The International Supply of Reserve Currency*

Pierpaolo Benigno
University of Bern

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Abstract

This paper provides insights into the historical inefficiencies and instabilities of the international monetary system, spanning from the Gold Standard to modern 'paper' currency systems. It analyzes the historical limited supply of safe assets within the reserve currency, as indicated by positive liquidity premia, both from government or private issuance. Additionally, it explores the macroeconomic and financial instabilities of shortages in the supply of safe assets with respect to the challenges of stabilizing inflation and economic activity. Innovations stemming from the competition of cryptocurrencies and the associated blockchain technology hold the potential for improving these outcomes.

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Throughout its history, the international monetary system has been plagued by inefficiencies and instabilities, primarily arising from the mechanisms of supply and demand of the dominant (reserve) currency.[1]

Evidence from the past demonstrates a recurring pattern of a dominant currency or, at most, two currencies, closely tied to the economic and political supremacy of the issuing nation (see Eichengreen, 2005). These currencies assume the traditional roles of serving as a medium of exchange, a store of value, and a unit of account within the international monetary system.[2] In the present day, the United States dollar stands as the hegemonic currency, playing a crucial role in facilitating bilateral transactions of goods in global markets. It serves as the unit of account for trade contracts and is widely demanded due to its liquidity and perceived safety, making it a highly desired safe asset.[3]

This paper presents an international macroeconomic model of a dominant currency that, through the government, is supplied as a safe asset worldwide. Safe assets are modeled to provide non-pecuniary benefits, with their marginal value increasing as the scarcity of the security grows. This framework implies both domestic and foreign demands for the securities, which are negatively correlated with their liquidity premium.

The model enables the characterization of alternative monetary regimes, ranging from the Gold Standard to pure paper currency regimes, defining the supply of safe assets and their connection with the overall management of monetary policy. Additionally, the framework is extended to study the coexistence of private and public liquidity.

The primary aim of this paper is to shed light on historical inefficiencies and instabilities within the international monetary system and to draw relevant conclusions. The inefficiencies relate to the conditions and reasons for the limited supply of international liquidity (safe assets), as reflected by positive liquidity premia. The instabilities encompass both macroeconomic and financial aspects, focusing on the stabilization of inflation and economic activity, as well as the implications of shortages in the supply of safe assets for economic activity.

The analysis begins with the classic Gold Standard, which establishes a link between the supply of international liquidity and the gold reserves held by the central bank of the dominant currency at a fixed gold parity. While this system succeeds in maintaining price level stability, it comes at a cost. One prominent limitation of the Gold Standard is the inefficiently low supply of international liquidity. This arises from the fact that the availability of liquidity is constrained by the quantity of gold

[1] Sargent (2010) discusses the dilemma between efficiency and stability in the supply of liquidity within the national borders.

[2] Gourinchas, Rey and Sauzed (2019) discuss extensively the roles of international curriencies in the international monetary system.

[3] Gopinath and Stein (2019) analyze the connection between trade invoicing in dollars, the dominant currency, and the creation of private safe assets in dollars.

reserves and the fixed gold price, leading to adverse macroeconomic consequences. When there is a heightened demand for liquidity from the rest of the world, but the supply remains limited, it can result in deflationary effects within the hegemonic country that can spread worldwide. These deflationary pressures, in turn, have the potential to trigger recessions or eventually depressions. Bernanke and James (1991) discuss the relevance of the deflationary pressures of the Gold Standard at the incipit of the Great Depression.[4] The model presented in this paper delves into the reasons behind the criticisms leveled against a rigid liquidity-backing system, as advocated by Keynes (1929) for Britain during the Gold Standard and by Triffin (1961) for the U.S. during the Bretton Woods system.

The analysis then considers an inconvertible 'paper' currency standard that completely decouples the supply of liquidity from commodities. A somehow surprising result is that this regime does not necessarily yield substantial gains in terms of efficiency and stability. A self-interested hegemonic country, in particular, may have incentives to keep the supply of liquidity low to benefit from relatively low borrowing costs, thereby sustaining high levels of consumption for its own economy. The international supply of liquidity can be even lower than under the Gold Standard. This inefficiency also entails macroeconomic costs. When the central bank of the reserve currency does not set interest rate policy by paying a rate on its liabilities (reserves), decisions regarding liquidity supply become interconnected with monetary policy stance and inflation objectives. Increasing the supply of liquidity leads to lower achievable inflation rates. A more modern monetary policy framework, in which the central bank conducts policies by paying an interest rate on reserves, untangles these linkages and enables independent decision-making regarding liquidity policy in relation to standard monetary policy stances. Nevertheless, the system remains fragile in conditions of excessive external demand for liquidity, which may compel the hegemonic country to adopt zero-lower bound policies.

The analysis then addresses the role of the private sector as an alternative provider of international liquidity. The insufficient supply of government liquidity naturally leads to private entities creating liquidity to exploit rents and profitable opportunities. There are conditions, albeit quite ideal, in which private liquidity can achieve an efficient global supply without compromising the stability of the system, as advocated by free banking theories, related to Smith (1776) and Hayek (1976). In a scenario characterized by a frictionless private market and perfect competition, intermediaries can provide the necessary liquidity for the world economy. This allows the central bank to remain insulated from fluctuations in the liquidity market and focus on macroeconomic stabilization, provided policies are set by paying an interest rate on reserves.

[4]Benati and Benigno (2023) has shown that the Gibson's paradox, i.e. the positive relationship between prices and interest rates observed during the Gold Standard, originates from fluctuations in the natural real rate of interest. An excess demand of liquid assets can lead to a fall in the natural real rate and, therefore, in the price level.

However, this ideal environment necessitates intermediaries to invest in either risk-free illiquid private securities or raise equity at market rates to absorb any potential balance sheet risks. Frictions in private intermediation activities can disrupt efficiency and potentially lead to macroeconomic instability. The model encompasses aspects of the 2007-2008 financial crisis, during which a liquidity crisis arose due to failures in private liquidity creation. This situation required government intervention through policies such as zero interest rates, increased government liquidity and swap intervention in international markets. The demand for dollar liquidity becomes highly inelastic, particularly during crises. However, there are advantages to provide liquidity assistance to tame the crisis, rather than exerting monopoly power, as discussed in Obstfeld (2023).[5]

The study of the international monetary system and its characteristics has been a fundamental topic in the field of international macroeconomics, with numerous noteworthy contributions. Aliber (1964) examined the advantages and disadvantages of the U.S. acting as a reserve currency, emphasizing that being the reserve currency allowed the U.S. to purchase more foreign goods due to the earnings from seigniorage profits. In this study, the seigniorage profits serve as the rationale for a self-interested hegemon to maintain a limited supply of international liquidity, akin to Mundell's (1972) discussion justifying an optimum balance of payment deficit. Kenen (1960) developed a model of the gold-exchange standard that discussed the instability of this framework when faced with a shortage of liquidity.[6] In a recent study, Fernández-Villaverde and Sanches (2023) found that the scarcity of the world gold stock exposes the dominant country to external shocks, while the creation of private money facilitates the international transmission of banking crises.

