),$$

showing the novel relationship between the policy rate and the rate directly influencing consumption/saving choices. Relatively to the more general framework that we present below, we assume here that the only asset that provides liquidity service to the households is bank deposit, so that $Q_t = D_t = R_t/\rho$.

Based on this simple structure, we can draw key implications on reserve effectiveness. Reserve can be an independent stabilization tool from the policy rate, even away from the effective lower bound. Variations in central bank reserves do affect the natural nominal interest rate independently of the movements of the policy rate: an increase in reserves ($\uparrow R_t$), ceteris paribus, lowers the liquidity services of deposits, lowering the natural nominal interest rate ($i_t^B \downarrow$) and having an expansionary effect on the economy, as long as the economy is not satiated with liquidity, i.e. $V_q(Q_t/P_t) > 0$. Similarly, movements in the policy rate, i^R, have amplifying effects on the natural nominal rate of interest, everything else being equal.

Reserves become ineffective when

1. there is full satiation of liquidity, so that $i_t^B = i_t^R = i_t^D$, or
2. there are no securities available that provide liquidity services or
3. even if some securities provide liquidity services, reserves are not in this class, like in the case in which they do not provide any collateral benefits ($\rho = 0$).

The further novel channel in our framework is that the supply of liquidity is bounded by, and connected to, the fiscal capacity of the government. Therefore the problem of output and inflation determination becomes a joint monetary and fiscal policy problem.

3 Model

We now present our model in a flexible-price endowment economy, underlining the transmission mechanism of monetary policy through the banking sector, via money-market rates. The purpose is to show how price and inflation determination results from the interaction between monetary and fiscal policy. Later, we add price rigidities and present the new framework in a log-linear approximation to study the optimal exit from zero-lower bound policies.

In this Section, we present the model in blocks starting from the banking sector, then households and, finally, the government, which includes the treasury and the central bank.

3.1 Banking Sector

At a generic time t, there is a potentially infinite number of intermediaries that can start intermediation activity without any entry cost. Each intermediary lives for two periods. Intermediaries entering at time t face the following balance sheet constraint:

$$R_t + A_t = D_t + N_t, \tag{4}$$

in which R_t are the holdings of central bank reserves which are remunerated at the rate i_t^R, A_t are the holdings of short-term private debt that carries an interest rate i_t^B. Intermediaries can finance their assets by issuing deposits D_t, at the interest rate i_t^D, and by raising equity N_t.[4]

Banks are subject to a reserve/collateral requirement of the form $R_t \geq \rho D_t \geq 0$ with $0 \leq \rho \leq 1$. The two extremes of the interval characterize two interesting cases. When $\rho = 1$, intermediaries need to back all deposits by reserves, like in a narrow banking system. When $\rho = 0$, there is no reserve/collateral requirement, but reserves should be non-negative, $R_t \geq 0$. Here we emphasize that the collateral requirement should be interpreted to capture not only the traditional reserve requirement but also other constraints that financial intermediaries may face.

Intermediaries can also invest in cash, which is going to be dominated by reserves. The economy is cashless in equilibrium but not without cash as a store of value. The possibility that reserves can be transformed into cash implies the existence of a zero-lower bound on the interest rate on reserves, $i_t^R \geq 0$.

Intermediaries' profits, Ψ_{t+1}, at time $t+1$ are given by

$$\Psi_{t+1} = (1 + i_t^B)A_t + (1 + i_t^R)R_t - (1 + i_t^D)D_t. \tag{5}$$

Intermediaries are subject to a limited-liability constraint, for their profits should be non-negative. This constraint can be written as

$$\Psi_{\min} = (1 + i_t^B)A_t + (1 + i_t^R)R_t - (1 + i_t^D)D_t \geq 0, \tag{6}$$

which is independent of the contingency at time $t+1$.[5] The banks' optimization problem consists

[4] We can also characterize a more general banking problem in which intermediaries supply also loans to the private sector, which are used to finance capital for production, see Benigno and Benigno (2021). That more general model would describe a credit channel, which would be orthogonal to the liquidity channel emphasized here, and ininfluential for the analysis.

[5] With risky assets, the limited-liability constraint is state-contingent.

in maximizing rents $\mathcal{R}$ (the expected discounted value of profits minus the value of equity)

$$\begin{aligned}\mathcal{R}_t &= E_t\left\{M_{t+1}\Psi_{t+1}\right\} - N_t \\ &= E_t\left\{M_{t+1}\left[(1+i_t^B)A_t + (1+i_t^R)R_t - (1+i_t^D)D_t\right]\right\} - N_t.\end{aligned} \tag{7}$$

with M_{t+1} being the households' stochastic discount since consumers own financial intermediaries.

Intermediaries choose A_t, R_t, D_t to maximize (7) under the budget constraint (4), the limited-liability constraint (6), and the collateral constraint $R_t \geq \rho D_t \geq 0$.

To solve the above-defined linear programming problem, first substitute the balance-sheet constraint (4) to replace A_t into (7) to obtain

$$\mathcal{R}_t = \left[\frac{(1+i_t^R)}{(1+i_t^B)} - 1\right] R_t - \left[\frac{(1+i_t^D)}{(1+i_t^B)} - 1\right] D_t. \tag{8}$$

In deriving equation (8), we have used $E_t\left\{M_{t+1}(1+i_t^B)\right\} = 1$, anticipating a result of the household's problem.

The rent function (8) should be maximized under the limited-liability constraint (6) and the collateral constraint. The first result is that raising equity bears no costs, which implies that the limited liability constraint is not binding for equity can be raised to satisfy it.[6]

We now discuss the implications for the supply of deposits by combining (8) with the collateral requirement $R_t \geq \rho D_t$. We distinguish the following two cases:

1. When $R_t > \rho D_t$, then $i_t^B = i_t^R$ otherwise rents will be positive in (8). Moreover, again the zero-rent condition applied to (8) implies that also the deposit rate will be fixed at the rate on reserves and, therefore, the Neo-Wicksellian framework is nested, i.e. $i_t^B = i_t^D = i_t^R$.

2. When $R_t = \rho D_t$, we can substitute it into (8) in place of R_t to obtain that

   $$(1+i_t^D) = \rho(1+i_t^R) + (1-\rho)(1+i_t^B), \tag{9}$$

   when rents are zero.

In case 2), the deposit rate at which intermediaries are willing to supply deposit is a weighted average of the policy rate and the natural nominal rate of interest, with a weight given by the parameter ρ. The collateral requirement, ρ, becomes crucial for characterizing the equilibrium relationship among the different interest rates.

- In a narrow banking regime, when $\rho = 1$, the deposit rate coincides with the policy rate, $i_t^D = i_t^R$, but in general $i_t^B > i_t^D = i_t^R$.

- When $\rho = 0$ and reserves no longer provide non-pecuniary benefits, it follows that $i_t^D = i_t^B$ and $i_t^B = i_t^R$ as long as reserves are positively supplied by the central bank. Therefore, when

[6] This result would also hold were intermediaries supplying risky loans.

$\rho = 0$, all interest rates are equalized, $i_t^B = i_t^R = i_t^D$, and the Neo-Wicksellian regime is nested again.

To conclude the characterization of the banking problem, the demand for equity, as discussed, is such as to make the limited-liability constraint not binding. Therefore, using (4) in (6), we obtain the following inequality:

$$\begin{aligned} N_t &\geq \frac{i_t^D - i_t^B}{1 + i_t^B} D_t + \frac{i_t^B - i_t^R}{1 + i_t^B} R_t \\ N_t &\geq 0 \end{aligned}$$

in which we have used (9) in moving from the first to the second line. Equity can be also zero in this simple model since there is no risk in the assets held by intermediaries.[7]

3.2 Households

We consider a representative consumer maximizing the following intertemporal utility:

$$E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} U\left(C_t, \frac{Q_t}{P_t}\right) \right\}, \tag{10}$$

in which E_{t_0} is the conditional-expectation operator at time t_0; β, with $0 < \beta < 1$, is the intertemporal discount factor in preferences, $U(\cdot, \cdot)$ is an increasing and concave function of their respective arguments, with $U(C_t, \cdot)$ reaching a satiation point at a generic value $\bar{q}$; C is a consumption good and Q are the nominal assets from which the households get non-pecuniary benefits (liquidity services). Q includes two types of securities: deposits, D, and treasury debt, B^h, therefore $Q_t = D_t + B_t^h$; P is the price level.

The household is subject to the following flow budget constraint:

$$P_t C_t + (D_t + B_t^h) + (1 + i_{t-1}^B) B_{t-1} + N_t + T_t \leq (1 + i_{t-1}^D)(D_{t-1} + B_{t-1}^h) + B_t + Y_t + \Psi_t. \tag{11}$$

She/He can invest its savings in two securities: D, deposit, and B^h, treasury notes, which are a perfect substitute for providing liquidity services and they both pay an interest rate i^D. She/He can borrow or lend through private risk-free bonds, B, which pay an interest rate i^B. All these securities are risk free, but deposit and treasury's notes provide liquidity services, whereas private bonds are illiquid.[8]

There is a subtle justification for why we are assuming that some risk-free securities, such as deposits and treasury debt, provide liquidity services while others do not. First note that central bank reserves are always free of risk and repaid, since the central bank issues those liabilities without being subject to a solvency constraint.[9] In attributing a liquidity role to treasury debt,

[7] If intermediaries supply risky loans, demand of equity is going to be positive to absorb the maximum loss on loans.

[8] Note that in the household's budget constraint a positive value for B denotes debt.

[9] See the discussion in Benigno (2020) and Woodford (2000).

we are implicitly assuming that the central bank always "backs" the treasury by extending the special risk-free properties of its liabilities to the treasury debt. For deposits, consider that they are issued by intermediaries and backed, although partially, by reserves. Given this guarantee, we also attribute a liquidity role to them. On the contrary, private debt needs to satisfy a solvency condition to be risk free, which makes it different from central bank reserves and from all the other securities that are implicitly backed by the central bank.

Households can finance intermediaries by supplying equity N; T are lump-sum taxes levied by the government. On the right-hand side of the budget constraint, households receive endowment Y, and intermediaries' nominal profits, Ψ.

Households maximize utility (10) by choosing stochastic sequences $\left\{C_t, B_t, D_t, B_t^h\right\}_{t=t_0}^{\infty}$ subject to the period budget constraint (11), an appropriate borrowing limit and initial conditions.

The following asset-pricing condition characterizes the choice with respect to the illiquid bonds, B_t:

$$E_t\left\{M_{t+1}\right\} = \frac{1}{1+i_t^B}, \tag{12}$$

where M_{t+1}, the nominal stochastic discount factor, is given by $M_{t+1} = \beta(U_c(C_{t+1}, q_{t+1})/P_{t+1})/(U_c(C_t, q_t)/P_t)$ with $q_t \equiv Q_t/P_t$. The expected value of the stochastic discount factor is equal to the price of the illiquid bonds – the inverse of the gross nominal interest rate. As noted, we label this interest rate, i^B, as the natural nominal rate of interest since it is the one that directly affects the saving-consumption choices.

The optimal choice with respect to the liquid securities, D_t and B_t^h, implies that

$$1 = \mu_t + (1+i_t^D)E_t\left\{M_{t+1}\right\}, \tag{13}$$

in which μ_t is the liquidity premium given by

$$\mu_t = \frac{U_q\left(C_t, q_t\right)}{U_c(C_t, q_t)},$$

with $U_q(\cdot,\cdot)$ and $U_c(\cdot,\cdot)$ the respective partial derivatives of the function $U(\cdot,\cdot)$, and $0 \leq \mu_t < 1$.

Note that we can combine (12) and (13) to obtain

$$(1+i_t^D) = (1-\mu_t)(1+i_t^B),$$

saying that the interest rate on deposit is lower, or almost equal, than the rate on illiquid bonds. Only when the economy is satiated with liquidity the two rates coincide. The optimal supply of equity, N, implies that its value is equal to the discounted value of intermediary profits:

$$N_t = E_t\left\{M_{t+1}\Psi_{t+1}\right\}.$$

Finally, the intertemporal budget constraint of the consumer holds with equality at all times.

3.3 Government

The government sector includes the treasury and the central bank. The treasury's budget constraint is

$$B_t^g = (1 + i_{t-1}^D)B_{t-1}^g - T_t - T_t^c, \tag{14}$$

in which the short-term debt issued by the treasury B^g carries the nominal interest rate i^D, the one on liquid securities; the treasury can pay debt via taxes, T, and use remittances, T^c, received from the central bank. The central bank has the following budget constraint:

$$B_t^c - R_t = (1 + i_{t-1}^D)B_{t-1}^c - (1 + i_{t-1}^R)R_{t-1} - T_t^c, \tag{15}$$

since it can issue interest-bearing reserves R at the rate i^R and it holds treasury notes as assets, denoted by B^c. Note that the central bank is not subject to any solvency condition since its liabilities define what a currency is. As mentioned, we are assuming that the central bank backs the treasury, which is, therefore, not subject to a solvency condition, too.

4 Equilibrium

We discuss the equilibrium conditions.

Asset-market equilibrium requires that all bonds issued by the treasury are held by the central bank and the households, therefore

$$B_t^g = B_t^c + B_t^h; \tag{16}$$

the debt issued by the private sector is held by intermediaries $A_t = B_t$; the supply and demand of deposits are in equilibrium, as well as the market of central bank reserves. Goods market equilibrium implies that output is equal to consumption

$$Y_t = C_t.$$

The exogeneity of output allows us to focus more neatly on the determination of money market rates, liquidity and inflation.

On the supply side of deposit, the banking equilibrium implies a link between the deposit rate, i^D, the policy rate, i^R, and the natural nominal rate of interest, i^B:

$$(1 + i_t^D) = \rho(1 + i_t^R) + (1 - \rho) \max\left\{(1 + i_t^B), (1 + i_t^R)\right\}, \tag{17}$$

with $i_t^B \geq i_t^R \geq 0$.

On the demand side, households hold deposits if the spread between deposit rate and natural nominal rate of interest satisfies

$$\frac{(1 + i_t^D)}{(1 + i_t^B)} = \left(1 - \frac{U_q\left(Y_t, q_t\right)}{U_c(Y_t, q_t)}\right). \tag{18}$$

Finally, equilibrium in demand and supply of illiquid securities relates the natural nominal rate of interest with liquidity and inflation

$$\frac{1}{1+i_t^B} = E_t\left\{\beta\frac{U_c(Y_{t+1},q_{t+1})}{U_c(Y_t,q_t)}\frac{P_t}{P_{t+1}}\right\}. \tag{19}$$

The three equations above show already the complexity in determining prices and inflation in this framework. Even abstracting from the issue of price determination, the policy rate, i^R, alone cannot determine the inflation rate without accounting for liquidity, q.

Note the three cases in which the Neo-Wicksellian framework is nested with the three money market rates equalized:

- Reserves in the banking sector are in excess, $R_t > \rho D_t$, therefore $i_t^B = i_t^R$ and from equation (17) it follows $i_t^D = i_t^B = i_t^R$;
- Reserves do not provide any non-pecuniary benefits to the banking sector, $\rho = 0$, therefore $i_t^D = i_t^B$ from equation (17), and $i_t^D = i_t^B = i_t^R$ given a positive supply of reserves;
- Liquidity premia are zero, i.e. $U_q(Y_t, q_t) = 0$, and therefore $i_t^D = i_t^B$ from equation (18) and $i_t^D = i_t^B = i_t^R$ from equation (17).

The first two cases follow from the banking equilibrium and depend on supply considerations on deposits. The last case depends on demand considerations on deposits, from the side of households.

To complete the equilibrium description, we need to characterize the supply of liquidity. Note that

$$q_t = \frac{D_t + B_t^h}{P_t}, \tag{20}$$

and

$$D_t \leq \frac{R_t}{\rho}. \tag{21}$$

Supply of treasury's debt and reserves depend on the flow budget constraints of the two authorities:

$$B_t^g = (1+i_{t-1}^D)B_{t-1}^g - T_t - T_t^c, \tag{22}$$

$$B_t^c - R_t = (1+i_{t-1}^D)B_{t-1}^c - (1+i_{t-1}^R)R_{t-1} - T_t^c, \tag{23}$$

which we have already described in details.

To complete the characterization of the equilibrium conditions, we need to state the intertemporal resource constraint of the economy, which holds in equilibrium as the mirror image of the intertemporal budget constraint of the private sector

$$\frac{(1+i_{t-1}^R)R_{t-1} + (1+i_{t-1}^D)B_{t-1}^h}{P_t} = E_t\left\{\sum_{T=t}^{\infty}\beta^{T-t}\frac{U_c(Y_T,q_T)}{U_c(Y_t,q_t)}\left[\frac{T_T}{P_T} + \frac{i_t^B - i_t^D}{1+i_t^B}\frac{B_t^h}{P_t} + \frac{i_t^B - i_t^R}{1+i_t^B}\frac{R_t}{P_t}\right]\right\}. \tag{24}$$

The outstanding overall real liabilities of the whole government (treasury and central bank), with respect to the private sector, should be equal, at each point in time, to the present-discounted value of real taxes and of the seigniorage revenues the government gets by selling liabilities, either reserves or treasury notes, at an interest rate lower than the natural nominal rate of interest.

Equilibrium is a set of stochastic sequences $\{i_t^B, i_t^D, i_t^R, q_t, P_t, D_t, B_t^h, B_t^g, B_t^c, R_t, T_t, T_t^c\}_{t=t_0}^{\infty}$ satisfying equilibrium conditions (16)–(24), for each $t \geq t_0$, with $i_t^B \geq i_t^R \geq 0$, given the stochastic sequence $\{Y_t\}_{t=t_0}^{\infty}$ and initial conditions $i_{t_0-1}^D, i_{t_0-1}^R$, $B_{t_0-1}^g, B_{t_0-1}^c, R_{t_0-1}$. There are four degrees of freedom to specify monetary and fiscal policy.