A contribution, closely connected to this research, is the work of Farhi and Maggiori (2018), which presents a model of the international monetary system that sheds light on historical evidence. Their study explores the fragility of the system resulting from the limited commitment of the reserve currency issuer to honor debt in real terms.[7] In contrast, this paper's model abstracts from strategic choices regarding the value of money and emphasizes that in a standard international macro model rooted in the "intertemporal approach to the current account" (as presented in Obstfeld and Rogoff, 1996), a self-oriented hegemon issuer already has a strategic incentive to manipulate the liquidity supply. There exists a trade-off between satisfying liquidity for domestic objectives and exploiting liquidity premia to reduce borrowing costs and enhance the current account. Furthermore, this paper's framework establishes a comprehensive link between liquidity choices and the conventional monetary policy

[5]This aligns with the concept discussed by Benigno and Robatto (2019) of raising the burden of taxation during challenging periods, serving in any case as a potential constraint on the supply of liquidity. Benigno and Robatto (2019) further discuss the rationale of the various types of government interventions undertaken during the financial crisis.

[6]Hagemann (1969) studies the implications of Kenen's model for the U.S. balance of payments.

[7]See also Obstfeld and Rogoff (2007) for an analysis of the real exchange rate adjustment required to put on a sustainable ground the U.S. current account position in the 2000s.

framework, illustrating the potential sources of macroeconomic instability resulting from an inefficient supply of liquidity.

This work is also closely related to the literature that has emphasized and quantified the exorbitant privilege of the country issuing the reserve currency, as discussed in the works of Gourinchas and Rey (2007), Gourinchas, Rey and Govillot (2017), He, Krishnamurthy and Milbradt (2019) and Maggiori, Neiman and Schreger (2020) and Akinci et al. (2022). The exorbitant privilege is here modelled with the liquidity services that the debt in the reserve currency provides both domestically and in the rest of the world. It is shown that financial market integration equalizes the marginal benefits of liquidity across countries.

A recent literature has studied the optimal supply of liquidity in a closed economy model in which taxes are distortionary, see the recent works of Angeletos, Collard and Dellas (2022), Benigno and Benigno (2022) and Sims (2022). This literature has shown the optimality of limiting the supply of liquidity below the satiation level because the liquidity premium allows the government to economize on distortionary taxes. Obstfeld (2011) has also emphasized the limits given by the fiscal capacity to the supply of international liquidity. Here, instead, taxes are not distortionary but it is still optimal to supply liquidity below satiation taking into account the higher consumption that the issuer country can afford.

Another pertinent literature is that initiated by Holmström and Tirole (1998), which explores the private supply of liquidity for the efficient functioning of the productive sector and its interaction with public liquidity provision.

This work is structured as it follows. Section 1 presents the model economy and Section 2 the equilibrium conditions. Section 3 discusses the optimal supply of liquidity from the global perspective. Section 4 analyzes the implications of the model under a gold-standard regime while Section 5 those under fiat money. Section 6 discusses the implications of an environment in which the international supply of liquidity is also created by financial intermediaries. Section 7 draws the conclusions.

1 Model

The world economy is composed by two countries: country H, the one in which the reserve currency is issued and country F, the rest of the world.

Consider preferences for households living in country H given by:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ C_t + L\left(g_t\right) + \theta_t V\left(q_t\right) \right\}, \tag{1}$$

in which β is the rate of time preferences, with $0 < \beta < 1$; C is consumption of a single good that is traded internationally. To simplify the analysis, utility is linear in consumption. Households also derive utility from holding gold, g, in the form of jewelry and from the real value, q, of holdings bonds denominated in units of the reserve currency: Q is the nominal value of these bonds, and P is the price of the traded good so that $q = Q/P$. The function $L(\cdot)$ has standard concave properties and is differentiable, while $V(\cdot)$ is also concave but has a satiation point at $\bar{q}$ such that $V_q(\cdot) = 0$ for $q \geq \bar{q}$, where $V_q(\cdot)$ is the first derivative of the function $V(\cdot)$ with respect to its argument. We are going to give later more details on the functional form that $V(\cdot)$ assumes. Finally, θ_t is a preference shock.

The non-pecuniary advantages obtained from bonds through direct utility benefits encompass the characteristics specific securities possess within the monetary system. These characteristics enable smooth transactions of goods and offer collateral, essential services provided by selected securities.

The modelling device through direct utility serves as a shortcut but has two important and realistic implications for asset pricing. Firstly, the return of the securities comprises both a pecuniary and a non-pecuniary component. Secondly, the demand for the security is no longer perfectly elastic concerning its price; instead, it follows a standard negative relationship. Both features imply that the non-pecuniary benefits are proportional to a liquidity premium that these securities have over other default-free but illiquid securities. Consequently, the demand for the liquid security is inversely related to the liquidity premium, and a higher supply should be negatively related to it.

In this respect, Krishnamurthy and Vissing-Jorgensen (2012) described a negative relationship between the debt-to-GDP ratio of the U.S. Treasury and the Aaa-Treasury corporate bond spread based on annual observations from 1919 to 2008, which the above modeling aims to capture. Interestingly, early literature on the characteristics of reserve currency, such as Aliber (1964), Mundell (1972), and Swoboda (1968), has argued for the seigniorage gains that the issuer of the currency can benefit from, implicitly relying on a downward-sloping demand schedule.

Modeling the flow services of gold stock and real liquidity directly in the utility function are common assumptions in the literature. For example, Barro and Misra (2016) assume complementarity between goods and the stock of gold in utility. Krishnamurthy and Vissing-Jorgensen (2012) include the stock of Treasury securities directly into the representative agent's utility function. Similarly, Stein (2012) assumes that private money provides additional benefits directly in utility. See also Sidrauski (1967) for an early model with money in the utility function, in which non-interest-bearing assets provide direct utility to the household.