In the next Section, we characterize the equilibrium through simple examples, showing the novel interaction between fiscal and interest-rate policies in determining prices and inflation.

4.1 Simple Examples

To understand the determination of prices and interest rates, we now make the simplifying assumptions that the economy is deterministic and that output is constant at $Y_t = Y$.

We discuss how monetary and fiscal policies are set. The central bank chooses the interest rate on reserves and their quantity, i.e. i_t^R and R_t. Let us assume a constant interest-rate policy, $i_t^R = i^R$ for each $t \geq t_0$, and an arbitrary positive sequences of reserves, $R_t > 0$ for each $t \geq t_0$. Consider the flow budget constraint of the central bank (23). First, note that i_t^D is not under direct control of the central bank. If i_t^R and R_t are set, there is still one degree of freedom to specify central bank's policy. We assume that the central bank rebates its profits to the treasury according to the remittances rule $T_t^c = i_{t-1}^D B_t^c - i_{t-1}^R R_t$. It follows from (23) that $B_t^c - R_t$ is constant over time.[10]

We assume that fiscal policy is set by the treasury according to the following rule

$$\frac{T_t}{P_t} = (1-\beta)\tau - (i_{t-1}^B - i_{t-1}^D)\frac{B_{t-1}^h}{P_t} - (i_{t-1}^B - i_{t-1}^R)\frac{R_{t-1}}{P_t}, \tag{25}$$

rebating to households the revenues the government gets by issuing liabilities at a lower rate than the natural rate of interest, in which τ is a positive parameter representing the fiscal policy stance. Given the tax policy (25) and the remittances policy of the central bank, the supply of treasury debt, B_t^g, is determined by the flow budget constraint of the treasury (22).

Specifying the tax policy as in (25) is interesting since it can determine the price level at time t_0. Insert (25) into (24) to obtain

$$\frac{(1 + i_{t_0-1}^B)(R_{t_0-1} + B_{t_0-1}^h)}{P_{t_0}} = \tau, \tag{26}$$

which shows that the price level P_{t_0} is determined by the fiscal policy stance, τ, given initial conditions R_{t_0-1}, $B_{t_0-1}^h$ and $i_{t_0-1}^B$.

Moreover, note that by aggregating the budget constraints of treasury (22) and central bank

[10] On the contrary, had policy been specified in terms of an exogenous path for B_t^c, R_t would have been endogenously determined.

(23), the consolidated government budget constraint can be written as

$$B_t^h + R_t = (1 + i_{t-1}^D)B_{t-1}^h + (1 + i_{t-1}^R)R_{t-1} - T_t.$$

By inserting the tax rule (25), we get

$$\frac{B_t^h + R_t}{P_t} = (1 + i_{t-1}^B)\frac{B_{t-1}^h + R_{t-1}}{P_t} - (1 - \beta)\tau$$

and therefore, using (26), that

$$\frac{B_t^h + R_t}{P_t} = \beta\tau \tag{27}$$

at all times $t \geq t_0$. The real value of government liabilities at any point in time is also proportional to the parameter τ of the fiscal policy rule.

Having determined the price level at time t_0, we now move to determine inflation and interest rates using equations (17) to (19). Recall them and exploit the simplifying assumption stated in this section to write

$$(1 + i_t^D) = \rho(1 + i_t^R) + (1 - \rho)\max\left\{(1 + i_t^B), (1 + i_t^R)\right\}. \tag{28}$$

$$\frac{(1 + i_t^D)}{(1 + i_t^B)} = \left(1 - \frac{U_q\left(Y, q_t\right)}{U_c(Y, q_t)}\right), \tag{29}$$

$$1 + i_t^B = \frac{1}{\beta}\frac{P_{t+1}}{P_t}, \tag{30}$$

with the further restriction that $i_t^B \geq i_t^R \geq 0$. Note that, given that the price level at time t_0 is determined by the fiscal policy, the natural nominal rate of interest, i^B, determines the inflation rate through equilibrium condition (30). However, this rate is not directly controlled by the central bank. To determine the inflation path, one needs to understand how the policy rate passes into the natural nominal rate of interest to affect the inflation rate. For this mechanism, it also key to account for the real value of the total liquidity in the economy, q, and its relationship with fiscal policy. Note that

$$q_t = \frac{D_t + B_t^h}{P_t} \leq \frac{R_t/\rho + B_t^h}{P_t},$$

where for the inequality we have used the collateral constraint $R_t \geq \rho D_t$. We proceed by studying the problem by means of three distinct cases: $\rho = 0, \rho = 1$ and $0 < \rho < 1$.

4.1.1 Case I: $\rho = 0$

We start with the simple case in which there is no reserve/collateral requirement. In Section 3.1, we have already shown that when $\rho = 0$ all money-market rates are equalized, $i_t^B = i_t^R = i_t^D$. Reserves are supplied in a positive amount, even negligible, and held by banks.[11] By moving the policy

[11] See also Woodford (2000) on how the central bank can control money-market rates with a negligible supply of reserves.

rate i^R, the central bank can control in a direct way the rate relevant for the consumption/saving choices of households, i^B. The Neo-Wicksellian paradigm emerges. There are no liquidity premia and deposits will be supplied and demanded to satiate liquidity in the economy.

Concerning deposits, we have assumed that they provide liquidity to households on the grounds that they have some backing from reserves. However, when $\rho = 0$ there is no such backing. There are two possible ways to interpret this case. One is to assume that intermediaries maintain special powers to transform illiquid risk-free securities into liquid deposits, since in any case $i_t^B = i_t^D$. Alternatively, we could assume that deposits do not provide any liquidity services in contrast with treasury bills, which are still backed by the central bank. In this case, the banking model would still implies that $i_t^B = i_t^R = i_t^D$ but, the interest rate on treasury debt could be lower if the supply of this debt is not enough to satiate liquidity. This result is interesting since it shows that the existence of securities carrying a convenience yield is not per se a sufficient condition for breaking the link between the policy rate and the natural nominal rate of interest. What matters is the transmission mechanism of the policy rate through the banking sector.

4.1.2 Case II: "Narrow" banking or central bank digital currency, $\rho = 1$

We now focus on the special case of full backing of deposits by reserves, i.e. $\rho = 1$. This framework is isomorphic to one in which households directly hold accounts at the central bank, a case that mimic 'central bank digital currency'.

Start by observing that

$$q_t = \frac{D_t + B_t^h}{P_t} \leq \frac{R_t + B_t^h}{P_t} = \beta\tau \tag{31}$$

where we have used $R_t \geq D_t$ and (27). From (28), assuming $\rho = 1$ implies $i_t^D = i^R$.

The full characterization of the equilibrium depends critically on the fiscal policy stance, that in our simple example is given by τ.

- When $\tau \geq \bar{q}/\beta$, where $\bar{q}$ is the satiation level of liquidity, then the Neo-Wicksellian equilibrium arises with $i_t^B = i^R = i^D$.

To be in the Neo-Wicksellian equilibrium, liquidity should be enough to satisfy the inequality $q_t \geq \bar{q}$. Using (31), we obtain the restriction on τ for such an equilibrium to exist, $\tau \geq \bar{q}/\beta$. Taxes should be set at a sufficiently high level to satiate liquidity. In this case, using equation (30), the inflation rate is determined at the constant $\Pi = P_{t+1}/P_t = \beta(1 + i^R)$ given the policy rate i^R.

- When $\tau < \bar{q}/\beta$ there is a disconnect between the policy rate and the natural nominal rate of interest. Using equations (29), (30) and (31), we obtain that

$$1 + i^B = \frac{1 + i^R}{1 - \frac{U_q(Y,\beta\tau)}{U_c(Y,\beta\tau)}},$$

$$\Pi = \frac{\beta(1+i^R)}{1-\frac{U_q(Y,\beta\tau)}{U_c(Y,\beta\tau)}}.$$

The spread between the natural nominal rate of interest and the policy rate decreases with τ.[12] A similar negative relationship also arises between the equilibrium inflation rate and the tax rate. The inflation rate is going to be higher than under the Neo-Wicksellian regime.

The final remark is on the irrelevance of reserves for the equilibrium inflation rate and the natural nominal rate of interest, irrespective of whether the policy rate coincides or not with the latter. For a given τ, the size of the central bank's balance sheet, or reserves, does not influence directly the inflation rate, although reserves remain an additional instrument of monetary policy. What matters is the overall government's balance sheet and the fiscal capacity, captured by the parameter τ. Reserves policy does matter, however, as shown in (26), to determine the initial price level, and will matter to pin down the price in each future contingency in a stochastic economy. Nevertheless, they will not matter to determine the expected inflation rate.

4.1.3 Case III: $0 < \rho < 1$

When $0 < \rho < 1$, reserves are going to be effective to determine inflation rate along with the policy rate. Rewrite total liquidity in the economy as

$$\frac{D_t + B_t^h}{P_t} \leq \frac{(\rho^{-1}-1)R_t + R_t + B_t^h}{P_t} = (\rho^{-1}-1)\frac{R_t}{P_t} + \beta\tau,$$

using $R_t \geq \rho D_t$ and (27). As previously derived, the real value of the total government liabilities is determined by the fiscal stance, τ, as

$$\frac{R_t + B_t^h}{P_t} = \beta\tau,$$

at each point in time. When $B_t^h \geq 0$,the previous equation implies that the real value of reserves is bounded by $\beta\tau$ and therefore

$$q_t = \frac{D_t + B_t^h}{P_t} \leq (\rho^{-1}-1)\frac{R_t}{P_t} + \beta\tau \leq \frac{\beta\tau}{\rho} \tag{32}$$

showing now a higher upper bound on the real value of the total liquidity in the economy.[13] Note again that total liquidity is bounded by the fiscal policy stance, τ, but, when $\rho < 1$, the bound is higher than in the case reserves fully back deposits, $\rho = 1$.

When τ is sufficiently low with $\tau \leq \rho\bar{q}/\beta$, reserves do not satiate the economy, but their variation influences the interest rates and the inflation rate. To see this, consider the relevant equilibrium conditions (28) – (30) and combine them under the assumption of no full satiation of

[12] We are assuming that liquidity and consumption are complements in utility.

[13] We are assuming that treasury's debt in the hand of the private sector is non-negative or equivalently that the treasury is not accumulating private assets.

liquidity to obtain

$$(1+i_t^B) = \frac{\rho}{\rho - \frac{U_q(Y,q_t)}{U_c(Y,q_t)}}(1+i^R),$$

$$\Pi_t = \beta(1+i_t^B).$$

The first equation shows that given the policy rate, the real value of liquidity influences the natural nominal rate of interest. An increase in the real value of liquidity, which can come now from two sources (a higher τ or an increase in reserves), as shown in (32), lowers the spread between the natural nominal rate of interest and the policy rate and, therefore, lowers the inflation rate. This simple example shows then a richer interaction between monetary and fiscal policy in determining prices and inflation. We elaborate more on it through an extension of the above model in the next Section.

5 Modeling Nominal Rigidities

We extend the benchmark model presented in the previous analysis to allow for endogenous production and nominal price rigidities. Since the purpose here is to study the role of reserves and interest-rate policies in a liquidity trap, we keep the analysis as simple as possible to compare the new framework with the existing literature, e.g. Eggertsson and Woodford (2003). Firms use labor, supplied by households, to produce goods. We set a simple stochastic structure of the model by considering a preference shock, such to bring the economy to the zero-lower bound.

Preferences of the household are now given by

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[U\left(C_t\right)+V\left(\frac{Q_t}{P_t}\right)-\int_0^1\frac{(H_t(j))^{1+\eta}}{1+\eta}dj\right]\right\},$$

where ξ is the preference shock, C is a Dixit-Stiglitz aggregator of a variety of measure one of goods produced by firms; $H(j)$ is labor of variety j, used by firm of type j; η, with $\eta \geq 0$, denotes the inverse of the Frisch elasticity of labor supply. The function $U(\cdot)$ is concave and increasing; $V(\cdot)$ is concave and non-decreasing, having a satiation level at a finite level $\bar{q} > 0$; $V_q(q_t) = 0$ for $q_t \geq \bar{q}$. Moreover, to have a well-defined demand of liquidity when q_t approaches $\bar{q}$ from below, we assume that $V_{qq}(q_t)$ remains negative in the limit.[14]

From the firms' side, we allow firms to be distributed on a unitary mass on the segment $[0,1]$; firms use labor to produce goods according to the technology $Y_t(j) = H_t(j)$ facing a demand function of the form $Y_t(j) = (P_t(j)/P_t)^{-\theta}Y_t$ in which $P(j)$ is the price of good j and θ is the elasticity of substitution among the variety of goods produced, with $\theta > 1$. Prices are sticky following the Calvo model in which a fraction $1-\alpha$ is allowed to change its prices maximizing the expected present discounted value of its profits. Firms that are not adjusting prices index them to the target Π. As it is standard in the literature, the described framework delivers an

[14] Appendix A.1 presents the model and all derivations in the general case of non-separable utility between goods and real liquidity.

Aggregate-Supply equation of the form

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1+\theta\eta}{\theta-1}} = \frac{F_t}{K_t},$$

in which F_t and K_t are given by

$$F_t = \xi_t U_c(Y_t)Y_t + \alpha\beta E_t\left\{\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1} F_{t+1}\right\},$$

$$K_t = \xi_t Y_t^{1+\eta} + \alpha\beta E_t\left\{\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta(1+\eta)} K_{t+1}\right\}.$$

The complete set of equilibrium conditions is presented in Appendix A.1. In the next Section, we discuss its novel features through a log-linear approximation.

6 A New Framework for Monetary Policy Analysis

In this Section, we present the general model with nominal rigidities through a log-linear approximation around the steady state, to compare it with the benchmark New-Keynesian Neo-Wicksellian model. Details of the log-linear approximation are left to Appendix A.3 while the steady state is analyzed in Appendix A.2.

Aggregate Supply

The aggregate supply (AS) block is the same as in the New-Keynesian framework with

$$\pi_t - \pi = \kappa\hat{Y}_t + \beta E_t(\pi_{t+1} - \pi), \tag{33}$$

for a positive parameter κ; $\pi_t \equiv \ln P_t/P_{t-1}$ and $\pi \equiv \ln \Pi$. Inflation deviations from the target depend positively on output $\hat{Y}_t$ and on the one-period ahead inflation expectations. As a general notation, variables with a hat denote log-deviations of the variable with respect to the steady state.

Aggregate Demand

The aggregate demand block is characterized by the same Euler equation as in the New Keynesian framework. The key difference is that in our framework the relevant nominal rate is the natural nominal rate of interest, i^B, and not the policy rate, see equation (19). In a log-linear approximation, we obtain

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t^B - E_t(\pi_{t+1} - \pi) - r_t^n), \tag{34}$$

in which r_t^n is the natural real rate of interest, which is a function of the preference shock.

Banking Sector and Money Market

From the banking sector we obtain the relationship among the relevant money-market rates

$$\hat{\imath}_t^B = \hat{\imath}_t^R + \frac{1-\nu}{\rho-\nu}(\hat{\imath}_t^B - \hat{\imath}_t^D), \tag{35}$$

as a first-order approximation to equation (17).[15]

The non-negative parameter v is given by the ratio of the marginal utility of liquidity versus that of consumption evaluated at the steady state, $v = V_q/U_c$. In a steady state in which liquidity is fully satiated, $v = 0$.

Equilibrium in the money market, see equation (18), implies that the real value of liquidity is positively related with output and negatively with respect to the liquidity premium through the relationship

$$\hat{q}_t = q_y \hat{Y}_t - q_i(\hat{\imath}_t^B - \hat{\imath}_t^D), \tag{36}$$

in which the elasticity of the demand of liquidity with respect to output is given by $q_y = \sigma_q/\sigma$ and that with respect to the money-market spread is $q_i = \sigma_q(1-v)/v$; σ_q is the intertemporal elasticity of substitution in liquidity, defined as $\sigma_q = -V_q/(V_{qq}q)$.

We briefly discuss two cases.

1. When $v = 0$, liquidity is fully satiated in steady state, $i_t^B = i_t^D = i_t^R$ and the aggregate demand equation becomes

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n), \tag{37}$$

in which the policy rate directly affects the real rate relevant for the consumption/saving choices like in the New Keynesian framework.

2. When $v > 0$, then we combine (35) and (36) into (34) to obtain

$$\hat{Y}_t = (1-\rho^{-1}v)E_t\hat{Y}_{t+1} - \sigma(1-\rho^{-1}v)(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) + q_y^{-1}\rho^{-1}v\hat{q}_t. \tag{38}$$

There are two important novel features shown by the AD equation: first, there is a role for liquidity in affecting the aggregate demand equation (liquidity channel); second, the coefficient $(1-\rho^{-1}v)$ in front of the expected level of output is positive and less than the unitary value, which has implications for the effectiveness of forward guidance.

To gauge the difference with respect to the standard AD equation, solve equation (38) forward

$$\hat{Y}_t = -(1-\rho^{-1}v)\sigma E_t\sum_{T=t}^{\infty}(1-\rho^{-1}v)^{T-t}(\hat{\imath}_T^R - (\pi_{T+1}-\pi) - r_t^n) + q_y^{-1}\rho^{-1}vE_t\sum_{T=t}^{\infty}(1-\rho^{-1}v)^{T-t}\hat{q}_T. \tag{39}$$

[15] Note that a requirement for the equilibrium, discussed in the Appendix, is that $\nu < \rho$.