Several works, such as Feenstra (1986) and Lucas and Stokey (1987), have shown equivalence results between modeling real money balances in the utility function, cash-in-advance constraints, and including money in liquidity costs, which appear instead in the budget constraint. These equivalence results have also shown how to rationalize an objective function like (1) with separability between goods and liquidity and a finite level of satiation for liquidity balances.[8]

[8]Refer to Woodford (2003, pp. 652-653). In models where the utility of liquidity strictly increases with real money balances, the liquidity premium approaches zero only as real liquidity tends to

Households are subject to the following budget constraint:

$$\begin{aligned} B_t + Q_t + S_t A_t + P_t C_t + P_{g,t} g_t &= (1+i_{t-1})B_{t-1} + (1+i^R_{t-1})Q_{t-1} + (1+i^*_{t-1})S_t A_{t-1} + \\ &\quad + P_t Y_t + P_{g,t} g_{t-1} - T_t + P_{g,t}(G_t - G_{t-1}). \end{aligned} \tag{2}$$

They can invest in three securities. B denotes the holdings of default-free bonds denominated in the home currency that pays an interest rate i; Q denotes holdings of bonds that are, as well, default free and denominated in the home currency, but they provide liquidity benefits in the utility function (1). For this reason, they might carry a different interest rate i^R. A are the holdings of default-free bonds denominated in units of foreign currency that pays the interest rate i^*. S is the nominal exchange rate between the home and foreign currency. Y is the exogenous endowment of the traded goods, T are lump-sum taxes levied by the government in country H; P_g is the price of gold and G_t is the stock of gold at time t, which accumulates for the household, with $G_t \geq G_{t-1}$.

The first-order conditions of the optimization problem of the household with respect to B_t, A_t and Q_t imply respectively:

$$1 + i_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t}, \tag{3}$$

$$1 + i^*_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t}\frac{S_t}{S_{t+1}}, \tag{4}$$

$$1 = \theta_t V_q\left(q_t\right) + \frac{1+i^R_t}{1+i_t}. \tag{5}$$

Equation (3) is a Fisher equation that relates the nominal interest to the real interest rate and the inflation rate. Due to linear preferences in consumption, the real rate is constant and equal to $1/\beta$. Equation (4) combined with (3) implies uncovered interest-rate parity saying that the cross-country differential between the nominal interest rates reflects variations over time of the nominal exchange rate:

$$\frac{1+i_t}{1+i^*_t} = \frac{S_{t+1}}{S_t}.$$

It is important to underline that UIP holds in reference to interest rates on securities that do not provide liquidity services. UIP does not hold when we put in relation the interest rate on the liquid securities issued in country H and that on illiquid securities issued in country F, which are the two policy rates. The final equation, (5), equates the cost of investing one unit of currency on the left-hand side to the non-pecuniary benefits provided by the liquid securities, represented as the first addendum on the right-hand side of the equation, and the discounted value of their payoff, which is the

infinity. However, in the context of this paper, the liquidity premium is zero at a finite level of real liquidity balances.

second addendum. Equation (5) determines the demand of liquid securities, implicitly given by

$$V_q(q_t) = \frac{1}{\theta_t}\frac{i_t - i_t^R}{1+i_t}. \tag{6}$$

Since $V_q(\cdot)$ is non-increasing with respect to its argument, then the real demand of liquid securities, q, is non-increasing in the spread between the nominal interest rate on illiquid and liquid securities, $i - i^R$. The higher the spread, the higher is the opportunity cost of holding liquidity. Note that $i_t \geq i_t^R$. Only when $i_t = i_t^R$, the demand of liquidity is at or beyond the satiation level.

The first-order condition with respect to gold holdings implies

$$\frac{P_{g,t}}{P_t} = L_g(g_t) + \beta\frac{P_{g,t+1}}{P_{t+1}}, \tag{7}$$

which says that the relative price of gold, on the left-hand side of the above equation, is equal to the marginal utility benefits provided by gold to the households and the discounted future relative price, the last term on the right-hand side of (7).

The optimization problem of the household is completed by the exhaustion of its intertemporal budget constraint, namely

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{C_t + \left(\frac{i_t - i_t^R}{1+i_t}\right)q_t\right\} = \frac{W_{t_0}^p}{P_{t_0}} + \sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\left(Y_t - \frac{T_t}{P_t}\right) + L_g(g_t)(G_t - g_t)\right\}.$$

in which private nominal wealth is given by

$$W_{t_0}^p = (1+i_{t_0-1}^*)S_{t_0}A_{t_0-1} + (1+i_{t_0-1})B_{t_0-1} + (1+i_{t_0-1}^R)Q_{t_0-1}.$$

The left-hand side of the intertemporal budget constraint of the household shows that real resources are paid to hold securities that provide liquidity services, when their interest rate is below the market rate i. On the right-hand side, the last addendum shows the resources that the households obtain by selling part of its gold endowment. These resources depend on the marginal utility that gold provides.[9]

Finally, we characterize the government's budget constraint as

$$Q_t^s + P_{g,t}g_{t-1}^c = (1+i_{t-1}^R)Q_{t-1}^s + P_{g,t}g_t^c - T_t \tag{8}$$

in which Q^s is the total supply of liquid securities, which are held both domestically and abroad. We make the assumption that liquidity is only provided by the government of country H. The government, specifically through the central bank, has the ability to hold gold reserves (represented as g^c) and can impose lump-sum taxes (T) through the treasury. It is important to note that the illiquid securities (B) held by households in (2) are privately issued and in zero-net supply within the private sector of country H. However, it is worth mentioning that even if the government were to issue these securities, the subsequent analysis would remain unaffected.

[9] Note that in deriving the intertemporal budget constraint of the household we have used (7).

1.1 The rest of the world

Country F denotes the rest of the world. Households derive utility from consumption and the liquidity services provided by the government securities of country H through the following functional form:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[C_t^* + \theta_t^* V\left(q_t^*\right)\right].$$

Variables have been previously defined, where an asterisk denotes the variable specific to country F. In particular, real liquidity is given by $q_t^* = Q_t^*/(S_t P_t^*)$ in which Q_t^* represents the foreign holdings of the liquid securities issued by the government in country H and P_t^* is the price of the traded good in units of foreign currency; S_t is the nominal exchange rate; θ_t^* is a preference shock. The abstract representation of the utility derived by foreign households from bonds issued in the reserve currency encompasses the various functions that the reserve currency serves within the international monetary system. These functions include acting as a vehicle currency for transactions, being the unit of account for trade invoicing, and serving as the preferred financing instrument for working capital within global value chains, as discussed in Gourinchas, Rey, and Sauzet (2019).