Not only the current real rate has less impact on output, for given intertemporal elasticity of substitution in consumption σ, but also movements in the expected future rates influence current output less and with a decaying weight. A similar argument applies to the effectiveness of the supply of liquidity in affecting current aggregate demand. Note that a rise in the supply of liquidity has an expansionary effect on output. Consistently with the discussion of Section 4.1.3, movements in liquidity can originate from variations in central-bank reserves or treasury's debt.

7 Optimal Monetary Policy Normalization

In this Section, we analyze how interest rate and reserve policies should be managed when the economy faces shocks that bring it to a liquidity trap. We study optimal policy using a linear-quadratic approach. We proceed through two steps. First, we analyze the model in which lump-sum taxes are available, then we consider the case of only distortionary taxation. As we have already discussed in previous section, fiscal policy and government constraints are key to determine inflation and output.

7.1 With lump-sum taxes

We first consider the case in which the government uses lump-sum taxes. To simplify the derivation of the loss function, we assume that a tax subsidy completely offset the monopolistic distortions in the steady state. Regarding liquidity, we assume that in the steady state the economy is close to the satiation level, i.e. V_q is non zero but of a small order. In the limit in which V_q becomes small, the demand of liquidity will be still of the same form as (36) with parameters $q_y = 0$ and $q_i = -U_c/(V_{qq}q)$.[16]

We show in the Appendix A that under these assumptions the second-order approximation of the utility of the consumers implies the following loss function

$$E_{t_0}\left\{\sum_{t=t_0}^{+\infty}\beta^{t-t_0}\left[\frac{1}{2}\hat{Y}_t^2+\frac{1}{2}\mu\left(\hat{q}_t-q^*\right)^2+\frac{1}{2}\frac{\theta}{\kappa}(\pi_t-\pi)^2\right]\right\}, \tag{40}$$

for some positive parameter μ defined in the Appendix. The policymaker should weigh deviations of output, real liquidity and inflation from their respective targets. The parameter q^* captures the liquidity distortions in the steady state, since liquidity is not fully satiated; q^* is such that $q^* = vq_i$ showing that it is of the same order as v. When liquidity is satiated we have that $\hat{q}_t = q^*$.[17]

We establish the following proposition

Proposition 1 *When lump-sum taxes are available, it is always optimal to increase liquidity (i.e. manage reserves) to reach satiation. This holds independently of the policy rate being or not being*

[16] This corresponds to an economy in which the parameter v is of small order. As discussed before, we assume that as q_t approaches from below the satiation level $\bar{q}$, the limiting value of V_{qq} from below is negative. The latter assumption corresponds to the existence of a well-defined interest-rate semi-elasticity of liquidity demand for values of q_t below the satiation level. See Woodford (2003, ch. 6, p.422) for an analysis in which cash provides utility.

[17] The loss function considers deviations of q_t which are bounded above by $\bar{q}$.

at the zero lower bound.

Proof. We start by considering first the case in which the economy is away from the zero lower bound, $\hat{\imath}_t^R > -\ln(1+i^R)$ in which i^R is the steady-state value of the policy rate. In this case the optimization problem consists in minimizing (40) by choosing $\{\hat{Y}_t, \pi_t, \hat{q}_t\}_{t=t_0}^{\infty}$ subject to the AS equation

$$\pi_t - \pi = \kappa \hat{Y}_t + \beta E_t(\pi_{t+1} - \pi). \tag{41}$$

When feasible, there is no trade-off between stabilizing inflation and output to their targets.[18] In the first best, liquidity should satiate the economy and therefore $\hat{q}_t = q^*$. Absent any constraint on the nominal interest rate, this first best can be achieved, and therefore the central bank can reach all three objectives in (40).

When the policy rate hits the zero-lower bound, the aggregate demand block becomes relevant to evaluate the trade-offs embedded in the optimal policy problem. Moreover, as stated, when v is of a small order, the equilibrium in the market of liquidity is still of the same form as in (36). The functional form of (34) is also unchanged while (35) holds imposing that v is equal to zero in a first-order approximation.

As shown in the Appendix, in the limit $\nu \to 0$, the AD equation (38) becomes

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) + \rho^{-1} q_i^{-1} \sigma \hat{q}_t. \tag{42}$$

The optimal policy problem solves the minimization of the loss function (40) under the constraints (41) and (42) by choosing the sequences $\{y_t, \pi_t, \hat{\imath}_t^R, \hat{q}_t\}_{t=t_0}^{\infty}$ given the inequality constraints $\hat{\imath}_t^R \geq -\ln(1+i^R)$ and $\hat{q}_t \leq q^*$. First-order conditions of the optimal policy problem are given by:

$$\hat{Y}_t - \kappa \varphi_{1,t} + \varphi_{2,t} - \beta^{-1} \varphi_{2,t-1} = 0 \tag{43}$$

$$\frac{\theta}{\kappa}(\pi_t - \pi) + \varphi_{1,t} - \varphi_{1,t-1} - \sigma \beta^{-1} \varphi_{2,t-1} = 0 \tag{44}$$

$$\mu(\hat{q}_t - q^*) - \rho^{-1} q_i^{-1} \sigma \varphi_{2,t} + \varphi_{3,t} = 0 \tag{45}$$

with the following Kuhn-Tucker conditions

$$\varphi_{2,t}(\hat{\imath}_t^R + \ln(1+i^R)) = 0$$

$$\varphi_{3,t}(\hat{q}_t - q^*) = 0$$

with $\hat{\imath}_t^R \geq -\ln(1+i^R)$, $q_t \leq q^*$, $\varphi_{2,t} \geq 0$, $\varphi_{3,t} \geq 0$, and $\varphi_{1,t}$ is the Lagrange multiplier associated with (41), $\varphi_{2,t}$ is the one associated with (42), and $\varphi_{3,t}$ with constraint $\hat{q}_t \leq q^*$.[19]

[18] A similar loss function can be derived when goods and liquidity are not separable in utility, as shown in Appendix *A*, in which case the target for output shifts with liquidity. However, in this case, this is exactly the same shifter that affects the AS equation, so that there is no trade-off between stabilizing inflation at the target and an appropriately-defined output gap.

[19] We are analyzing optimal policy from a timeless perspective, see Woodford (2003).

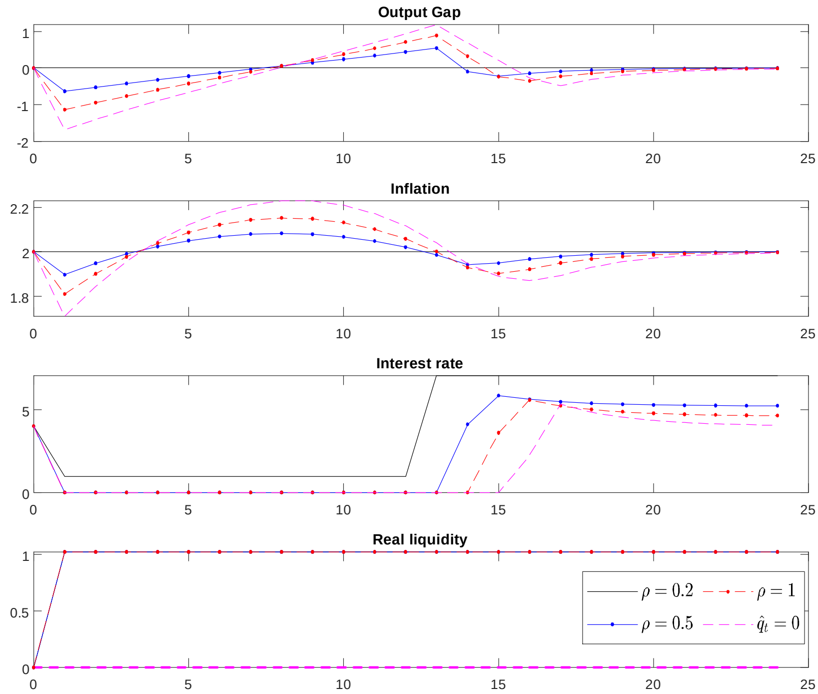

Figure 1: Optimal policy responses following a negative shock to the natural rate of interest, which brings it to -4% at annual rates for 12 quarters. Output gap is in %, Inflation and interest rates are in % and at annual rates. Real liquidity is in % deviations from the steady state. When $\hat{q}_t = 0$, real liquidity does not move. In the other cases, real liquidity moves optimally and the parameter ρ, the reserve-to-deposit ratio, is calibrated to the values $0.2, 0.5$ and 1, respectively.

The first-order conditions (43)–(45) show formally the result that when the zero-lower bound constraint is never binding, i.e. $\varphi_{2,t} = 0$ at all times, the first best can be achieved and liquidity is set to satiate the economy, see equation (45) which is validated by $\hat{q}_t = q^*$ and $\varphi_{3,t} = 0$. We now prove that this result holds even when the economy is at the zero lower bound. Consider first-order condition (45), which can be written as

$$\mu(\hat{q}_t - q^*) = \frac{\sigma}{\rho q_i}\varphi_{2,t} - \varphi_{3,t}. \tag{46}$$

To prove that constraint $q_t \leq q^*$ is going to bind at the zero-lower bound, suppose by contradiction that $\varphi_{3,t} = 0$ and $\hat{q}_t < q^*$, then since $\varphi_{2,t} > 0$ equation (46) implies $\hat{q}_t > q^*$, which contradicts the assumption made. Therefore $\varphi_{3,t} > 0$ and $\hat{q}_t = q^*$. ■

Our proposition establishes that, no matter whether the economy is or is not at the zero-lower bound, it is always optimal to increase liquidity to reach satiation and mimic the Neo-Wicksellian equilibrium in which there are no spreads in the money market. We note that reserves are still a tool of policy but, once they are supplied to satiate the economy, any increase becomes irrelevant for the equilibrium inflation and output, and this irrelevance result is again independent of whether the policy rate is or is not at the zero-lower bound. Clearly, reserves and tax policy, the drivers of q, become relevant for output and inflation determination, when satiation is not reached.

Figure 1 shows the equilibrium outcome for inflation, output, interest rate and the path of real liquidity in an economy which is hit by a negative shock to the natural rate of interest that pushes it to negative values, at -4% for twelve quarters. The calibration is discussed in Appendix C. The Figure displays the optimal policy when liquidity satiates the economy for the following values $\{0.2, 0.5, 1\}$ of the parameter ρ, and compares it with the optimal policy constrained on no variations of liquidity. In the latter case, it is optimal to set the policy rate to stay at the zero-lower bound for 15 quarters, 3 quarters beyond the duration of the shock. As in Eggertsson and Woodford (2003), inflation overshoots the target at the time the natural real rate returns to normal values while the economy experiences a boom after the initial contraction.[20] With liquidity set optimally, the economy recovers early, fluctuations of output and inflation with respect to their targets are smaller, the stay at the zero-lower bound is shorter. The increase in liquidity pushes aggregate demand up, requiring less need of forward guidance and, therefore, lower overshoot of the inflation rate with respect to the target. When ρ is small, the zero lower bound constraint is even overcome by the increase in real liquidity, still the nominal interest rate falls to fully accommodate the drop in the natural rate of interest.

The AD equation (42) is useful to get intuition for the latter result. The increase in liquidity can offset the negative shock to the natural real rate of interest.[21] When $\hat{q}_t = q^* = vq_i$ in (42), the drop in the natural rate of interest r_t^n is lowered by a factor $\rho^{-1}\nu$. The offsetting force is higher the lower ρ is. Indeed, considering a lower ρ, the equilibrium in money markets

$$\hat{\imath}_t^B - \hat{\imath}_t^D = \rho(\hat{\imath}_t^B - \hat{\imath}_t^R),$$

requires the spread in the liquidity market, $\hat{\imath}^B - \hat{\imath}^D$, see equation (36), to be only a fraction of the spread between the natural nominal rate of interest, $\hat{\imath}^B$, and the policy rate, $\hat{\imath}^R$. Therefore, for the same increase in liquidity to reduce the spread in the liquidity market $(\hat{\imath}^B - \hat{\imath}^D)$, a larger drop in the natural nominal rate of interest, $\hat{\imath}^B$, with respect to the policy rate, $\hat{\imath}^R$, is required, which is therefore more stimulative on aggregate demand.

In the Appendix, we also discuss the case in which liquidity and goods are complement in utility, showing that the first-order condition (45) depends also on the lagged value of the Lagrange multiplier φ_2. In general, it does not change the main message from this Section, that, when lump-sum taxes are available, liquidity should be set to satiate the economy. Starting from a situation in which it is sub-optimally supplied, its increase can shorten the stay at the zero lower bound and alleviate the costs in terms of inflation and output for the economy. Next Section considers the case when only distortionary taxes are available, a feature that implies an optimal supply of liquidity below full satiation and costs of varying liquidity.

7.2 With only distortionary taxation

We now consider the case in which lump-sum taxes are not available. For simplicity we focus on the case in which deposits are fully backed by reserves, i.e. $D_t \leq R_t$. This implies that the deposit rate coincides with the policy rate, $i^D = i^R$, but not necessarily with the natural nominal rate of

[20] With a sub-optimal policy, inflation and output will fall by more.

[21] Note that since $q^* = q_i\nu$, the calibration of the parameter q_i is ininfluential for the results.

interest. Moreover, the overall liquidity, q_t, coincides with the consolidated government liquidity in the hands of the households, i.e.

$$q_t = \frac{R_t + B_t^h}{P_t}.$$

It follows that the supply of liquidity, q_t, is now directly related to fiscal policy through the intertemporal budget constraint of the government:

$$\frac{(1+i_{t-1}^R)}{\Pi_t} q_{t-1} = E_t \sum_{T=t}^{\infty} R_{t,T} \left[(\tau_T Y_T - Tr_T) + \frac{i_T^B - i_T^R}{1+i_T^B} q_T \right]. \tag{47}$$

In equilibrium the outstanding real value of government debt should be equal to the present discounted value of revenues from taxes less exogenous government transfers, Tr, the first term on the right-hand side of (47), and "seigniorage", the last term on the right-hand side of (47).[22] The latter represents the revenues the government gets by issuing debt at lower cost with respect to the natural nominal rate of interest, i^R rather than i^B, because government debt provides liquidity benefits.

Supplying liquidity entails now distortions in terms of taxation that can affect welfare. In this context, we study the optimal policy problem using again linear-quadratic approximations following the method expounded in Benigno and Woodford (2003). The approximation is taken around an optimal steady state. Details are in Appendix B.

In the steady state, the optimal supply of liquidity is determined by the following condition:

$$V_q(q) = -\frac{\phi_q}{1+\phi_q}(V_{qq}(q)q) \tag{48}$$

in which ϕ_q is a non-negative Lagrange multiplier associated to the constraint (47). The Lagrange multiplier is zero when lump-sum taxes are available, therefore $V_q = 0$. It is optimal to supply liquidity up to the point of satiating the economy and driving the marginal benefit of liquidity to zero, consistently with the analysis of Section 7.1. With distortionary taxation, instead, it is optimal to reduce liquidity below its satiation level, consistently with a well-behaved liquidity demand in the limiting case $V_q \to 0$. As mentioned, V_{qq} should remain negative as q approaches the satiation level from below. Under this assumption, equation (48) shows that, in the steady state, the optimal supply of liquidity is below the satiation level.

We now discuss the optimal policy problem. In Appendix B, we show that a quadratic approximation of the loss function has the following form

$$L_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2}\lambda_y y_t^2 + \frac{1}{2}\lambda_\pi(\pi_t - \pi)^2 + \frac{1}{2}\lambda_q \hat{q}_t^2 \right\} \tag{49}$$

for positive parameters λ_y, λ_π and λ_q. The policymaker should care about deviations of an appropriately-defined output gap, y, inflation, π, and real liquidity, $\hat{q}$, from their steady state

[22] See also Benigno and Woodford (2003) and Eggertsson and Woodford (2004) for similar specification in which they allow for real (lump-sum) government transfers Tr, exogenously given.

values. The optimal policy problem is subject to a modified aggregate supply equation

$$(\pi_t - \pi) = \kappa[y + \psi(\tilde{\tau}_t - \tilde{\tau}_t^*)] + \beta E_t(\pi_{t+1} - \pi),$$

which accounts for the time-varying effects of the distortionary tax on firm's revenue, for a positive parameter ψ discussed in the Appendix. The variable $\tilde{\tau}_t$ is defined as $\tilde{\tau}_t = \tau_t - \tau$ in which τ is the steady state value of the tax rate, whereas $\tilde{\tau}_t^*$ represents a combination of the shocks such that when $\tilde{\tau}_t$ achieves that value, output and inflation can be stabilized at their respective targets.

Under the assumption (on the collateral requirement) $\rho = 1$, the AD equation (38) simplifies to:

$$y_t = (1 - v)E_t y_{t+1} - \sigma(1 - v)(\hat{i}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) + \sigma\sigma_q^{-1} v\hat{q}_t, \quad (50)$$

for an appropriately defined natural real rate of interest, r_t^n.

An additional constraint of the optimal policy problem is the first-order approximation of (47), which can be written as

$$\hat{q}_{t-1} - (\pi_t - \pi) - \sigma^{-1} y_t + (\hat{i}_{t-1}^R - r_{t-1}^n) = -f_t + E_t \sum_{T=t}^{\infty} \beta^{T-t}[b_y y_t + b_\tau(\tilde{\tau}_t - \tilde{\tau}_t^*) + b_q \hat{q}_t], \quad (51)$$

for parameters b_y, b_τ and b_q defined in Appendix B; the variable f, as in Eggertsson and Woodford (2004), captures the "fiscal stress," which measures the extent to which full stabilization of output, inflation and liquidity at their targets implied by the loss function (49), is not compatible with the intertemporal budget constraint of the government. When $f_t = 0$ at all times, it is feasible to reach all three targets provided the movements in the natural real rate of interest, r^n, do not imply violation of the zero-lower bound for the nominal interest rate.[23] Indeed, when all targets in (49) are achieved $\hat{i}_t^R = r_t^n$ all times. When the natural real rate of interest, r^n, falls substantially, there could be violation of the zero-lower bound for the policy rate, i^R, therefore a trade-off emerges between stabilizing the relevant variables.

We consider, therefore, how policy should be set when the only constraint on the full stabilization of the relevant variables in (49) is given by the existence of the zero-lower bound on the policy rate.[24]

We consider a shock that brings the natural real rate of interest, r^n, from the steady-state level of 2% to -4% at annual rates for twelve quarters. Given that the steady-state policy rate is set at 4% accounting for a 2% inflation target, the shock to the natural rate of interest could be fully accommodated only if the policy rate could fall at -2%. The zero-lower bound prevents this fall and creates an interesting trade-off among stabilizing the relevant macroeconomic variables.

[23] In this reasoning, we are considering zero values for the initial conditions $\hat{q}_{t_0-1}$, $\hat{i}_{t_0-1}^R$,$r_{t_0-1}^n$. We could also allow for different initial conditions requiring, in the case, f_{t_0} to adjust appropriately.

[24] Note that when the optimal supply of liquidity is close to eliminate the distortions in the money market, i.e. $v \to 0$, the problem collapses to exactly that analyzed by Eggertsson and Woodford (2004) in the standard New-Keynesian model with absence of lump-sum taxes. Indeed, the AD equation boils down to the standard one in which liquidity does not affect, directly, aggregate demand. The AS equation is already the same as in their framework, as well as the parameters λ_y and λ_π in the loss function (49). With $v \to 0$, λ_q goes instead to zero as well as b_q in the

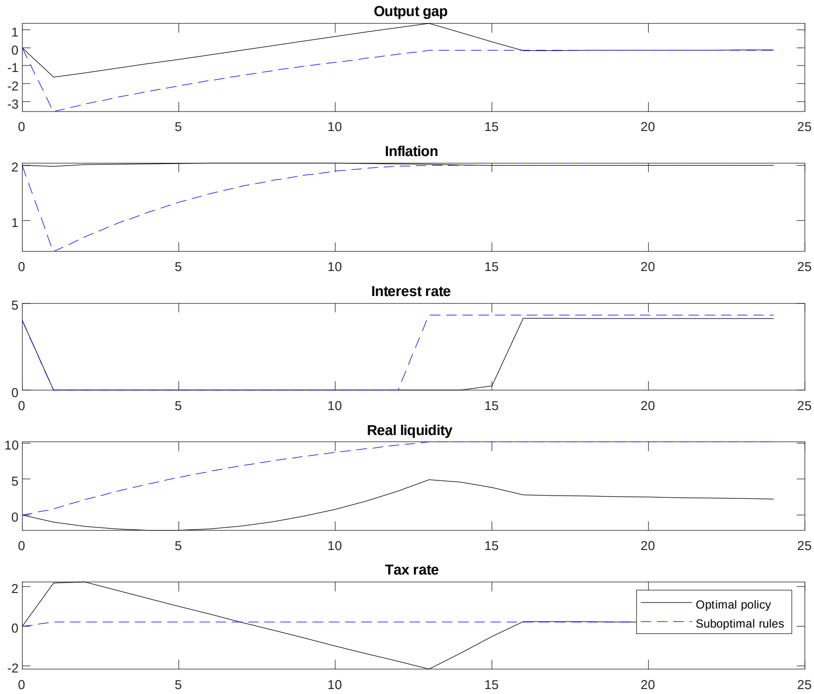

Figure 2: Comparison between optimal policy when taxes are distortionary and sub-optimal rules. Impulse responses following a negative shock to the natural rate of interest, which brings it to -4% at annual rates for 12 quarters. Output gap is in %, Inflation and interest rates are in % and at annual rates. Real liquidity is in % deviations from the steady state. Tax rate is in percentage points and in deviations from the steady-state value.

In Figure 2 we compare the optimal policy with sub-optimal policies in which (i) the central bank sets inflation at the target, i.e. $\pi_t = \pi$, whenever it is feasible, otherwise it sets the policy rate to zero and (ii) the fiscal authority keeps the tax gap $\tilde{\tau}_t - \tilde{\tau}_t^*$ at a level that it expects to maintain indefinitely without violating the intertemporal government budget constraint; that is, an expected path of the tax gap such that $E_t(\tilde{\tau}_T - \tilde{\tau}_T^*) = \tilde{\tau}_t - \tilde{\tau}_t^*$ for all $T \geq t$ is consistent with (51).[25]

The Figure shows the costs of the sub-optimal policy with respect to the optimal in terms of contraction in the output gap and inflation below the target. The liftoff of the policy rate from the zero-lower bound occurs exactly at the time in which the shock vanishes. Optimal policy, instead, succeeds to stabilize inflation while keeping moderate variations in the output gap.

There are three important features of the optimal policy that we discuss. First, in line with the literature, optimal policy requires a stay at the zero-lower bound longer with respect to the duration of the shock. In the Figure, the interest rate remains at the zero-lower bound for two additional quarters. What is interesting to note is that the liquidity channel in the AD equation does not imply a shorter stay at the zero-lower bound. We are going to elaborate more on this soon. The second result, as well in line with Eggertsson and Woodford (2004), is the use of the tax policy to stabilize the economy. Note that in the case of disturbances not severe enough for the zero lower bound to bind, the tax gap, $\tilde{\tau} - \tilde{\tau}^*$ would not move at all. Instead, in the case of a

constraint (51); b_y and b_τ also approach same values as in Eggertsson and Woodford (2004).

[25] Appendix C provides details on the calibration used.

larger shock, optimal policy involves raising tax rates during the liquidity trap, while committing to cut them after the shock has vanished. As in Eggertsson and Woodford (2004), the tax rate operates through the AS equation and its increase acts to push up inflation at the early stage of the liquidity trap, when the deflationary pressures are stronger, while putting downward pressure when the shock vanishes. The last feature of optimal policy is the path followed by liquidity. As it is shown in the Figure, optimal policy requires a lower increase in liquidity with respect to the sub-optimal policy. The main reason for this counter-intuitive result is in the success of the optimal policy in stabilizing inflation and output. Indeed, the fall in the output gap under the sub-optimal rules produces lower revenues from taxes, which lead to a large accumulation of public liabilities. An interesting feature of the path of liquidity under optimal policy is its rise towards the end of the liquidity trap with a pick just after the shock ends and around the time of the liftoff of the policy rate. The withdrawal starts already before the liftoff of rates and then proceeds very slowly. The return to the initial optimal steady state happens only after a very long period of time, not shown in the picture. In contrast to Eggertsson and Woodford (2004), public liabilities do return to the initial steady state in this framework, but at a very slow pace so that the increase in the public sector balance sheet remains for long, and definitely longer than the return to normal of conventional policies. Such a slow return of liabilities to their initial steady state is also responsible of their limited rise at the beginning of the trap, since in any case deviations from the initial steady state are costly according to the loss function (49).

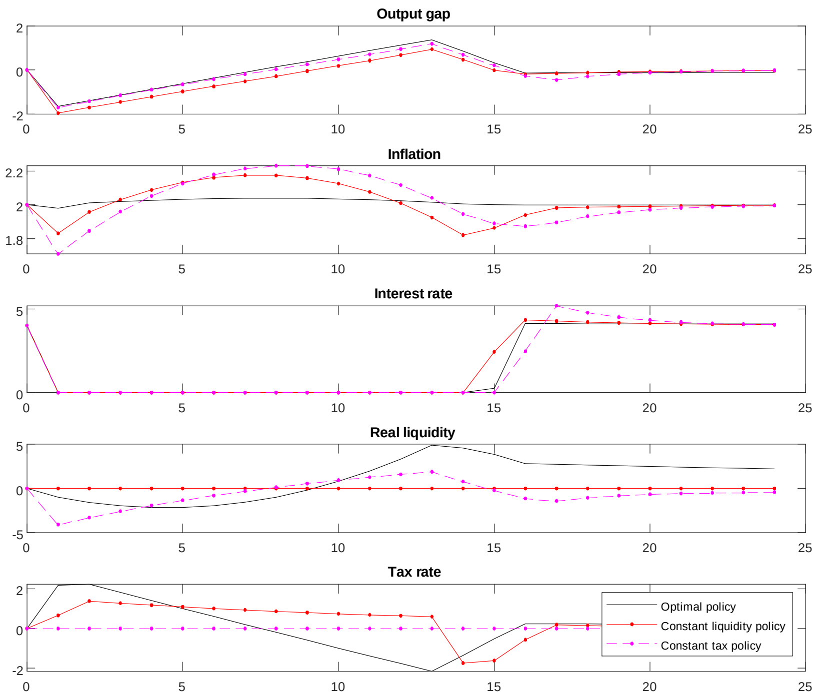

Figure 3: Comparison between optimal policy when taxes are distortionary, optimal policies when liquidity is kept constant and optimal policy when the expected tax gap is constant. Impulse responses following a negative shock to the natural rate of interest, which brings it to -4% at annual rates for 12 quarters. Output gap is in %, Inflation and interest rates are in % and at annual rates. Real liquidity is in % deviations from the steady state. Tax rate is in percentage points and in deviations from the steady-state value.

We now elaborate more on the characteristics of the optimal policy by comparing it with two other sub-optimal policies, but of a special kind. In the first policy, that we label "constant tax policy", we assume that the fiscal authority is able to move the tax gap $\tilde{\tau}_t - \tilde{\tau}_t^*$ in a way that it expects to maintain it in the future, i.e. $E_t(\tilde{\tau}_T - \tilde{\tau}_T^*) = \tilde{\tau}_t - \tilde{\tau}_t^*$ for all $T \geq t$ and consistently with the intertemporal budget constraint (51). At the same time, we assume that the monetary authority minimizes the loss function (49) under the same constraints as in the general optimal policy problem, but considering as given the path of the fiscal variables $\tilde{\tau} - \tilde{\tau}^*$ and the fact that the intertemporal solvency of the government is ensured by the tax policy. In this way, we aim to characterize the equilibrium outcome when the tax gap acting on the AS equation is expected to be constant. This combination of policies is going to emphasize the role of the interest-rate policy in managing the shock.

In the second policy, that we label "constant liquidity policy", fiscal policy moves the tax gap to fully stabilize liquidity at the steady state while the monetary authority minimizes the loss function (49) under the same constraints as in the general optimal policy problem, but considering as given the path of the fiscal variables $\tilde{\tau} - \tilde{\tau}^*$ and the fact that the intertemporal solvency of the government is ensured by the tax policy. In this way, we aim at characterizing how optimal policy would cope with the shock when liquidity is not used.

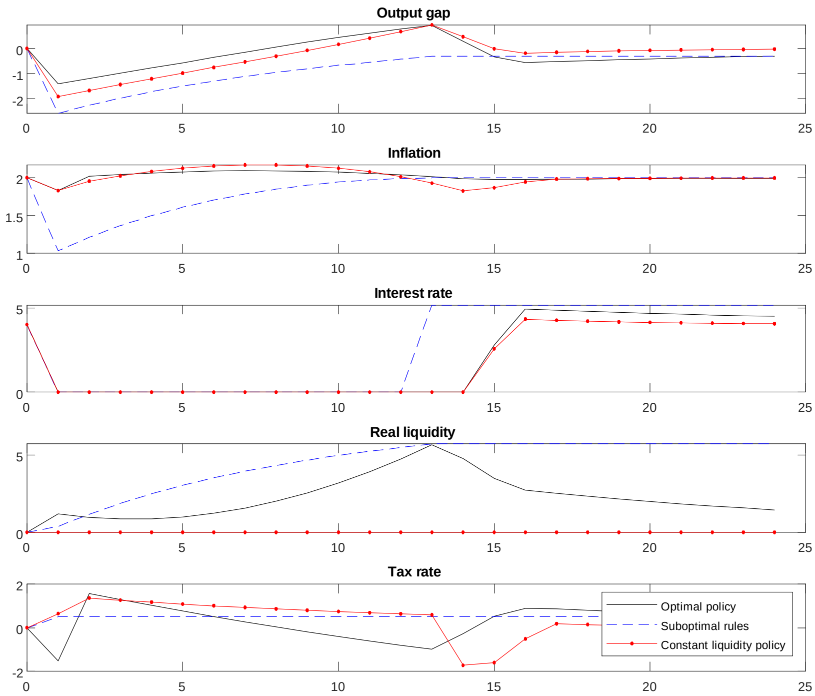

Figure 4: Comparison between optimal policy when taxes are distortionary, optimal policy with constant liquidity and sub-optimal rules, when money-market spread is high. Impulse responses following a negative shock to the natural rate of interest, which brings it to -4% at annual rates for 12 quarters. Output gap is in %, Inflation and interest rates are in % and at annual rates. Real liquidity is in % deviations from the steady state. Tax rate is in percentage points and in deviations from the steady-state value.

Figure 3 compares the three policies. Let's start from the comparison between optimal policy and the "constant tax policy". Figure 3 shows the importance of moving the tax rate under the optimal policy to stabilize inflation at the target, avoiding the disinflation at the onset of the trap and the overshooting at the end. Under the "constant tax policy", the interest rate is forced to stay one quarter longer at the zero lower bound to be able to mitigate, alone, the disinflationary pressure of the shock and, after its liftoff, it also overshoots the long-run level for a while. The comparison between optimal policy and the "constant liquidity policy" shows again the imperfect inflation stabilization of the latter with respect to the former but less than with a "constant tax policy", since taxes here move in the right direction. There are, however, larger losses in terms of output gap. What is interesting is that the liftoff of the policy rate occurs at the same time as under optimal policy, but with a larger first hike that is responsible of pushing down inflation below the target.

Higher spread in money markets

In the previous analysis, the parameter ν, which captures the spread in money markets between liquid and illiquid securities, has been calibrated to the average, in the sample 1971-2005, of the spread between the three-month commercial paper rate and the same maturity treasury bills rate for the US economy. This value corresponds to a sixty basis point spreads at annual rates. According to the AD equation (50) a one-percent once-and-for-all increase in liquidity raises output, everything else being equal, by $\sigma\sigma_q^{-1}\upsilon$ percentage points. Since $\sigma = 0.5$, $\sigma_q = 0.2$ and $\upsilon = 0.0015$, it corresponds to an increase of output of just 0.00375 percentage points. Figure 4 considers instead a 4% spread, more in line with what observed at the onset of the 2007-2008 financial crisis through several indicators in money markets, and compares the optimal policy with the sub-optimal rules of Figure 1 and the "constant liquidity policy". The important difference in the optimal policy, with respect to previous Figures, is in the path of liquidity, which now increases even at the beginning of the trap supported by a fall in the tax rate rather than the hike of Figure 1. Then, liquidity gradually increases to peak at the time in which the shock vanishes. Its withdrawal occurs before the liftoff of rates and, after that, in a very gradual way. The increase in liquidity is now larger but still moderate, reflecting two contrasting forces with respect to a lower calibration of the parameter ν. On the one side, a higher value of ν implies a stronger liquidity channel in the AD equation (50); on the other side, the relative cost of varying liquidity rises in the loss function. The ratio λ_q/λ_π is now 6.5 times higher than in the previous case.

Figure 4 confirms that there are some output costs (line "constant liquidity policy"), but moderate in size, of a non-active use of liquidity with respect to the optimal policy. This result might depend on the fact that the benefits of an increase in liquidity are still relatively small, captured by the combination of parameters $\sigma\sigma_q^{-1}\upsilon$ in the AD equation (50), with respect to the costs of raising liquidity and with respect to the benefits of stabilizing the output gap. As it is common in the literature, our calibration implies a high costs of inflation stabilization with the ratio λ_y/λ_π taking a value of 0.0021. Indeed, in all Figures optimal policy is geared towards stabilizing inflation at the target rather than closing the output gap.