Foreign households are subject to the following flow budget constraint:

$$A_t^* + \frac{Q_t^*}{S_t} + P_t^* C_t^* = (1 + i_{t-1}^*) A_{t-1}^* + (1 + i_{t-1}^R) \frac{Q_{t-1}^*}{S_t} + P_t^* Y_t^* - T_t^*,$$

in which variables have been already defined.

The household's optimization problem implies a Fisher equation of the form

$$(1 + i_t^*) = \frac{1}{\beta} \frac{P_{t+1}^*}{P_t^*}. \tag{9}$$

The foreign demand of the liquid security is implicitly given by the first-order conditions of the household's problem with respect to q_t^*:

$$1 = \theta_t^* V_q\left(q_t^*\right) + \beta(1 + i_t^R) \left(\frac{P_t^*}{P_{t+1}^*} \frac{S_t}{S_{t+1}}\right). \tag{10}$$

We assume that there are no frictions in trading goods, so that the law of one price holds, $P = SP^*$. Using it, we can write (10) as

$$1 = \theta_t^* V_q\left(q_t^*\right) + \frac{1 + i_t^R}{1 + i_t}. \tag{11}$$

When θ_t^* increases, indicating a larger preference for the safe securities denominated in the reserve currency, the foreign demand increases for a given spread $i_t - i_t^R$. Comparing it with equation (5), it follows that

$$\theta_t V_q\left(q_t\right) = \theta_t^* V_q\left(q_t^*\right).$$

Integrated financial markets for the liquid securities imply that the marginal benefits of liquidity are equated across countries. This feature of the model depends on the assumption of perfect foresight and frictionless financial markets and would also hold in a stochastic economy under complete financial markets.

The intertemporal budget constraint of the foreign household is

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{C_t^* + \left(\frac{i_t - i_t^R}{1+i_t}\right)q_t^*\right\} = \frac{W_{t_0}^{*p}}{P_{t_0}^*} + \sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\left(Y_t^* - \frac{T_t^*}{P_t^*}\right)\right\},$$

with

$$W_{t_0}^{*p} = (1+i_{t_0-1}^*)A_{t_0-1}^* + (1+i_{t_0-1}^R)\frac{Q_{t_0-1}}{S_{t_0}}.$$

The household in country F does also pay resources to hold foreign liquidity securities, as the above intertemporal budget constraint shows.

The government budget constraint in country F is simply given by

$$A_t^s = (1+i_{t-1}^*)A_{t-1}^s - T_t^*,$$

in which A^s is the the total government's supply of foreign assets. As mentioned, these assets do not provide liquidity services.

2 Equilibrium

In equilibrium, assets markets clear for each security. The illiquid private securities B are in zero net supply within the private sector in country H, therefore

$$B_t = 0.$$

The liquid securities issued by the government in country H are held domestically and abroad

$$Q_t^s = Q_t + Q_t^*. \tag{12}$$

The securities issued by the government in country F are also held domestically and abroad

$$A_t^s = A_t + A_t^*.$$

Goods and gold markets are in equilibrium

$$Y_t^* + Y_t = C_t + C_t^*, \tag{13}$$

$$g_t + g_t^c = G_t. \tag{14}$$

The current account of country H follows from combining the flow budget constraint of the household and that of the government to obtain

$$\frac{Q_t^*}{P_t} - \frac{A_t}{P_t^*} = (1+i_{t-1}^R)\frac{Q_{t-1}^*}{P_t} - (1+i_{t-1}^*)\frac{A_{t-1}}{P_t^*} + C_t - Y_t, \tag{15}$$

in which we have used the law of one price.

Equilibrium is the following set of sequences $\left\{P_t, P_t^*, P_{g,t}, i_t, i_t^R, i_t^*, Q_t^*, Q_t, Q_t^s, T_t, g_t, g_t^c\right\}$ satisfying (3), (5), (7), (8) (9), (11), (12) and (14), given exogenous sequences $\{Y_t, Y_t^*, G_t, \theta_t, \theta_t^*\}$ and initial conditions $Q_{t_0-1}^s$, $g_{t_0-1}^c$, in which monetary and fiscal policies in country H specify three restrictions on the policy variables and monetary policy in country F specifies one restriction. The difference in the number of degrees of freedom between country H and F is because the government in country H supplies the liquid securities Q^s and also holds gold g^c. It is worth noting that consumption in both countries is indeterminate because of the assumption of linear utility. Later, we are going to show that we can still calculate, under certain boundary conditions, the present-discounted value of consumption when evaluating a utility-based welfare criterion.

3 The optimal supply of liquidity from the global economy perspective

We first analyze the optimal supply of liquidity from the perspective of the world economy considering aggregate utility as the welfare benchmark. This is simply given by

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{Y_t + Y_t^* + L\left(g_t\right) + \theta_t V\left(q_t\right) + \theta_t^* V\left(q_t^*\right)\right\}, \tag{16}$$

having used equilibrium in the goods market to substitute out for the aggregate consumption in the world economy. Alternatively one could use the two households' intertemporal budget constraints to obtain the same result. In order to analyze what is the optimal supply of liquidity viewed from the world-economy perspective, we need to understand the constraints to its supply. In a fiat money regime, there is no limit to this supply except to be finite at any finite time horizon. The reason is that central bank's liabilities define what a currency is and this feature allows the central bank to create money as needed, without the requirement of selling its assets to meet obligations, even if those obligations carry interest payments. Furthermore, this default-free nature of the central bank's liabilities extends to the treasury when it is implicitly or explicitly supported by the central bank. Based on these factors, the model presented above does not necessitate a solvency condition for the government's liabilities. Additionally, it is not implied by the households' transversality condition.

An implication of previous observations is that the following inequality can hold with a strict positive sign

$$\lim_{t\to\infty} \beta^{t-t_0} \left(q_t^s - \frac{P_{g,t}}{P_t} g_t^c\right) \geq 0, \tag{17}$$

in which $q_t^s = Q_t^s / P_t$. In principle the government in country H, following the arguments given above, could run a Ponzi scheme on its liabilities. Although this is possible, we rule it out by limiting the supply of liquidity through the resources the government has to pay it back with certainty. Using the above condition with strict equality and iterating the flow budget constraint of the government, equation (8), we obtain an intertemporal resource constraint for the government of the form:

$$\frac{(1+i^R_{t_0-1})Q^s_{t_0-1} - P_{g,t_0} g^c_{t_0-1}}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{T_t}{P_t} + \left(\frac{i_t - i^R_t}{1+i_t} \right) q^s_t - L_g(g_t) g^c_t \right\}. \quad (18)$$

Assets, such as gold held in the central bank's balance sheet, can be used to offset the outstanding debt in real terms on the left-hand side of the equation. On the right-hand side, the first term represents the resources obtained through lump-sum taxes. By issuing liabilities at a premium (i.e., when the interest rate i_t is greater than the interest rate i^R_t), additional real resources can be generated to support government liabilities, in addition to the standard revenues derived from taxes. Lastly, the final term on the right-hand side of the equation accounts for the costs associated with holding gold, as it offers a lower return compared to the real interest rate. It is important to note that when the amount of gold held remains constant, the benefits on the left-hand side of the equation (18) precisely offset the costs on the right-hand side (18).

Using the first-order conditions (5) and (7), we can write (18) as

$$K_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \tau_t + \theta_t V_q(q_t) q^s_t + L_g(g_t)(g^c_{t_0-1} - g^c_t) \right\}, \quad (19)$$

where $\tau_t \equiv T_t/P_t$ and

$$K_{t_0} = \frac{(1+i^R_{t_0-1})Q^s_{t_0-1}}{P_{t_0}}.$$

In the Appendix, we solve the optimization problem of maximizing (16) under the constraint (19), given an initial condition of the form $K_{t_0} = \bar{K}_{t_0}$.[10]

Upon examination, it becomes apparent that, owing to the lump-sum non-distortive nature of taxes, it is optimal to utilize taxes to fulfill the intertemporal resource constraint (19), thereby maximizing utility by satiating liquidity in both countries and allowing the household to retain all gold holdings.

Proposition 1 *The optimal supply of liquidity from the global perspective is to achieve satiation in both countries so that the marginal benefits of liquidity are zero, i.e.* $V_q(\cdot) = V_{q^*}(\cdot) = 0$.

[10] The condition $K_{t_0} = \bar{K}_{t_0}$, in which $\bar{K}_{t_0}$ is a constraint on the variables in K_{t_0} such that they are of the same functional form of state variables as they have at future dates in the optimal policy problem, aligns with a characterization of the optimal policy problem from a timeless perspective (see Woodford, 2023).

The proof is in the Appendix. In the first best, the government issuing the reserve currency foregoes the real resources it would have gained from reduced liquidity and a positive liquidity premium, thereby benefiting from lower borrowing costs. A central planner recognizes that these gains actually represent losses for other countries.

A crucial assumption that leads to the complete satiation outcome is the non-distortive nature of taxes. If taxes were distortionary, it would be optimal to reduce them and at the same time reduce liquidity below satiation. In the Appendix, we analyze the case in which it is costly to raise taxes, in a way that the government has to pay real resources $\psi(\tau_t)$ to increase real taxes $\tau_t = T_t/P_t$ above a non-negative threshold k. The function $\psi(\cdot)$ is such that $\psi(\cdot) = 0$ for $\tau_t \leq k$; $\psi'(\cdot) > 0$ and $\psi''(\cdot) > 0$ for $\tau_t > k$. Because of the real resources paid by the government, the optimization problem changes to take into account the factor $\psi(\tau_t)$ to be subtracted from the objective function (16) as well as from the right-hand side of the constraint (19).

The Appendix shows that, by a second-best argument, it is optimal to reduce liquidity below satiation so that the costs of relying on distortionary taxes is appropriately reduced. At the optimum $(\tilde{q}_t, \tilde{q}_t^*, \tilde{\tau})$, the elasticity of the utility with respect to liquidity should be proportional to the marginal cost of taxation

$$-\frac{V_q\left(\tilde{q}_t\right)}{V_{qq}(\tilde{q}_t)\tilde{q}_t} = \Phi(\tilde{q}_t, \tilde{q}_t^*, \theta_t/\theta_t^*)\psi^{'}(\tilde{\tau}),$$

for a function $\Phi(\tilde{q}_t, \tilde{q}_t^*, \theta_t/\theta_t^*)$ that is equal to one when $\theta_t = \theta_t^*$.

However, the resource constraint (19) also shows that there are ways to relax any tax constraints through the investment in assets, perhaps held in the central bank's balance sheet. In the case of gold, shown in equation (19), the real value of gold holdings can back the overall debt issued by the government, therefore requiring less taxes to repay the debt, circumventing the tax constraint. Considering the costs on the right-hand side, the real benefits of holding gold materialize when it is sold at some point in time.

An alternative is for the central bank to hold privately-issued securities in which case, setting gold holdings to zero, $g_t^c = 0$, the intertemporal budget constraint of the government is given by

$$\frac{(1 + i_{t_0-1}^R)Q_{t_0-1}^s - (1 + i_{t_0-1})B_{t_0-1}^c}{P_{t_0}} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{\tau_t - \psi(\tau_t) + \theta_t V_q(q_t) q_t^s\right\}.$$

Private assets holdings, denoted as B^c, can serve as collateral for government obligations, thereby reducing the reliance on taxes, even if they are distortive. A similar outcome can be achieved by granting the central bank the ability to hold foreign assets.

Taking into consideration the optimal benchmark of Proposition 1 in the context of the global economy, we proceed to examine whether it can be attained through alternative international monetary regimes. We begin by analyzing the classic gold standard.

4 Gold standard

The gold standard is an international monetary regime in which one, or more, currencies are exchanged at a fix parity with respect to gold. We define the currency of country H as the one in which the overall government liabilities Q^s are backed by gold. The value of these liabilities, at each point in time, should be equal to the value of gold held by the central bank

$$Q_t^s = P_{g,t} g_t^c,$$

where, for simplicity, we normalize the fixed parity of gold to the unitary value, $P_{g,t} = 1$. It follows that $q_t^s = g_t^c / P_t$ in which $q_t^s = Q_t^s / P_t$. We further assume that liquidity does not pay an interest rate to capture the traditional features of the gold standard, therefore $i_t^R = 0$.

Note that equilibrium in the gold market implies that $g_t^c = G_t - g_t$. Since $q_t^s = q_t + q_t^*$, with $q_t = Q_t / P_t$ and $q_t^* = Q_t^* / P_t$, it follows that

$$\frac{G_t - g_t}{P_t} = q_t + q_t^*. \tag{20}$$

Consider the asset pricing condition (7) under the fixed parity of gold $P_{g,t} = 1$,

$$\frac{1}{P_t} = L_g (g_t) + \beta \frac{1}{P_{t+1}}. \tag{21}$$

Equations (3) and (5) imply that

$$\frac{1}{P_t} = \theta_t V_q (q_t) \frac{1}{P_t} + \beta \frac{1}{P_{t+1}}. \tag{22}$$

Comparing (21) and (22), it follows that the marginal benefits of liquidity are equalized to the marginal benefits of gold

$$L_g (g_t) = \frac{1}{P_t} \theta_t V_q (q_t). \tag{23}$$

Moreover, the marginal benefits of liquidity are equalized across countries through asset markets, as we have already mentioned:

$$\theta_t V_q (q_t) = \theta_t^* V_q (q_t^*). \tag{24}$$

Equations (20), (21), (23) and (24) determine the equilibrium for the sequences of variables $\{g_t, q_t, q_t^*, 1/P_t\}_{t=t_0}^{\infty}$.