Larger weight on output-gap stabilization

Consider now an extreme case in which the ratio λ_q/λ_π is fifty times higher than the one

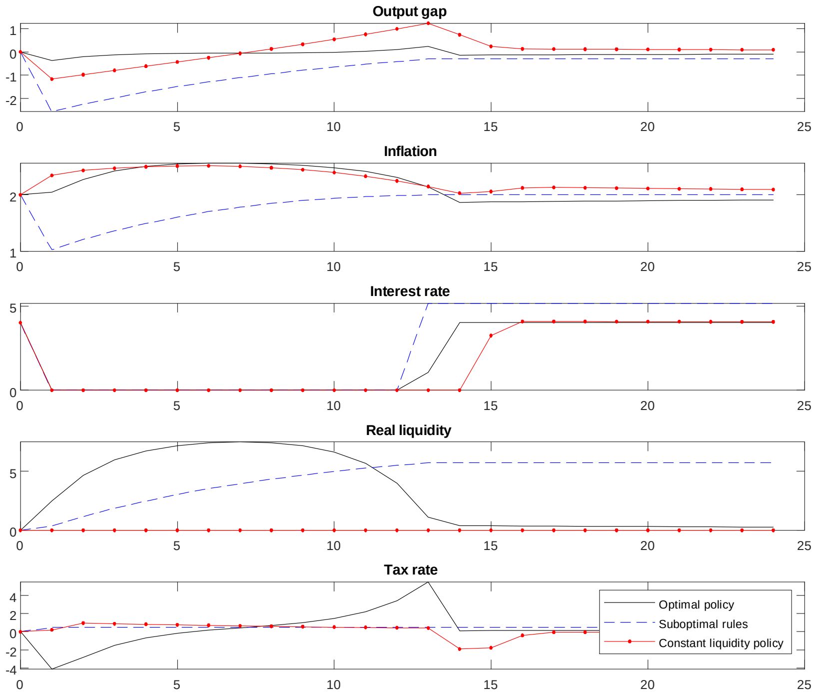

Figure 5: Comparison between optimal policy when taxes are distortionary, optimal policy with constant liquidity and sub-optimal rules, when λ_y/λ_π is fifty times higher than the benchmark calibration and money-market spread is high. Impulse responses following a negative shock to the natural rate of interest, which brings it to -4% at annual rates for 12 quarters. Output gap is in %, Inflation and interest rates are in % and at annual rates. Real liquidity is in % deviations from the steady state. Tax rate is in percentage points and in deviations from the steady-state value.

calibrated in Figure 4, maintaining a higher value for v. Figure 5 shows the impulse responses in this case. There are several important features to underline. The optimal policy now stabilize more the output gap and the difference with respect to the "constant liquidity policy" is larger. Optimal stabilization happens with a sizeable rise of liquidity, which picks in the middle of the liquidity trap. The withdrawal starts earlier than the end of the trap and liquidity is completely absorbed once policy rates are normalized. Liquidity injection lowers the stay at the zero lower bound, two quarter less than when liquidity is not used and exactly at the time in which the shock vanishes. There is no need of communicating a longer stay at zero interest rate, in contrast with other cases. Instead, in a "constant liquidity policy" a longer stay compensates for the non-use of liquidity, but it does not help much to stabilize the output gap. Note also that inflation, which is less costly in terms of welfare, stays above the target for all the duration of the trap.

Summing up

The analysis would then suggest that when the aim is to stabilize inflation at the target, this could be better achieved by a combination of an appropriate interest-rate policy, which implies a longer stay at the zero-lower bound than the duration of the shock, and a tax policy in which taxes are raised at the beginning of the trap and lowered at the end. An active liquidity policy is optimal but with marginal benefits on the output gap, except for when spreads in money market are higher. Liquidity should peak at the end of the trap and withdrawn before the liftoff of the policy rates, but only gradually. Instead, when the aim is to reduce the fluctuations in the output gap, liquidity policies are more desirable and the policy rate does not need to stay longer at the zero-lower bound than the duration of the shock. Liquidity should peak during the middle of the liquidity trap and then withdraw at a faster pace to be completely absorbed as policy rates are normalized.

8 Conclusion

We have proposed a new framework for monetary policy analysis that encompasses, as a special case, the Neo-Wicksellian paradigm. The nominal interest rate relevant for saving/consumption decisions can only be controlled by the central bank's simultaneous targeting of the interest rate on reserves and their quantity. The Neo-Wicksellian model is nested when liquidity is fully satiated.

The new framework shows an important interaction between monetary and fiscal policy in controlling inflation and output. We have applied it to the study of optimal policy in a liquidity trap, showing the role of tax policy and liquidity in influencing the optimal response of the policy rate to a natural real interest rate shock.

In this version, we have focused on the liquidity channel as the key mechanism through which reserve policies are effective. In subsequent research it would be interesting to study the interplay between the credit channel, as in Benigno and Benigno (2021), and the liquidity one emphasized in this work.

Finally, the model has been kept as simple as possible for tractability and to compare it with existing analysis in the literature. It requires thorough extension in order to provide realistic quantitative analysis.

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A Model with lump-sum taxation

We describe in a compact way the general model of Section 5. Moreover, we also generalize the assumption of that Section using the following household's preference specification

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[U(C_t, q_t) - \int_0^1 \frac{(H_t(j))^{1+\eta}}{1+\eta} dj \right],$$

which has non-separability between consumption goods and real liquidity.

A.1 Equilibrium conditions

Starting from the household problem, we have the following equilibrium conditions derived from the optimal consumption/saving decisions:

$$E_t \left\{ \beta \frac{\xi_{t+1} U_c(Y_{t+1}, q_{t+1})}{\xi_t U_c(Y_t, q_t)} \frac{P_t}{P_{t+1}} \right\} = \frac{1}{1+i_t^B} \tag{A.1}$$

$$1 = \frac{U_q\left(Y_t, q_t\right)}{U_c\left(Y_t, q_t\right)} + \frac{1+i_t^D}{1+i_t^B}. \tag{A.2}$$

From the intermediary sector we obtain that the spread between the deposit rate, the policy rate and the natural nominal rate of interest is

$$(1+i_t^D) = \rho(1+i_t^R) + (1-\rho)\max((1+i_t^B), (1+i_t^R)), \tag{A.3}$$

with $i_t^R \geq 0$. Turning to the production side, we have the following AS equation derived from the problem of the final-good producer:

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha} \right)^{\frac{1+\theta\eta}{\theta-1}} = \frac{F_t}{K_t}, \tag{A.4}$$

in which F_t and J_t are given by

$$F_t = (1-\tau)\xi_t U_c\left(Y_t, q_t\right) Y_t + \alpha\beta E_t \left\{ \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1} F_{t+1} \right\}, \tag{A.5}$$

$$K_t = \mu_\theta \xi_t Y_t^{1+\eta} + \alpha\beta E_t \left\{ \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta(1+\eta)} K_{t+1} \right\}, \tag{A.6}$$

with $\mu_\theta \equiv \theta/(\theta-1)$. Note that we have defined

$$q_t = \frac{D_t + B_t^h}{P_t} \tag{A.7}$$

and that

$$D_t \leq \frac{1}{\rho} R_t. \tag{A.8}$$

Finally, the consolidated budget constraint of the government implies that

$$\frac{B_t^h + R_t}{P_t} = \frac{(1 + i_{t-1}^D)}{\Pi_t} \frac{B_{t-1}^h}{P_{t-1}} + \frac{(1 + i_{t-1}^R)}{\Pi_t} \frac{R_{t-1}}{P_{t-1}} - \frac{T_t}{P_t} - \tau Y_t. \tag{A.9}$$

The above set of nine equilibrium conditions determine the equilibrium allocation for the following variables, $Y_t, i_t^B, i_t^D, i_t^R, q_t, D_t/P_t, B_t^h/P_t, \Pi_t, K_t, F_t, R_t/P_t, T_t$, considering also that a transversality condition holds with respect to the overall government liabilities as a mirror image of the transversality condition of households.

A.2 Steady state

We consider a steady state in which the interest rate on reserves is constant at $i_t^R = i^R$ and the real value of reserves and government liabilities are also constant. First, note that in the steady state

$$1 + i^B = \frac{\Pi}{\beta}$$

$$\frac{1 + i^D}{1 + i^B} = 1 - \frac{U_q(Y, q)}{U_c(Y, q)}$$

$$1 + i^D = \rho(1 + i^R) + (1 - \rho) \max\left(1 + i^B, 1 + i^R\right).$$

In what follows, we define

$$\nu \equiv \frac{U_q(Y, q)}{U_c(Y, q)}$$

with $\nu \geq 0$; $\nu = 0$ when there is full satiation of liquidity. Therefore, we can also write

$$\frac{1 + i^B}{1 + i^R} = \frac{\rho}{\rho - \nu}$$

and

$$\Pi = \frac{\beta\rho}{\rho - \nu}(1 + i^R).$$

Note that it should be the case that $\nu \leq \rho$.

The following equation determine the steady-state values of Y

$$\frac{1 - \tau}{\mu_\theta} U_c(Y, q) = H_l = Y^\eta,$$

given q. When lump-sum taxes are available, we assume that τ is set to eliminate the monopolistic distortions, i.e. $\tau = 1 - \mu_\theta$.

A.3 Approximation of equilibrium conditions

Considering first the AD demand side of the model, we have the following first-order approximations of the equilibrium conditions (A.1), (A.2) and (A.3)

$$E_t\hat{Y}_{t+1} = \hat{Y}_t + \sigma(\hat{\imath}_t^B - E_t(\pi_{t+1} - \pi) - r_t^n) - \sigma\chi(\hat{q}_t - E_t\hat{q}_{t+1}) \tag{A.10}$$

$$\hat{q}_t = q_y\hat{Y}_t - q_i(\hat{\imath}_t^B - \hat{\imath}_t^D) \tag{A.11}$$

$$(1-v)\hat{\imath}_t^D = (\rho - v)\hat{\imath}_t^R + (1-\rho)\hat{\imath}_t^B \tag{A.12}$$

in which we have defined variables with hat as the log-deviations of the respective variables with respect to the steady state; $\pi_t \equiv \ln P_{t+1}/P_t$, $r_t^n = \hat{\xi}_t - E_t\hat{\xi}_{t+1}$ $\pi \equiv \ln \Pi$, $\sigma \equiv -U_c/(U_{cc}Y)$, $\sigma_q \equiv -U_q/(U_{qq}q)$, $\chi \equiv U_{cq}q/U_c$, $\delta \equiv Y/q$, $q_y \equiv (v\sigma^{-1}+\delta\chi)/(v(\sigma_q^{-1}+\chi))$, $q_i = (1-v)/(v(\sigma_q^{-1}+\chi))$. Note that we have defined $q_t = [(\rho^{-1}-1)R_t + R_t + B_t^h]/P_t$.

We now turn to the approximation of the AS equation, given by (A.4) to (A.6). We obtain

$$\pi_t - \pi = \kappa\left(\hat{Y}_t - \frac{\chi}{(\sigma^{-1}+\eta)}\hat{q}_t\right) + \beta E_t(\pi_{t+1} - \pi), \tag{A.13}$$

with

$$k \equiv \frac{(1-\alpha)(1-\alpha\beta)(\sigma^{-1}+\eta)}{\alpha(1+\theta\eta)}.$$

Equations (A.10), (A.11), (A.12), (A.13), are four equations in the following six stochastic sequences $\left\{\pi_t, \hat{Y}_t, \hat{\imath}_t^B, \hat{\imath}_t^R, \hat{\imath}_t^D, \hat{q}_t\right\}$, given the stochastic process $\{r_t^n\}$.

A.4 Derivation of the loss function

Consider the expected utility of the consumers

$$U_{t_0} = E_{t_0}\left\{\sum_{t=t_0}^{+\infty}\beta^{t-t_0}\xi_t\left[(U(Y_t,q_t) - H(Y_t)\Delta_t)\right]\right\},$$

in which

$$H(Y_t) = \frac{Y_t^{1+\eta}}{1+\eta},$$

and

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{-\theta(1+\eta)} dj.$$

Note the following second-order Taylor approximation around the steady state

$$\begin{aligned}
U(Y_t, q_t) &= U(Y, q) + U_c(Y_t - Y) + U_q(q_t - q) + \frac{1}{2}U_{qq}(q_t - q)^2 \\
&\quad + U_{cq}(q_t - q)(Y_t - Y) + \frac{1}{2}U_{cc}(Y_t - Y)^2 + \mathcal{O}(||\xi||^3) \\
&= U(Y, q) + U_c Y \hat{Y}_t + U_q q \hat{q}_t + \frac{1}{2}U_c Y \left(1 - \frac{1}{\sigma}\right)\hat{Y}_t^2 + \\
&\quad + U_{cq} q Y \hat{q}_t \hat{Y}_t + \frac{1}{2}U_q q \left(1 - \frac{1}{\sigma_q}\right)\hat{q}_t^2 + \mathcal{O}(||\xi||^3) \qquad \text{(A.14)}
\end{aligned}$$

where $\mathcal{O}(||\xi||^3)$ collects terms of order higher than the second and where, in the second line, we have used the following approximation:

$$\left(\frac{Y_t - Y}{Y}\right) = \hat{Y}_t + \frac{1}{2}\hat{Y}_t^2 + \mathcal{O}(||\xi||^3),$$

and similarly for $q_t - q$. We have also used the definition $\sigma = -U_c/U_{cc}Y$ and $\sigma_q = -U_q/U_{qq}q$, which are evaluated at the steady state.

Similarly, we can write

$$\begin{aligned}
H(Y_t)\Delta_t &= H(Y) + H_l(Y_t - Y) + \frac{1}{2}H_{ll}(Y_t - Y)^2 + \\
&\quad + H(Y)(\Delta_t - 1) + \mathcal{O}(||\xi||^3) \\
&= H(Y) + H_l Y \hat{Y}_t + \frac{1}{2}H_l Y (1 + \eta)\hat{Y}_t^2 + \\
&\quad + H(Y)(\Delta_t - 1) + \mathcal{O}(||\xi||^3), \qquad \text{(A.15)}
\end{aligned}$$

having used the definition $\eta \equiv H_{ll}Y/H_l$ and already used the fact that the expansion of Δ_t is of second-order magnitude, as it will be shown.

Combining (A.14) and (A.15), we obtain

$$\begin{aligned}
U(Y_t, q_t) - \Delta_t H(Y_t) &= U(Y, q) - \Delta H(Y) + U_c \hat{Y}_t + \frac{1}{2}U_c Y \left(1 - \frac{1}{\sigma}\right)\hat{Y}_t^2 \\
&\quad + U_q q \hat{q}_t + \frac{1}{2}U_q q \left(1 - \frac{1}{\sigma_q}\right)\hat{q}_t^2 + \chi U_c Y \hat{q}_t \hat{Y}_t - H_l Y \hat{Y}_t \\
&\quad - \frac{1}{2}H_l Y (1 + \eta)\hat{Y}_t^2 - H(Y)(\Delta_t - 1) + \mathcal{O}(||\xi||^3), \qquad \text{(A.16)}
\end{aligned}$$

and therefore

$$\begin{aligned}
U(Y_t, q_t) - \Delta_t H(Y_t) &= U_c Y \left[\hat{Y}_t + \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\hat{Y}_t^2 + \chi \hat{q}_t \hat{Y}_t\right] + U_q q \left[\hat{q}_t + \frac{1}{2}\left(1 - \frac{1}{\sigma_q}\right)\hat{q}_t^2\right] + \\
&\quad - H_l Y \left[\hat{Y}_t + \frac{1}{2}(1 + \eta)\hat{Y}_t^2\right] - H(Y)(\Delta_t - 1) + \mathcal{O}(||\xi||^3),
\end{aligned}$$

by neglecting constant terms.

We consider a steady-state in which output is at the efficient level, and therefore $U_c Y = H_l Y$

while we consider that liquidity is just close to be fully satiated, meaning that U_q is positive but small and of the same order as $\mathcal{O}(||\xi||)$. We assume, however, a well-defined demand of liquidity in the limit, by assuming that U_{qq} remains non-negligible. Therefore $U_q \cdot \sigma_q^{-1}$ for small v converges to $-U_{qq}q$. Assuming $U_cY = H_lY$ and $U_q = \mathcal{O}(||\xi||)$, we can simplify the above expression by also neglecting all terms independent of policy to

$$U(Y_t, q_t) - \Delta_t H(Y_t) = U_cY\left[-\frac{\hat{\Delta}_t}{1+\eta} - \frac{\sigma^{-1}+\eta}{2}\hat{Y}_t^2 + \chi\hat{q}_t\hat{Y}_t + \frac{1}{2}\frac{U_{qq}q^2}{U_cY}\hat{q}_t^2 + \frac{U_qq}{U_cY}\hat{q}_t\right] + \mathcal{O}(||\xi||^3),$$

Use now (A.11), which boils down to

$$-\frac{U_{qq}q}{U_c}\hat{q}_t = \delta\hat{Y}_t - (\hat{\imath}_t^B - \hat{\imath}_t^D),$$

when v is of order $\mathcal{O}(||\xi||)$. Therefore, $q_i = -U_c/U_{qq}q$ and $q_y = \delta\chi q_i$ and we can write:

$$U(Y_t, q_t) - \Delta_t H(L_t) = U_cY\left[-\frac{\hat{\Delta}_t}{1+\eta} - \frac{\sigma^{-1}+\eta}{2}\hat{Y}_t^2 + \chi\hat{q}_t\hat{Y}_t - \frac{1}{2}\frac{1}{\delta q_i}\hat{q}_t^2 + \frac{v}{\delta}\hat{q}_t\right] + \mathcal{O}(||\xi||^3),$$

which can be written as

$$\begin{aligned} U(Y_t, q_t) - \Delta_t H(L_t) \quad &= \quad U_cY\left[-\frac{\hat{\Delta}_t}{1+\eta} - \frac{\sigma^{-1}+\eta}{2}(\hat{Y}_t - \hat{Y}_t^*)^2 - \frac{1}{2}z\,(\hat{q}_t - q^*)^2\right] + \\ &\quad +\mathcal{O}(||\xi||^3) \end{aligned} \tag{A.17}$$

in which

$$\hat{Y}_t^* = \frac{\chi}{\sigma^{-1}+\eta}\hat{q}_t$$

$$q^* = \frac{v}{z\delta}$$

and

$$z \equiv \frac{1}{q_i\delta} - \frac{\chi^2}{\sigma^{-1}+\eta}.$$

Consider now the following approximation

$$\begin{aligned} \xi_t[U(Y_t, q_t) - \Delta_t H(Y_t)] \quad &= \quad (\xi_t - \xi)[U(Y, q) - \Delta H(Y)] + \xi[U(Y_t, q_t) - \Delta_t H(Y_t)]^{2^{nd}} + \\ &\quad\; (\xi_t - \xi)[U_c(Y_t - Y) + U_q(q_t - q) - H_l(Y_t - Y)] + \mathcal{O}(||\xi||^3) \end{aligned}$$

in which the second addendum on the right-hand side of the first line is meant to represent the second-order approximation found above. Given the steady-state assumptions made above, it follows that

$$\xi_t[U(Y_t, q_t) - \Delta_t H(Y_t)] = [U(Y_t, q_t) - \Delta_t H(Y_t)]^{2^{nd}} + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

Recall the law of motion of Δ_t

$$\Delta_t \equiv \alpha \Delta_{t-1} \left(\frac{\Pi_t}{\Pi}\right)^{\theta(1+\eta)} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta(1+\eta)}{\theta-1}} \tag{A.18}$$

and take a second-order Taylor expansion around the steady state in which $\Delta_t = 1$ and $\Pi_t = \Pi$ to obtain

$$\hat{\Delta}_t = \alpha \hat{\Delta}_{t-1} + \frac{\alpha}{1-\alpha}\theta(1+\eta)(1+\eta\theta)\frac{(\pi_t-\pi)^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3).$$

Now note that

$$\hat{\Delta}_t = \alpha^{t-t_0+1}\hat{\Delta}_{t_0-1} + \frac{1}{2}\frac{\alpha\theta}{(1-\alpha)}(1+\eta)(1+\eta\theta)\sum_{s=t_0}^{t}\alpha^{t-s}\,(\pi_s-\pi)^2 + \mathcal{O}(||\xi||^3)$$

and therefore

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{\Delta}_t = \frac{1}{2}\frac{\alpha\theta(1+\eta)(1+\eta\theta)}{(1-\alpha)(1-\alpha\beta)}\sum_{t=t_0}^{\infty}\beta^{t-t_0}(\pi_t-\pi)^2 + \mathcal{O}(||\xi||^3), \tag{A.19}$$

neglecting initial condition $\hat{\Delta}_{t_0-1}$.