Proposition 2 *Under a gold-standard regime, liquidity is below the satiation level in both countries.*

The result in the above proposition can be proved by inspecting equation (23). The positivity of the marginal utility of gold implies a positive marginal utility of liquidity in country H and, therefore, through equation (24), in country F.

To get more insight into the solution, we characterize a special case.

Assumption 1 *Assume* $\theta_t = 1$, $\theta_t^* = \theta^* > 0$ *and* $G_t = G > 0$.

The simplification is done for the purpose of characterizing variations of the equilibrium due to changes in the foreign demand of assets, through θ^*, and gold stock, through G, while we maintain θ at the unitary value.

Assumption 2 *Preferences are specified as follows:*

$$V(q) = \begin{cases} \ln\left(\frac{q_t}{\bar{q}}\right) - \frac{q_t}{\bar{q}} & q_t < \bar{q} \\ -1 & q_t \geq \bar{q} \end{cases},$$

$$L(g) = \ln g.$$

The function $V(q)$ is appropriately designed to be non-decreasing in q; it has a satiation point at $\bar{q}$ and moreover $V_{qq}(q)$ remains non zero, negative, as q approaches the satiation point. The latter assumption is needed to have a well defined demand of liquidity as liquidity approaches the satiation point.

To solve for equilibrium, we first note that with a constant preference shock and supply of gold, the equilibrium endogenous variables will be constant over time. In this equilibrium, (21) determines the real value of gold held by the private sector at the level $g_t/P_t = 1/(1-\beta)$.

Using this result and the above assumptions into (23), we obtain that the equilibrium liquidity in country H is constant at

$$q_t = \frac{\bar{q}}{1 + \bar{q}(1-\beta)} < \bar{q}. \tag{25}$$

Liquidity is below satiation level in country H. Liquidity in country F can be expressed as a function of that in country H using (24):

$$\frac{q_t^*}{\bar{q}} = \frac{\theta^* \frac{q_t}{\bar{q}}}{1 + (\theta^* - 1)\frac{q_t}{\bar{q}}} < 1. \tag{26}$$

Liquidity in country F is also below the satiation level and is increasing in θ^*. Note the special case in which $q_t^* = q_t$ when $\theta^* = 1$.

To determine the value of money, consider the equilibrium in the liquidity market, (20), which can be written as:

$$\frac{G}{P_t} = \frac{1}{(1-\beta)} + q_t + q_t^*$$

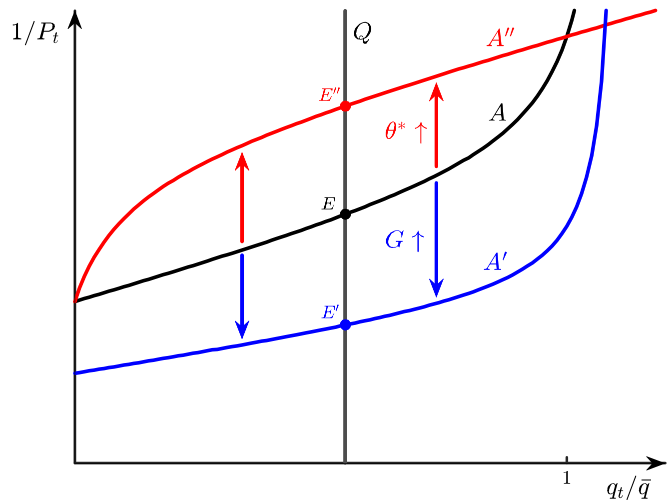

Figure 1: Gold Standard regime. Equilibrium liquidity (q_t) in country H and value of money ($1/P_t$). Initial equilibrium at E. When the stock of gold increases, $G \uparrow$, the A schedule shifts downward to A' and the equilibrium moves to E'. When there is a higher demand of liquidity abroad, $\theta^* \uparrow$, then the A schedule shifts upward to A'' and the equilibrium moves to E''.

and using (26) as

$$\frac{1}{P_t} = \frac{1}{G(1-\beta)} + \frac{\bar{q}}{G} F\left(\frac{q_t}{\bar{q}}, \theta^*\right) \frac{q_t}{\bar{q}}, \tag{27}$$

in which the function $F(\cdot, \cdot)$ is given by

$$F\left(\frac{q_t}{\bar{q}}, \theta^*\right) = \frac{1 + \theta^* + (\theta^* - 1)\frac{q_t}{\bar{q}}}{1 + (\theta^* - 1)\frac{q_t}{\bar{q}}}.$$

Equation (27) determines the value of money, that is $1/P_t$, given the equilibrium liquidity in country H.

Figure 1 plots (25) and (27) in a diagram of coordinates $q_t/\bar{q}$ and $1/P_t$. The schedule (27), labelled with A, is upward sloping, while (25), labelled with Q, is vertical. The two curves intersect at equilibrium E, which shows that liquidity is below the satiation point. Furthermore, some interesting comparative-static analysis can help to understand the determinant of liquidity and the value of money.

Suppose there is an increase in the supply of gold (G). This leads to a downward shift of the schedule (27) to A' and causes it to flatten. The liquidity levels in both countries remain unchanged, but the value of money decreases. It is interesting to note that although the discovery of gold has inflationary effects, it does not alter the overall supply of liquidity. This result is unexpected since one might assume that the supply of the commodity would be relevant to the supply of liquidity. However, the equilibrium liquidity is determined by the asset-pricing condition condition (21) and (22). When transitioning from one equilibrium with a constant value of money to another constant value, the marginal utility of gold changes proportionally to the value of money through equation (21). As a result, the marginal utility of liquidity, $V_q(q_t)$, remains unchanged in equation (23). The increase in the endowment of gold is entirely absorbed by an increase in the central bank's holdings of gold, leading to inflationary consequences.