Combining and inserting this result into the expected discounted value of the approximated utility flow, (A.17), we finally obtain

$$\begin{aligned} U_{t_0} \quad = \quad & -(\sigma^{-1}+\eta)Y^{1+\eta}E_{t_0}\left\{\sum_{t=t_0}^{+\infty}\beta^{t-t_0}\left[\frac{1}{2}(\hat{Y}_t-\hat{Y}_t^*)^2 + \frac{\mu}{2}\left(\hat{q}_t-q^*\right)^2 + \frac{1}{2}\frac{\theta}{\kappa}(\pi_t-\pi)^2\right]\right\} \\ & +\mathcal{O}(||\xi||^3), \end{aligned}$$

where

$$\mu \equiv \frac{z}{\sigma^{-1}+\eta}$$

from which loss function (40) follows.

A.5 Optimal Policy

First consider the constraints of the optimal policy problem (A.10) – (A.12) under the assumption of a small v. They respectively imply that

$$E_t\hat{Y}_{t+1} = \hat{Y}_t + \sigma(\hat{\imath}_t^B - E_t(\pi_{t+1}-\pi) - r_t^n) - \sigma\chi(\hat{q}_t - E_t\hat{q}_{t+1}) \tag{A.20}$$

$$\hat{q}_t = q_y\hat{Y}_t - q_i(\hat{\imath}_t^B - \hat{\imath}_t^D) \tag{A.21}$$

$$\hat{\imath}_t^D = \rho\hat{\imath}_t^R + (1-\rho)\hat{\imath}_t^B. \tag{A.22}$$

Combining equation (A.22) with (A.21), we can write

$$\hat{\imath}_t^B = \hat{\imath}_t^R + \frac{q_y}{q_i\rho}\hat{Y}_t - \frac{1}{q_i\rho}\hat{q}_t,$$

which we can be substituted into (A.20) to get

$$E_t\hat{Y}_{t+1} = \left(1 + \frac{\sigma q_y}{q_i\rho}\right)\hat{Y}_t - \frac{\sigma}{q_i\rho}\hat{q}_t + \sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) - \sigma\chi(\hat{q}_t - E_t\hat{q}_{t+1}).$$

We now define

$$y_t \equiv \hat{Y}_t - \hat{Y}_t^* = \hat{Y}_t - \frac{\chi}{\sigma^{-1} + \eta}\hat{q}_t$$

to re-write the above equation as

$$E_t y_{t+1} = \left(1 + \frac{q_y\sigma}{q_i\rho}\right) y_t + \left(\frac{q_y\sigma}{q_i\rho}\frac{\chi}{\sigma^{-1} + \eta} - \frac{\sigma}{q_i\rho}\right)\hat{q}_t + \sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) - \frac{\eta\chi}{\sigma^{-1} + \eta}(\hat{q}_t - E_t\hat{q}_{t+1}).$$

Note that

$$q_y = \delta\chi q_i$$

and therefore we can write:

$$E_t y_{t+1} = \left(1 + \frac{\sigma\delta\chi}{\rho}\right) y_t - \frac{\sigma\delta z}{\rho}\hat{q}_t + \sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) - \frac{\sigma\eta\chi}{1 + \sigma\eta}(\hat{q}_t - E_t\hat{q}_{t+1}).$$

The Lagrangian of the optimal policy problem is given by

$$\begin{aligned}\mathcal{L}_{t_0} \quad = \quad & E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}\left(y_t^2 + \mu(q_t - q^*)^2 + \frac{\theta}{\kappa}(\pi_t - \pi)^2\right) + \varphi_{1,t}\left(\pi_t - \pi - \kappa y_t - \beta(\pi_{t+1} - \pi)\right) + \right.\right. \\ & +\varphi_{2,t}\left(-y_{t+1} + \left(1 + \frac{\sigma\delta\chi}{\rho}\right) y_t - \frac{\sigma\delta z}{\rho}\hat{q}_t + \sigma(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) - \frac{\sigma\eta\chi}{1 + \sigma\eta}(\hat{q}_t - E_t\hat{q}_{t+1})\right) + \\ & \left.\left. +\varphi_{3,t}(q_t - q^*)\right]\right\}\end{aligned}$$

The first-order conditions with respect to y_t, π_t and $\hat{q}_t$ are given respectively by

$$y_t - \kappa\varphi_{1,t} + \left(1 + \rho^{-1}\sigma\delta\chi\right)\varphi_{2,t} - \beta^{-1}\varphi_{2,t-1} = 0$$

$$\frac{\theta}{\kappa}(\pi_t - \pi) + \varphi_{1,t} - \varphi_{1,t-1} - \sigma\beta^{-1}\varphi_{2,t-1} = 0$$

$$\mu(q_t - q^*) - \rho^{-1}\sigma\delta z\varphi_{2,t} - \frac{\eta\chi\sigma}{1 + \sigma\eta}(\varphi_{2,t} - \beta^{-1}\varphi_{2,t-1}) + \varphi_{3,t} = 0$$

with the following Kuhn-Tucker conditions

$$\varphi_t(\hat{\imath}_t^R + \ln(1 + i^R)) = 0$$

$$\gamma_t(q_t - q^*) = 0$$

with $\hat{\imath}_t^R \geq -\ln(1+i^R)$, $q_t \leq q^*$, $\varphi_t \geq 0$ and $\gamma_t \geq 0$.

A nice way to characterize the optimal policy in this case, as in Eggertsson and Woodford (2003), is to define a time-varying price target p_t^*. Optimal policy sets the target variable $\tilde{p}_t$, defined as a combination of the price index and the output gap $\tilde{p}_t \equiv \theta p_t + \hat{Y}_t$, at p_t^* whenever it is feasible, otherwise the policy rate is set to zero. The target price p_t^* adjusts following the law of motion

$$p_{t+1}^* = \pi + p_t^* + \frac{(1+\kappa\sigma)}{\beta\varsigma}(p_t^* - \tilde{p}_t) - \frac{1}{\beta\varsigma}(p_{t-1}^* - \tilde{p}_{t-1}),$$

in which we have defined $\varsigma \equiv \left(1+\rho^{-1}\sigma\delta\chi\right)$. We can also write the last first-order condition of the optimal policy problem as

$$\mu(\hat{q}_t - q^*) = \frac{\sigma}{\rho q_i \varsigma}(p_t^* - \tilde{p}_t) - \frac{\eta\chi}{\beta(1+\sigma\eta)\varsigma}(p_{t-1}^* - \tilde{p}_{t-1}) + \varphi_{3,t}. \tag{A.23}$$

By inspection, we can derive the following results. If the economy is out of the liquidity traps for two periods, the current and previous one, then it is optimal to set $\hat{q}_t = q^*$. Indeed in this case, $\tilde{p}_t = p_t^*$ at t and $t-1$, and $\varphi_{3,t} = 0$ in (A.23). Instead, if the economy is just one period out of the trap, it is optimal to set $\hat{q}_t < q^*$ since $\tilde{p}_t = p_t^*$ at t and $\tilde{p}_t < p_t^*$ at $t-1$. During the trap, instead, whether the deviation $(p_t^* - \tilde{p}_t)$ dominates $(p_{t-1}^* - \tilde{p}_{t-1})$ in (A.23) depends on parameter values. With a small degree of complementarity between goods and liquidity χ, the deviation $(p_t^* - \tilde{p}_t)$ dominates and, therefore, it will be optimal to set $\hat{q}_t = q^*$.

B Optimal policy with distortionary taxation

In this Appendix, we consider the optimal policy problem when there are no lump-sum taxes. We assume separable utility of the form:

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[\frac{C_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} + V(q_t) - \int_0^1 \frac{(H_t(j))^{1+\eta}}{1+\eta}dj\right].$$

and that deposits are fully backed by reserves, $R_t \geq D_t$. Note that in equilibrium we can write the above utility as

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[\frac{Y_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} + V(q_t) - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t,\right]$$

given the definition of Δ_t given in (A.18). Under the assumption that $\rho = 1$ for which the deposit rate is equal to the policy rate, i.e. $i_t^D = i_t^R$, the flow budget constraint of the government is

$$\frac{B_t^h + R_t}{P_t} = \frac{(1+i_{t-1}^R)}{\Pi_t}\frac{B_{t-1}^h + R_{t-1}}{P_{t-1}} - (\tau_t Y_t - Tr_t)$$

in which G is an exogenous transfer. The above flow budget constraint can be written as

$$q_t = \frac{(1+i_{t-1}^R)}{\Pi_t} q_{t-1} - (\tau_t Y_t - Tr_t)$$

and therefore

$$\frac{(1+i_t^R)q_t}{1+i_t^B} + \frac{i_t^B - i_t^R}{1+i_t^B} q_t = \frac{(1+i_{t-1}^R)}{\Pi_t} q_{t-1} - (\tau_t Y_t - Tr_t).$$

Since

$$\frac{1}{1+i_t^B} = \beta E_t \left(\frac{U_c(Y_{t+1})\xi_{t+1}}{U_c(Y_t)\xi_t} \frac{1}{\Pi_{t+1}} \right)$$

we can iterate the equation forward using the transversality condition to obtain

$$\frac{(1+i_{t-1}^R)}{\Pi_t} U_c(Y_t)\xi_t q_{t-1} = E_t \sum_{T=t}^{\infty} \beta^{T-t} U_c(Y_T)\xi_T \left[(\tau_T Y_T - Tr_T) + \frac{i_T^B - i_T^R}{1+i_T^B} q_T \right]$$

and therefore

$$\frac{(1+i_{t-1}^R)}{\Pi_t} U_c(Y_t)\xi_t q_{t-1} = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[U_c(Y_T)\xi_T(\tau_T Y_T - Tr_T) + \xi_T V_q(q_T) q_T \right]$$

having used

$$1 = \frac{V_q\left(q_t\right)}{U_c\left(Y_t\right)} + \frac{1+i_t^R}{1+i_t^B}.$$

B.1 The deterministic steady state

Here we compute the steady state of the optimal monetary and fiscal policy problem in a deterministic problem in which the exogenous disturbances ξ_t and G_t takes constant values $\xi = 1$ and $G_t = G$, for all $t \geq t_0$.

We thus consider the problem of maximizing

$$U_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{Y_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} + V(q_t) - \frac{Y_t^{1+\eta}}{1+\eta} \Delta_t \right) \tag{B.24}$$

subject to the constraints

$$K_t p \left(\frac{\Pi_t}{\Pi} \right)^{\frac{1+\eta\theta}{\theta-1}} = F_t, \tag{B.25}$$

$$F_t = (1-\tau_t) Y_t^{1-\sigma^{-1}} + \alpha\beta \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\theta-1} F_{t+1}, \tag{B.26}$$

$$K_t = \mu_\theta Y_t^{1+\eta} + \alpha\beta \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\theta(1+\eta)} K_{t+1}, \tag{B.27}$$

$$W_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\tau_t Y_t^{1-\sigma^{-1}} - Tr_t Y_t^{-\sigma^{-1}} + V_q(q_t) q_t), \tag{B.28}$$

$$\Delta_t = \alpha\Delta_{t-1}\left(\frac{\Pi_t}{\Pi}\right)^{\theta(1+\eta)} + (1-\alpha)p\left(\frac{\Pi_t}{\Pi}\right)^{-\frac{\theta(1+\eta)}{1-\theta}}, \tag{B.29}$$

given specified initial conditions $\Delta_{t_0-1}, F_{t_0}, K_{t_0}, W_{t_0}$ where we have defined

$$p\left(\frac{\Pi_t}{\Pi}\right) \equiv \left(\frac{1-\alpha(\Pi_t/\Pi)^{\theta-1}}{1-\alpha}\right).$$

We introduce Lagrange multipliers $\phi_{1,t}$ through $\phi_{5,t}$ corresponding to constraints (B.25) through (B.29) respectively. Note that the lagrange multiplier ϕ_4 is constant. We also introduce multipliers dated t_0 corresponding to the constraints implied by the initial conditions F_{t_0}, K_{t_0}; the latter multipliers are normalized in such a way that the first-order conditions take the same form at date t_0 as at all later dates. The first-order conditions of the maximization problem are then the following. The one with respect to Y_t is

$$\begin{aligned} Y_t^{-\sigma^{-1}} - \Delta_t Y_t^{\eta} - (1-\tau_t)(1-\sigma^{-1})Y_t^{-\sigma^{-1}}\phi_{2,t} - (1+\eta)\mu_\theta Y_t^{\eta}\phi_{3,t} + \tau_t Y_t^{-\sigma^{-1}}\phi_4 \\ -\sigma^{-1}Y_t^{-\sigma^{-1}}\tau_t\phi_4 + \sigma^{-1}Y_t^{-\sigma^{-1}-1}Tr_t\phi_4 = 0; \end{aligned} \tag{B.30}$$

that with respect to Δ_t is

$$-\frac{Y_t^{1+\eta}}{1+\eta} + \phi_{5,t} - \alpha\beta\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta(1+\eta)}\phi_{5,t+1} = 0; \tag{B.31}$$

that with respect to Π_t is

$$\begin{aligned} \frac{1+\theta\eta}{\theta-1}p\left(\frac{\Pi_t}{\Pi}\right)^{\frac{1+\theta\eta}{\theta-1}-1} p_\pi\left(\frac{\Pi_t}{\Pi}\right)K_t\phi_{1,t} - \alpha(\theta-1)\left(\frac{\Pi_t}{\Pi}\right)^{\theta-2}\frac{F_t}{\Pi}\phi_{2,t-1} \\ -\theta(1+\eta)\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta(1+\eta)-1}\frac{K_t}{\Pi}\phi_{3,t-1}+ \\ -\theta(1+\eta)\alpha\Delta_{t-1}\left(\frac{\Pi_t}{\Pi}\right)^{\theta(1+\eta)-1}\frac{1}{\Pi}\phi_{5,t} - \frac{\theta(1+\eta)}{\theta-1}(1-\alpha)p\left(\frac{\Pi_t}{\Pi}\right)^{\frac{(1+\eta\theta)}{\theta-1}} p_\pi\left(\frac{\Pi_t}{\Pi}\right)\phi_{5t} = 0; \end{aligned} \tag{B.32}$$

that with respect to τ_t is

$$\phi_{2,t} + \phi_4 = 0; \tag{B.33}$$

that with respect to F_t is

$$-\phi_{1,t} + \phi_{2,t} - \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}\phi_{2,t-1} = 0; \tag{B.34}$$

that with respect to K_t is

$$p\left(\frac{\Pi_t}{\Pi}\right)^{\frac{1+\eta\theta}{\theta-1}}\phi_{1t} + \phi_{3t} - \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta(1+\eta)}\phi_{3,t-1} = 0; \tag{B.35}$$

that with respect to q_t is

$$V_q(q_t) = -\phi_4(V_q(q_t) + V_{qq}(q_t)q_t). \tag{B.36}$$

We search for a solution to these first-order conditions in which $\Pi_t = \Pi$, $\Delta_t = \Delta$, $Y_t = Y$, $\tau_t = \tau$, and $q_t = q$ at all times. A steady-state solution of this kind also requires that the Lagrange multipliers take constant values. We furthermore conjecture the existence of a solution in which $\Delta = 1$, $p(\cdot) = 1$, $p_\pi(\cdot) = -(\theta-1)\alpha/[(1-\alpha)\Pi]$, and $K = F$. Using these substitutions, we find that (the steady-state version of) each of the first-order conditions (B.30) – (B.36) is satisfied if the steady-state values satisfy

$$1 - Y_t^{\eta+\sigma^{-1}} = [(1-\sigma^{-1}) + \sigma^{-1}g - (1+\eta)\mu_\theta Y_t^{\eta+\sigma^{-1}}]\phi_2, \tag{B.37}$$

$$(1-\alpha\beta)\phi_5 = \frac{Y^{1+\eta}}{1+\eta},$$

$$\phi_4 = -\phi_2, \tag{B.38}$$

$$\phi_1 = (1-\alpha)\phi_2,$$

$$\phi_3 = -\phi_2,$$

$$V_q(q) = -\phi_4(V_q(q) + V_{qq}(q)q). \tag{B.39}$$

We have defined $g = Tr/Y$. Similarly, (the steady-state versions of) the constraints (B.25) – (B.29) are satisfied if

$$\frac{(1-\tau)}{\mu_\theta} = Y^{\eta+\sigma^{-1}}, \tag{B.40}$$

$$(\tau Y - gY) + V_q(q)qY^{\sigma^{-1}} = (1-\beta)q\frac{(1+i^R)}{\Pi}, \tag{B.41}$$

$$F = K = (1-\alpha\beta)^{-1}\mu_\theta Y^{1+\eta},$$

$$W = \frac{Y^{-\sigma^{-1}}(1+i^R)q}{\Pi}.$$

We can use (B.40) and (B.38) into (B.37) to obtain

$$\phi_4 = \frac{1 - \frac{(1-\bar{\tau})}{\mu_\theta}}{(1+\eta)(1-\tau) - (1-\sigma^{-1}) - \sigma^{-1}g} \tag{B.42}$$

which is positive provided $\tau < (\eta + \sigma^{-1}(1-g))/(1+\eta)$. Note that the multiplier ϕ_4 is function of τ and that output is a decreasing function of τ using (B.40). Moreover

$$\frac{(1+i^R)}{\Pi} = \frac{1+i^R}{1+i^B}\frac{(1+i^B)}{\Pi} = \left(1 - V_q(q)Y^{\sigma^{-1}}\right)\frac{1}{\beta}.$$

Therefore we can write (B.41) as

$$(\tau - g)Y(\tau) + \frac{V_q(q)q}{\beta Y(\tau)^{-\sigma^{-1}}} = \frac{(1-\beta)}{\beta}q$$

which togheter with

$$V_q(q) = -\phi_4(\tau)(V_q(q) + V_{qq}(q)q).$$

represents a set of two equations to solve for q and τ. Note that we should require $\tau < (\eta+\sigma^{-1}(1-g))/(1+\eta)$ and $V_q(q)Y(\tau)^{\sigma^{-1}} < (1-\beta)$. The former restriction for q to be positive, the latter for τ to be positive.