If there is an increase in the demand for liquidity from the rest of the world, represented by a rise in θ^*, it leads to a shift in the function $F\left(\frac{q_t}{\bar{q}}, \theta^*\right)$ and results in a steeper schedule (27) that moves from A to A''.[11] The liquidity level in country H remains unchanged, but it increases in country F. Since the overall stock of gold remains the same, this generates deflationary pressures in country H, and the equilibrium moves to E''.

Proposition 3 *Under Assumptions 1 and 2, in a gold-standard regime, new discoveries of gold have inflationary consequences, whereas an increase in foreign demand for liquidity has deflationary effects.*

There are some important conclusions to draw from this analysis in terms of the objectives of efficiency and stability. On the one side, the gold standard, by linking the supply of money to that of a commodity, is able to stabilize the price level. However, strict price stability comes with other costs. In terms of efficiency, backing liquidity with a commodity in limited supply economizes on the supply of liquidity and does not allow for the achievement of the desirable full satiation equilibrium from a global perspective. Despite the benefits of achieving a stable price level, there are costs in terms of macroeconomic stability. Firstly, fluctuations in the supply of gold can be a source of variations in the price level. Secondly, and most importantly, higher external demand for the reserve currency can create deflationary pressures with tangible macroeconomic costs. In a more detailed model incorporating a non-traded sector and price rigidity, the higher demand for domestic securities leads to an appreciation of the currency. This appreciation leads to a deflationary pressure on the price of traded goods, raising the relative price of non-traded versus traded goods, inducing deflationary pressures in the non-traded sector, and ultimately leading to a recession.[12]

These results are consistent with the concerns many economists had on the stability of the gold-standard system. Keynes (1923) argued against the gold standard to free up monetary policy for stabilization purposes. Bernanke and James (1991) emphasized the disruptive effect of deflation on the financial system. They argue that the worldwide deflation of the early 1930s was the result of a monetary contraction

[11] When moving from $\theta^* = 1$ to $\theta^* > 1$, the A schedule becomes strictly concave.

[12] In the model of this section, the law of one price holds. For constant prices of traded goods in the foreign country, an appreciation of the exchange rate is equivalent to a fall in the domestic price of traded goods.

transmitted via the gold standard. In this regard, the unavoidable devaluation of the dollar, as predicted by Triffin (1961), is merely the outcome of the unbacked liquidity provided by the U.S. during the Bretton Woods system to meet the surge in external demand of dollars. Without the increase in liquidity, the U.S. would have faced the deflationary pressures resulting from a strict commodity peg, as discussed earlier.

5 Self-oriented hegemon in a 'paper' currency regime

Let's examine an inconvertible 'paper' currency monetary standard where the availability of liquidity is not necessarily tied to a specific commodity. This arrangement aims to alleviate the constraint of having a limited supply of liquidity. However, when considering the viewpoint of a policymaker primarily concerned with their self-interest, it becomes apparent that they prefer to restrict the supply of liquidity, possibly even to a lesser extent than what the gold standard would suggest. Additionally, we explore the consequences in relation to macroeconomic stability.

To evaluate the choice of a self-oriented policymaker, we use as a criterion the utility (1) of the households of country H. In few steps, we show how to evaluate welfare in a simple way. First, consider the current account equation (15) in real terms

$$q_t^* - a_t = \frac{1+i_{t-1}^R}{\Pi_t} q_{t-1}^* - \frac{1+i_{t-1}^*}{\Pi_t^*} a_{t-1} + C_t - Y_t,$$

in which $q_t^* = Q_t^*/P_t$, $a_t = A_t/P_t^*$, Π_t^* and Π_t are inflation rates in both countries for the respective traded-good prices. Defining

$$\tilde{q}_t = \frac{1+i_{t-1}^R}{\Pi_t} q_{t-1}^* - \frac{1+i_{t-1}^*}{\Pi_t^*} a_{t-1},$$

we can write it as

$$\beta \tilde{q}_{t+1} = \tilde{q}_t - \frac{i_t - i_t^R}{1+i_t} q_t^* + C_t - Y_t,$$

and using (11) as:

$$\beta \tilde{q}_{t+1} = \tilde{q}_t - \theta_t^* V_q\left(q_t^*\right) q_t^* + C_t - Y_t.$$

Integrating it forward, and assuming an appropriate borrowing limit on $\tilde{q}_t$, we obtain[13]

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} C_t = -\tilde{q}_{t_0} + \sum_{t=t_0}^{\infty} \beta^{t-t_0} Y_t + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \theta_t^* V_q(q_t^*) q_t^*.$$

We can use the above expression into (1) to substitute for the discounted value of consumption to obtain

$$U_{t_0} = -\tilde{q}_{t_0} + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ Y_t + L\left(g_t\right) + \theta_t V\left(q_t\right) + \theta_t^* V_q(q_t^*) q_t^* \right\}. \tag{28}$$

[13] The limit condition on $\tilde{q}_t$ follows from the transversality condition of the households and equation (17) with equality, which we have assumed to hold.

We add also constraints so that the solution of the optimal commitment problem delivers stationary policy rules. To this end, we assume that the policymaker considers an additional constraint on $\tilde{q}_{t_0}$ that is going to be self-consistent with the equilibrium functional form it will take at a future date, as in a timeless-perspective commitment.

As a consequence the self-oriented government in country H maximizes the following objective:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ L\left(g_t\right) + \theta_t V\left(q_t\right) + \theta_t^* V_q(q_t^*) q_t^* \right\}. \tag{29}$$

There are important differences with respect to welfare viewed from the global perspective (16). A self-oriented policymaker does not care about the benefits that liquidity provides to the rest world, but just about its own benefits, the second addendum of (29), and the rents that it can derive by supplying liquidity at a premium to the rest of the world, the last addendum of (29). The only constraint to optimal policy is the equalization of the marginal utility of liquidity across countries, equation (24).

The first-order conditions with respect to q_t and q_t^* imply respectively that

$$V_q\left(q_t\right) = -V_{qq}\left(q_t\right)\lambda_t,$$

$$V_q(q_t^*) + V_{qq}(q_t^*)q_t^* = V_{qq}\left(q_t^*\right)\lambda_t,$$

in which λ_t is the Lagrange multiplier attached to the constraint (24).

One solution of the above equations is to have q_t and q_t^* to be equal or greater than the satiation level. Indeed, in this case, all derivatives of the function $V(\cdot)$ are zeros and the above equations are satisfied. We will show shortly that this solution is not the global optimum.