The remaining equations can then be solved (uniquely) for $K = F$ and for W.

B.2 A second-order approximation to utility

We use the previous steps to find the second-order approximation of the utility

$$U_{t_0} = E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[\frac{Y_t^{1-\sigma^{-1}}-1}{1-\sigma^{-1}} + V(q_t) - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t\right]\right\}. \tag{B.43}$$

Note that

$$\begin{aligned}\xi_t\left[\frac{Y_t^{1-\sigma^{-1}}-1}{1-\sigma^{-1}} + V(q_t) - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t\right] &= U_cY\left[\hat{Y}_t + \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\hat{Y}_t^2\right] + U_cY\hat{Y}_t\hat{\xi}_t - \\ &+V_qq\left[\hat{q}_t + \frac{1}{2}\left(1-\frac{1}{\sigma_q}\right)\hat{q}_t^2\right] + V_qq\hat{q}_t\hat{\xi}_t - H_lY\hat{Y}_t\hat{\xi}_t \\ &-H_lY\left[\hat{Y}_t + \frac{1}{2}(1+\eta)\hat{Y}_t^2\right] - H(Y)(\Delta_t - 1) + \mathcal{O}(||\xi||^3),\end{aligned}$$

Note that in the steady $H_l = (1-\Phi)U_c$ where

$$\Phi \equiv 1 - \frac{(1-\bar{\tau})}{\mu_\theta} < 1$$

measures the inefficiency of steady-state output $\bar{Y}$. We can then write

$$\begin{aligned}\xi_t\left[\frac{Y_t^{1-\rho}-1}{1-\rho} + V(q_t) - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t\right] &= U_cY\left[\Phi\hat{Y}_t + \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\hat{Y}_t^2\right] + \Phi U_cY\hat{Y}_t\hat{\xi}_t - \\ &+V_qq\left[\hat{q}_t + \frac{1}{2}\left(1-\frac{1}{\sigma_q}\right)\hat{q}_t^2\right] + V_qq\hat{q}_t\hat{\xi}_t \\ &-\frac{1}{2}(1-\Phi)U_cY(1+\eta)\hat{Y}_t^2 - \frac{(1-\Phi)}{1+\eta}U_cY(\Delta_t - 1) \\ &+\mathcal{O}(||\xi||^3),\end{aligned}$$

We can therefore write

$$\begin{aligned}U_{t_0} &= U_cY\cdot E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\Phi\widehat{Y}_t - \frac{1}{2}u_{yy}\widehat{Y}_t^2 + \Phi\widehat{Y}_t\hat{\xi}_t - u_\Delta\hat{\Delta}_t + \\ &+v\delta^{-1}\left[\hat{q}_t(1+\hat{\xi}_t) + \frac{1}{2}\left(1-\sigma_q^{-1}\right)\hat{q}_t^2\right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3),\end{aligned} \tag{B.44}$$

where

$$u_{yy} \equiv -(1-\sigma^{-1}) + (1-\Phi)(1+\eta),$$
$$u_{\Delta} \equiv \frac{(1-\Phi)}{1+\eta}.$$

We substitute (A.19) into (B.44) to obtain

$$U_{t_0} = \overline{Y}\bar{u}_c \cdot E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\Phi \widehat{Y}_t - \frac{1}{2} u_{yy} \widehat{Y}_t^2 + \Phi \widehat{Y}_t \hat{\xi}_t - \frac{1}{2} u_{\pi} (\pi_t - \pi)^2 + $$
$$+ v\delta^{-1} \left[\hat{q}_t (1 + \hat{\xi}_t) + \frac{1}{2} \left(1 - \sigma_q^{-1}\right) \hat{q}_t^2) \right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3),$$

where we have further defined

$$\kappa \equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \frac{(\eta + \sigma^{-1})}{(1+\eta\theta)}, \qquad u_{\pi} \equiv \frac{\theta(\eta+\sigma^{-1})(1-\Phi)}{\kappa}.$$

We can also write it as

$$U_{t_0} = \overline{Y}\bar{u}_c \cdot E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [a_x' x_t - \frac{1}{2} x_t' A_x x_t - \frac{1}{2} x_t' A_{\varepsilon} \varepsilon_t - \frac{1}{2} a_{\pi} (\pi_t - \pi)^2] +$$
$$+ \text{t.i.p.} + \mathcal{O}(||\xi||^3),$$

where we have used the following definitions

$$x_t \equiv \begin{bmatrix} \hat{\tau}_t \\ \hat{Y}_t \\ \hat{q}_t \end{bmatrix},$$

$$\varepsilon_t \equiv \begin{bmatrix} \hat{\xi}_t \\ \hat{G}_t \end{bmatrix}$$

$$a_x' \equiv \begin{bmatrix} 0 & \Phi & v\delta^{-1} \end{bmatrix}$$

$$A_x \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(1-\sigma^{-1}) + (1-\Phi)(1+\eta) & 0 \\ 0 & 0 & -v\delta^{-1}\left(1-\sigma_q^{-1}\right) \end{bmatrix}$$

$$A_{\varepsilon} \equiv \begin{bmatrix} 0 & 0 \\ -\Phi & 0 \\ -v\delta^{-1} & 0 \end{bmatrix}$$

$$a_{\pi} \equiv \frac{\theta(\eta+\sigma^{-1})(1-\Phi)}{\kappa}.$$

B.3 A second-order approximation of the AS equation

We follow Benigno and Woodford (2003) to obtain that a second-order approximation of the AS equation is:

$$\begin{aligned} V_t &= \frac{1-\alpha}{\alpha}\frac{(1-\alpha\beta)}{(1+\theta\eta)}\Big((\eta+\sigma^{-1})\hat{Y}_t + \omega_\tau\hat{\tau}_t + \frac{1}{2}\frac{\omega_\tau}{(1-\bar{\tau})}\hat{\tau}_t^2 + \frac{1}{2}[(\hat{\xi}_t + (1+\eta)\hat{Y}_t)^2 - \\ &(-\omega_\tau\hat{\tau}_t + \hat{\xi}_t + (1-\sigma^{-1})\hat{Y}_t)^2]\Big) + \frac{\theta(1+\eta)}{2}(\pi_t-\pi)^2 + \beta E_t V_{t+1} + \text{t.i.p.} + \mathcal{O}(||\xi||^3). \end{aligned}$$

In a more compact way, we can write

$$\begin{aligned} V_t &= \kappa(c_x' x_t + \frac{1}{2}x_t' C_x x_t + x_t' C_\varepsilon \varepsilon_t + \frac{1}{2}c_\pi(\pi_t-\pi)^2) + \beta E_t V_{t+1} \\ &+ \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned} \tag{B.45}$$

We have defined

$$\omega_\tau \equiv \bar{\tau}/(1-\bar{\tau})$$

$$\psi \equiv \omega_\tau/(\eta+\sigma^{-1}),$$

$$c_x' \equiv \begin{bmatrix} \psi & 1 & 0 \end{bmatrix},$$

$$C_x \equiv \begin{bmatrix} \psi & (1-\sigma^{-1})\psi & 0 \\ (1-\sigma^{-1})\psi & (2+\eta-\sigma^{-1}) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C_\varepsilon \equiv \begin{bmatrix} \psi & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$c_\pi \equiv \frac{\theta(1+\eta)}{\kappa}$$

We can also integrate (B.45) forward from time t_0 to obtain

$$\begin{aligned} V_{t_0} &= E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\kappa(c_x' x_t + \frac{1}{2}x_t' C_x x_t + x_t' C_\varepsilon \varepsilon_t + \frac{1}{2}c_\pi(\pi_t-\pi)^2) \\ &+ \text{t.i.p.} + \mathcal{O}(||\xi||^3). \end{aligned} \tag{B.46}$$

Note that in a first-order approximation, (B.45) can be written as simply

$$(\pi_t - \pi) = \kappa[\hat{Y}_t + \psi\hat{\tau}_t] + \beta E_t(\pi_{t+1}-\pi), \tag{B.47}$$

since $V_t = (\pi_t - \pi) + \mathcal{O}(||\xi||^2)$.

B.4 A second-order approximation of the government's intertemporal budget constraint

We now derive a second-order approximation to the intertemporal government budget constraint

$$W_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} [\xi_T Y_T^{1-\sigma^{-1}} \tau_T - \xi_T Y_T^{-\sigma^{-1}} Tr_T + \xi_T V_q(q_T) q_T], \tag{B.48}$$

and

$$W_t = \frac{(1+i_{t-1}^R) q_{t-1}}{\Pi_t} \xi_t Y_t^{-\sigma^{-1}}. \tag{B.49}$$

First, we take a second-order approximation of the term $\xi_t Y_t^{1-\sigma^{-1}} \tau_t$ obtaining

$$\begin{aligned}
\xi_t Y_t^{1-\sigma^{-1}} \tau_t &= Y^{1-\sigma^{-1}} \tau + (1-\sigma^{-1}) Y^{-\sigma^{-1}} \tau \tilde{Y}_t + Y^{1-\sigma^{-1}} \tilde{\tau}_t + Y^{1-\sigma^{-1}} \tau \tilde{\xi}_t + \\
&\quad -\frac{1}{2} \sigma^{-1} (1-\sigma^{-1}) Y^{-\sigma^{-1}-1} \tau \tilde{Y}_t^2 + (1-\sigma^{-1}) Y^{-\sigma^{-1}} \tilde{Y}_t \tilde{\tau}_t + \\
&\quad + (1-\sigma^{-1}) Y^{-\sigma^{-1}} \tau \tilde{Y}_t \tilde{\xi}_t + Y^{1-\sigma^{-1}} \tilde{\tau}_t \tilde{\xi}_t + \mathcal{O}(||\xi||^3), \\
&= Y^{1-\sigma^{-1}} \tau + (1-\sigma^{-1}) Y^{1-\sigma^{-1}} \tau \hat{Y}_t + Y^{1-\sigma^{-1}} \tau \left(\hat{\tau}_t + \frac{1}{2} \hat{\tau}_t^2 \right) + Y^{1-\sigma^{-1}} \tau \hat{\xi}_t \\
&\quad + \frac{1}{2} (1-\sigma^{-1})^2 \tau Y^{1-\sigma^{-1}} \hat{Y}_t^2 + (1-\sigma^{-1}) \tau Y^{1-\sigma^{-1}} \hat{Y}_t \hat{\tau}_t + \\
&\quad + (1-\sigma^{-1}) Y^{1-\sigma^{-1}} \tau \hat{Y}_t \hat{\xi}_t + Y^{1-\sigma^{-1}} \tau \hat{\tau}_t \hat{\xi}_t + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3) \\
&= Y^{1-\sigma^{-1}} \tau + Y^{1-\sigma^{-1}} \tau [(1-\sigma^{-1}) \hat{Y}_t + \hat{\tau}_t + \frac{1}{2} \hat{\tau}_t^2 + \hat{\xi}_t + \frac{1}{2} (1-\sigma^{-1})^2 \hat{Y}_t^2 \\
&\quad + (1-\sigma^{-1}) \hat{Y}_t \hat{\tau}_t + (1-\sigma^{-1}) \hat{Y}_t \hat{\xi}_t + \hat{\tau}_t \hat{\xi}_t] + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3),
\end{aligned}$$

where a tilde variable denote the deviation of the variable with respect to the steady state. Considering a second-order approximation of the term

$$\begin{aligned}
\xi_t Y_t^{-\sigma^{-1}} Tr_t &= Y^{-\sigma^{-1}} Tr - \sigma^{-1} Y^{-\sigma^{-1}-1} Tr \tilde{Y}_t + Y^{-\sigma^{-1}} \cdot \tilde{T}r_t + Y^{-\sigma^{-1}} Tr \cdot \tilde{\xi}_t + \\
&\quad + \frac{1}{2} \sigma^{-1} (1+\sigma^{-1}) Y^{-\sigma^{-1}-2} Tr \cdot \tilde{Y}_t^2 - \sigma^{-1} Y^{-\sigma^{-1}-1} \cdot \tilde{Y}_t \tilde{T}r_t + \\
&\quad - \sigma^{-1} Y^{-\sigma^{-1}-1} Tr \cdot \tilde{Y}_t \tilde{\xi}_t + \mathcal{O}(||\xi||^3), \\
&= Y^{-\sigma^{-1}} Tr - \sigma^{-1} Y^{-\sigma^{-1}} Tr \hat{Y}_t + Y^{-\sigma^{-1}} Tr \cdot \hat{T}r_t + Y^{-\sigma^{-1}} Tr \cdot \tilde{\xi}_t \\
&\quad + \frac{1}{2} \sigma^{-2} Tr Y^{-\sigma^{-1}} \hat{Y}_t^2 - \sigma^{-1} Y^{-\sigma^{-1}} Tr \cdot \hat{Y}_t \hat{T}r_t + \\
&\quad - \sigma^{-1} Y^{-\sigma^{-1}} Tr \cdot \hat{Y}_t \hat{\xi}_t + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3) \\
&= Y^{1-\sigma^{-1}} g + Y^{1-\sigma^{-1}} g [-\sigma^{-1} \hat{Y}_t + \hat{T}r_t + \hat{\xi}_t + \frac{1}{2} \sigma^{-2} \hat{Y}_t^2 \\
&\quad - \sigma^{-1} \hat{Y}_t \hat{T}r_t - \sigma^{-1} \hat{Y}_t \hat{\xi}_t] + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3),
\end{aligned}$$

We now take a second-order approximation of the term

$$\begin{aligned}
\xi_t V_q(q_t) q_t &= V_q q + V_{qq} q \tilde{q}_t + V_q \tilde{q}_t + V_q q \tilde{\xi}_t + \frac{1}{2}(V_{qqq} q + 2V_{qq})\tilde{q}_t^2 + (V_q + V_{qq} q)\tilde{q}_t \tilde{\xi}_t \\
&\quad + \mathcal{O}(||\xi||^3) \\
&= V_q q + (V_{qq} q^2 + V_q q)\hat{q}_t + V_q q \hat{\xi}_t + \frac{1}{2}(V_{qqq} q^3 + 3V_{qq} q^2 + V_q q)\hat{q}_t^2 + \\
&\quad + (V_q q + V_{qq} q^2)\hat{q}_t \hat{\xi}_t + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3), \\
&= V_q q[1 + (1 - \sigma_q^{-1})\hat{q}_t + \hat{\xi}_t + \frac{1}{2}(\tilde{\sigma}_q^{-1}\sigma_q^{-1} - 2\sigma_q^{-1} + 1)\hat{q}_t^2 + \\
&\quad + (1 - \sigma_q^{-1})\hat{q}_t \hat{\xi}_t] + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3)
\end{aligned}$$

in which we have defined $1 + \tilde{\sigma}_q^{-1} = -V_{qqq} q / V_{qq}$.

We can then write

$$\begin{aligned}
\tilde{W}_t &= \tau[(1 - \sigma^{-1})\hat{Y}_t + \hat{\tau}_t + \frac{1}{2}\hat{\tau}_t^2 + \hat{\xi}_t + \frac{1}{2}(1 - \sigma^{-1})^2 \hat{Y}_t^2 + (1 - \sigma^{-1})\hat{Y}_t \hat{\tau}_t + \\
&\quad + (1 - \sigma^{-1})\hat{Y}_t \hat{\xi}_t + \hat{\tau}_t \hat{\xi}_t] - g[-\sigma^{-1}\hat{Y}_t + \hat{T}r_t + \hat{\xi}_t + \frac{1}{2}\sigma^{-2}\hat{Y}_t^2 \\
&\quad - \sigma^{-1}\hat{Y}_t \hat{T}r_t - \sigma^{-1}\hat{Y}_t \hat{\xi}_t] \\
&\quad v\delta^{-1}[(1 - \sigma_q^{-1})\hat{q}_t + \hat{\xi}_t + \frac{1}{2}(\tilde{\sigma}_q^{-1}\sigma_q^{-1} - 2\sigma_q^{-1} + 1)\hat{q}_t^2 + \\
&\quad + (1 - \sigma_q^{-1})\hat{q}_t \hat{\xi}_t] + \beta E_t \tilde{W}_{t+1} + \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3)
\end{aligned}$$

and in a more compact way

$$\begin{aligned}
\tilde{W}_t &= [b_x' x_t + b_\varepsilon' \varepsilon_t + \frac{1}{2} x_t' B_x x_t + x_t' B_\varepsilon \varepsilon_t] + \beta E_t \tilde{W}_{t+1} \\
&\quad \text{s.o.t.i.p.} + \mathcal{O}(||\xi||^3)
\end{aligned} \tag{B.50}$$

where $\tilde{W}_t \equiv (W_t - \bar{W})/(U_c Y)$ and

$$b_x' = \begin{bmatrix} \tau & \tau(1 - \sigma^{-1}) + g\sigma^{-1} & v\delta^{-1}(1 - \sigma_q^{-1}) \end{bmatrix},$$

$$b_\varepsilon' = \begin{bmatrix} (\tau - g + v\delta^{-1}) & -g \end{bmatrix}$$

$$B_x = \begin{bmatrix} \tau & \tau(1 - \sigma^{-1}) & 0 \\ (1 - \sigma^{-1}) & \tau(1 - \sigma^{-1})^2 - g\sigma^{-2} & 0 \\ 0 & 0 & v\delta^{-1}(\tilde{\sigma}_q^{-1}\sigma_q^{-1} - 2\sigma_q^{-1} + 1) \end{bmatrix},$$

$$B_\xi = \begin{bmatrix} \tau & 0 \\ \tau(1 - \sigma^{-1}) + \sigma^{-1} g & g\sigma^{-1} \\ v\delta^{-1}(1 - \sigma_q^{-1}) & 0 \end{bmatrix}.$$

Moreover integrating forward (B.50), we obtain that

$$\tilde{W}_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [b_x' x_t + \frac{1}{2} x_t' B_x x_t + x_t' B_\varepsilon \varepsilon_t] + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \tag{B.51}$$

where we have moved ε_t in t.i.p.

Note that up to first-order terms, we can write

$$\begin{aligned} \tilde{W}_t &= \Big\{ [\tau(1-\sigma^{-1}) + g\sigma^{-1}]\hat{Y}_t + \tau\hat{\tau}_t - g\hat{T}r_t + (\tau - g + v\delta^{-1})\hat{\xi}_t)] \\ &\quad + v\delta^{-1}(1-\sigma_q^{-1})\hat{q}_t \Big\} + \beta E_t \tilde{W}_{t+1}. \end{aligned}$$

Note that $\bar{W} = (1-\beta)^{-1}(U_c Y)(\tau - g + v\delta^{-1})$ and $\hat{W}_t \equiv (W_t - \bar{W})/\bar{W} = \tilde{W}_t \cdot (U_c Y/\bar{W})$. Moreover note that

$$\frac{(1-\beta)}{\beta} q(1-v) = (\tau - g)Y + vq$$

and therefore

$$\frac{(1-\beta)}{\beta} \delta^{-1}(1-v) = (\tau - g) + v\delta^{-1}.$$

It also follows that $\bar{W}/U_c Y = \beta^{-1}\delta^{-1}(1-v)$. Define $\omega \equiv (\tau - g)/[(1-\beta)\bar{W}/(U_c Y)]$, therefore $v\delta^{-1} = (1-\omega)[(1-\beta)\bar{W}/(U_c Y)]$. Define also $\varrho = \beta\delta/(1-v)$. We can then write:

$$\hat{W}_t = \varrho\tilde{\tau}_t - \varrho\tilde{T}r_t + (\varrho\tau - (1-\beta)\omega\sigma^{-1})\hat{Y}_t + (1-\beta)[\hat{\xi}_t + (1-\omega)(1-\sigma_q^{-1})\hat{q}_t] + \beta E_t \hat{W}_{t+1},$$

in which we have used the definition $\tilde{\tau}_t = \tau_t - \tau$ and from now onwards $\tilde{T}r_t = (Tr_t - Tr)/Y$. Moreover

$$\hat{W}_t \equiv \hat{q}_{t-1} - (\pi_t - \pi) - \sigma^{-1}\hat{Y}_t + \hat{\xi}_t + \hat{\imath}_{t-1}^R$$

We can write

$$\begin{aligned} \hat{q}_{t-1} - (\pi_t - \pi) - \sigma^{-1}\hat{Y}_t + \hat{\imath}_{t-1}^R &= [b_y \hat{Y}_t + \varrho\tilde{\tau}_t - \varrho\tilde{T}r_t + b_q \hat{q}_t] \\ &\quad + \beta E_t[\hat{q}_t - (\pi_{t+1} - \pi) - \sigma^{-1}\hat{Y}_{t+1} + \hat{\imath}_t^R - \tilde{r}_t^n], \end{aligned}$$

in which we have defined $\tilde{r}_t^n = \hat{\xi}_t - E_t\hat{\xi}_{t+1}$ in which we have defined

$$\begin{aligned} b_y &\equiv (\varrho\tau - (1-\beta)\omega\sigma^{-1}), \\ b_q &\equiv (1-\beta)(1-\omega)(1-\sigma_q^{-1}). \end{aligned}$$

B.5 A quadratic approximation to the policy objective function

Using the above derivations, we can now derive a quadratic approximation to the policy objective function. To this end, we combine equation (B.46) and (B.51) in a way to eliminate the linear terms in (B.44). Indeed, we find ϑ_1, ϑ_2 such that

$$\vartheta_1 b_x' + \vartheta_2 c_x' = a_x' \equiv [0\ \Phi\ v\delta^{-1}].$$

The solution is given by

$$\begin{aligned} \vartheta_1 &= -\frac{\Phi}{\Gamma}, \\ \vartheta_2 &= \frac{\Phi(1-\tau)(\sigma^{-1}+\eta)}{\Gamma}, \end{aligned}$$

where

$$\Gamma = (1-\tau)(1+\eta) - (1-\sigma^{-1}(1-g)).$$

Note that the lagrange multiplier ϕ_4, given in (B.42), is such that $\phi_4 = -\vartheta_1$ and, therefore, given the first-order condition (B.39) it also follows that

$$\vartheta_1 v\delta^{-1}(1-\sigma_q^{-1}) = v\delta^{-1}.$$

We can, therefore, write

$$\begin{aligned} E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Phi\hat{Y}_t &= E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\vartheta_1 b_x' + \vartheta_2 c_x']x_t = \\ & -E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\frac{1}{2}x_t' D_x x_t + x_t' D_\varepsilon \varepsilon_t + \frac{1}{2}d_\pi(\pi_t-\pi)^2] \\ & +\vartheta_1\tilde{W}_{t_0} + \vartheta_2\kappa^{-1}V_{t_0} + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \end{aligned}$$

where

$$D_x \equiv \vartheta_1 B_x + \vartheta_2 C_x, \quad \text{etc.}$$

Hence

$$\begin{aligned} U_{t_0} &= \Omega E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{a_x' x_t - \frac{1}{2}x_t' A_x x_t - x_t' A_\varepsilon \varepsilon_t - \frac{1}{2}a_\pi(\pi_t-\pi)^2\right\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \\ &= -\Omega E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{1}{2}x_t'\Delta_x x_t + x_t'\Delta_\varepsilon\varepsilon_t + \frac{1}{2}\lambda_\pi(\pi_t-\pi)^2\right\} + \\ & +X_{t_0} + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \\ &= -\Omega E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{1}{2}\lambda_y\hat{Y}_t^2 - \lambda_g\hat{T}r_t\hat{Y}_t + \lambda_q\hat{q}_t^2 + \frac{1}{2}\lambda_\pi(\pi_t-\pi)^2\right\} + X_{t_0} + \\ & +\text{t.i.p.} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{B.52}$$

In particular, we obtain that $\Omega = \bar{u}_c\bar{Y}$ and that

$$\lambda_y \equiv (1-\Phi)(\sigma^{-1}+\eta) + \Phi(\sigma^{-1}+\eta)\frac{(1-\tau)(1+\eta)}{\Gamma} + \frac{\Phi}{\Gamma}\sigma^{-1}g;$$

$$\lambda_q = \frac{\Phi}{\Gamma} v\delta^{-1}\sigma_q^{-1}(\sigma_q^{-1} - \tilde{\sigma}_q^{-1})$$

$$\lambda_g = \frac{\Phi}{\Gamma} g\sigma^{-1}$$

moreover we have defined

$$\lambda_\pi = \frac{\Phi\theta(1-\tau)(\sigma^{-1}+\eta)(1+\eta)}{\Gamma\kappa} + \frac{(1-\Phi)\theta(\sigma^{-1}+\eta)}{\kappa}.$$

Finally,

$$X_{t_0} \equiv \bar{Y}\bar{u}_c[\vartheta_1 \tilde{W}_{t_0} + \vartheta_2 \kappa^{-1} V_{t_0}]$$

is a transitory component.

Therefore the loss function is given by

$$L_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2}\lambda_y y_t^2 + \frac{1}{2}\lambda_\pi(\pi_t - \pi)^2 + \frac{1}{2}\lambda_q q_t^2 \right\}.$$

in which the output gap is defined by $y_t = \hat{Y}_t - \hat{Y}_t^*$ with $\hat{Y}_t^* \equiv \lambda_y^{-1}\lambda_g \tilde{T}r_t/g$.

B.6 Optimal policy problem

Before solving the optimal policy problem, we recall the constraints to appropriately modify them. The AS equation is given by

$$(\pi_t - \pi) = \kappa[y_t + \psi_\tau(\tilde{\tau}_t - \tilde{\tau}_t^*)] + \beta E_t(\pi_{t+1} - \pi),$$

in which we have defined $\psi_\tau = \psi/\tau$ and $\tilde{\tau}_t^* = -\psi_\tau^{-1}\hat{Y}_t^*$. The AD equation is now

$$y_t = (1-v)E_t y_{t+1} - \sigma(1-v)(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) + \sigma\sigma_q^{-1} v\hat{q}_t$$

in which

$$r_t^n = \tilde{r}_t^n + \frac{1}{\sigma}E_t\hat{Y}_{t+1}^* - \frac{1}{\sigma(1-v)}\hat{Y}_t^*$$

The intertemporal budget constraint of the government is given by

$$\hat{q}_{t-1} - (\pi_t - \pi) - \sigma^{-1}\hat{Y}_t + \hat{\imath}_{t-1}^R = E_t \sum_{T=t}^{\infty} \beta^{T-t}[b_y \hat{Y}_T + \varrho\tilde{\tau}_T - \varrho\tilde{T}r_T + b_q\hat{q}_T - \beta\tilde{r}_T^n]$$

which can be written as

$$\hat{q}_{t-1} - (\pi_t - \pi) - \sigma^{-1}y_t + (\hat{\imath}_{t-1}^R - r_{t-1}^n) = -f_t + E_t \sum_{T=t}^{\infty} \beta^{T-t}[b_y y_t + \varrho(\tilde{\tau}_T - \tilde{\tau}_T^*) + b_q\hat{q}_T]$$

having define the fiscal-stress variable f_t

$$f_t = -E_t \sum_{T=t}^{\infty} \beta^{T-t}[b_y \hat{Y}_T^* + \varrho \tilde{\tau}_T^* - \varrho \tilde{T} r_T - \beta \tilde{r}_T^n] - r_{t-1}^n.$$

In the evaluation of the optimal policy, we assume a zero fiscal stress at all times. The optimal policy problem minimizes the quadratic loss function

$$L_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2}\lambda_y y_t^2 + \frac{1}{2}\lambda_\pi(\pi_t - \pi)^2 + \frac{1}{2}\lambda_q q_t^2 \right\}$$

under the log-linear approximation of the equilibrium conditions:

$$(\pi_t - \pi) = \kappa[y_t + \psi_\tau(\tilde{\tau}_t - \tilde{\tau}_t^*)] + \beta E_t(\pi_{t+1} - \pi).$$

$$y_t = (1-v)E_t y_t - \sigma(1-v)(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) + \sigma\sigma_q^{-1} v \hat{q}_t.$$

$$\begin{aligned} \hat{q}_{t-1} - (\pi_t - \pi) - \sigma^{-1} y_t + \hat{\imath}_{t-1}^R - r_{t-1}^n &= b_y y_t + \varrho(\tilde{\tau}_T - \tilde{\tau}_T^*) + b_q \hat{q}_T \\ &\quad + \beta E_t[\hat{q}_t - (\pi_{t+1} - \pi) - \sigma^{-1} y_{t+1} + \hat{\imath}_t^R - r_t^n], \end{aligned}$$

First-order conditions with respect to $\hat{Y}_t, \pi_t, \hat{\tau}_t, \hat{\imath}_t^R$ and $\hat{q}_t$ are given respectively by

$$\lambda_y y_t - \kappa\phi_{1,t} + \phi_{2,t} - \beta^{-1}(1-v)\phi_{2,t-1} - \sigma^{-1}(\phi_{3,t} - \phi_{3,t-1}) - b_y \phi_{3,t} = 0$$

$$\lambda_\pi(\pi_t - \pi) + \phi_{1,t} - \phi_{1,t-1} - \sigma(1-v)\beta^{-1}\phi_{2,t-1} - (\phi_{3,t} - \phi_{3,t-1}) = 0$$

$$-\kappa\psi_\tau\phi_{1,t} - \varrho\phi_{3,t} = 0$$

$$\sigma(1-v)\phi_{2,t} + \beta(E_t\phi_{3,t+1} - \phi_{3,t}) - \phi_{4,t} = 0$$

$$\lambda_q \hat{q}_t - \sigma\sigma_q^{-1} v \phi_{2,t} - b_q \phi_{3,t} + \beta(E_t\phi_{3,t+1} - \phi_{3,t}) = 0$$

in which $\phi_{4,t}$ is the lagrange multiplier associated to the zero-lower bound constraint

$$(\hat{\imath}_t^R + \ln(1 + i^R)) \geq 0$$

with $\phi_{4,t} \geq 0$.

C Calibration

We calibrate the model parameters as in the following table:

Table 1: Calibration of parameters

$\beta = 0.995$	$\kappa = 0.02$
$\sigma = 0.5$	$\delta = 0.598$
$\eta = 0.47$	$v = 0.0015$
$\theta = 10$	$\Pi = 1 + 0.02/4$
$\tau = 0.3$	

The intertemporal elasticity of substitution in consumption σ is set to 0.5; the inverse of the Frisch elasticity of labor supply is set to $\eta = 0.47$; the elasticity of substitution among the varieties of goods in the consumption basket is set to $\theta = 10$; the slope of the AS equation is the to $\kappa = 0.02$, the tax rate is set at $\tau = 0.3$. All the above calibration, except for the tax rate, is taken from Eggertsson and Woodford (2003). The gross inflation rate Π is set to be consistent with an inflation target of 2% at annual rates. The rate of time preference is set to $\beta = 0.9975$ so that the steady state nominal interest rate is at 2% at annual rates. The parameter g is determined by

$$g = \tau - \delta^{-1} \left[\frac{1 - \beta - \chi}{\beta} \right]$$

Given the calibration of τ and θ, then $\Phi = 1 - (1 - \tau)(\theta - 1)/\theta$. The parameter $\delta = Y/q$ is build using the average ratio between total liquidity and nominal GDP for the period 1971 Q1 to 2005 Q4. The total liquidity is computed as the sum of M1, without currency in circulation, plus the public debt in the hands of private investors. Data are taken from FRED: M1 corresponds to the series with the code M1NS; currency corresponds to the series with the code MBCURRCIR, the federal government debt in the hands of private investors corresponds to the series HBPIGDQ188S, which is expressed as a percentage of nominal GDP. The average of the ratio q/Y is equal to 41.8% using GDP at annual rates, therefore $\delta = 1/(0.418 \times 4)$.

The spread between risk-free illiquid and liquid securities, v, is calibrated as the average of the log-difference between the 3-month AA non financial commercial paper gross interest rate (series CP3M and CPN3M) and the 3-month treasury bills gross interest rate (series TB3MS), for the period 1971 Q1 to 2005 Q4.

The elasticity σ_q is determined by the first-order condition of the optimal policy problem, equation (B.39):

$$\sigma_q = \frac{\phi_4}{1 + \phi_4} = \frac{-\vartheta_1}{1 - \vartheta_1} = \frac{\Phi}{\Gamma + \Phi}.$$

The elasticity $\tilde{\sigma}_q$ is set at $\tilde{\sigma}_q = \sigma_q + 0.05$. Note that the elasticity of liquidity demand with respect to the interest-rate spread can be retrieved from $q_i = \sigma_q(1 - v)/v$, but it is irrelevant for the simulations presented in the text. The parameter ρ is instead set to take values 0.2, 0.5 and 1 in the simulation of Figure 1 and the unitary value in all other Figures.