Let us focus now on the case $q_t < \bar{q}$, we can combine the above two equations to eliminate the Lagrange multiplier λ_t and obtain

$$\frac{V_q\left(q_t\right)}{V_{qq}\left(q_t\right)} + \frac{V_q(q_t^*) + V_{qq}(q_t^*)q_t^*}{V_{qq}\left(q_t^*\right)} = 0 \tag{30}$$

which describes the trade-off between varying liquidity across countries. Two objectives are encompassed in (30) weighted by $1/V_{qq}\left(q_t\right)$ and $1/V_{qq}\left(q_t^*\right)$, respectively. The first, captured by the first addendum on the left of the equation, refers to the satiation of liquidity in the hegemon country, which can be obtained when $V_q\left(q_t\right) = 0$. The second captures the maximization of rents by supplying liquidity abroad, which is maximized when $V_q(q_t^*) + V_{qq}(q_t^*)q_t^* = 0$. Equation (30) together with (24) determines q_t and q_t^*.

We can get further insights into the solution by utilizing Assumptions 1 and 2. Additionally, let's begin by assuming that $\theta^* = 1$. Using (24), we can observe that $q_t = q_t^*$ and, therefore, we can write (30) as:

$$2V_q\left(q_t\right) + V_{qq}(q_t)q_t = 0,$$

which simplifies under the preference specification assumed to

$$2\left(\frac{1}{q_t}-\frac{1}{\bar{q}}\right)-\frac{1}{q_t}=0.$$

The solution is $q_t = \bar{q}/2$, with liquidity supplied half of the satiation level. We now show that this dominates in terms of welfare the full satiation solution. We can write (29) disregarding utility from gold as

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\left[\ln\left(\frac{q_t}{\bar{q}}\right)-\frac{q_t}{\bar{q}}\right]+\left(1-\frac{q_t}{\bar{q}}\right)\right\}.$$

Note that in the solution with full satiation, the terms in the curly brackets is equal to -1, while when $q_t = \bar{q}/2$ it is equal to $\ln(1/2)$ which is a higher value.

In a fiat money system, a self-oriented issuer of international liquidity finds advantageous to provide liquidity below the satiation point. The rationale behind this is the existence of a trade-off between reaching the satiation point of liquidity and maintaining a higher level of consumption. This balance can be achieved by retaining profits from issuing liquidity, which result in lower borrowing costs.[14] A central planner would, instead, recognize that the advantages of lowering borrowing costs for country H come at the expense of lower consumption for country F, with no gains when viewed from the global perspective. Therefore, the optimal supply of liquidity from the global perspective would be to supply liquidity up to the satiation level, as demonstrated in Section 3.

Examining the efficiency aspect, it is important to note that a 'paper' monetary standard does not necessarily guarantee a greater supply of liquidity compared to a gold standard. Indeed, for reasonable parametrization, the level $\bar{q}/2$ is even below that implied in (25).

Proposition 4 *Under Assumptions 1 and 2 and $\theta^* = 1$, in a 'paper' monetary standard, a self-oriented supplier of international liquidity finds it advantageous to restrict liquidity supply below the satiation level.*

In the more general case, with θ^* different from the unitary value, the optimal supply of liquidity is determined by equation (26) together with (30), which can be written under Assumptions 1 and 2 as:

$$\left(\frac{q_t}{\bar{q}}\right)=\left(\frac{q_t}{\bar{q}}\right)^2+\left(\frac{q_t^*}{\bar{q}}\right)^2. \tag{31}$$

Figure 2 plots (26), labelled with the letter B, and (31), labelled with C, in a diagram with coordinates $(q_t/\bar{q}, q_t^*/\bar{q})$. The schedule (26) is a semi-circle, which

[14] In a closed-economy model, distortionary taxation is a reason for optimally limiting the supply of liquidity.

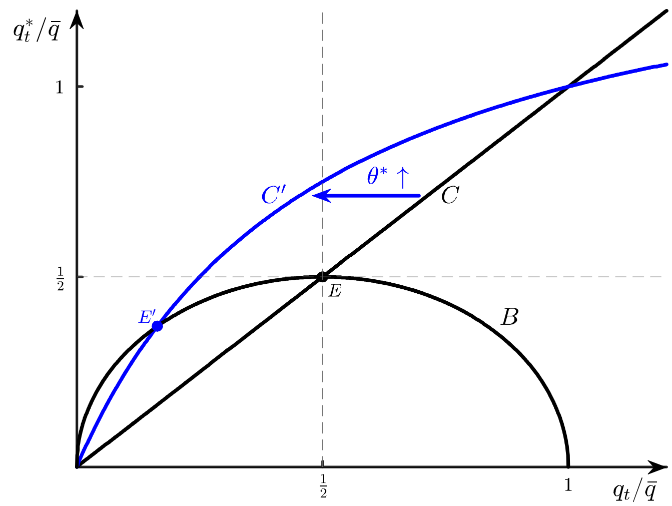

Figure 2: Fiat-money regime. Equilibrium liquidity (q_t) in country H and (q_t^*) in country F. Initial equilibrium at E when $\theta^* = 1$. When there is a higher demand of liquidity abroad, $\theta^* \uparrow$, then the C schedule shifts to the left to C' and the equilibrium moves to E'.

is increasing in q_t^* and q_t for $q_t \leq \bar{q}/2$ and decreasing q_t^* afterward. The schedule (31) is upward sloping. When $\theta^* = 1$, they intersect at the equilibrium E in which $q_t = q_t^* = \bar{q}/2$. When instead θ^* increases above the unitary value, the schedule C shifts to the left to C', the equilibrium liquidity in both countries falls reaching the point E'. If θ^* were below one and then increasing, we would have still observed a fall in liquidity in country H, but an increase in F. This different behavior depends on the fact that, given the assumed preferences, the marginal benefits of increasing q_t are positive for $q_t < \bar{q}/2$ and negative for $q_t > \bar{q}/2$ while it is always marginally costly to increase liquidity for the foreign economy, because of the foregone rents in the liquidity market. In general, when there is an increase in the foreign demand of liquidity, a self-oriented hegemon accommodates it by reducing liquidity domestically. In some cases, it might even reduce the overall supply of liquidity.

5.1 Implications for macroeconomic stability

Let's consider the implications for macroeconomic stability of an international monetary system based on a self-oriented hegemon. We derive first the implications for the interest-rate policy and the equilibrium inflation rate, distinguishing between a system in which liquidity does not pay an interest rate, like in the pre-financial crisis where the Federal Reserve was not remunerating reserves, and one in which it does, like in the recent monetary-policy framework. In the first case $i_t^R = 0$, in the latter case i_t^R is a policy choice of the central bank and can be positive. In what follows, we refer to the interest rate i on illiquid securities as the market nominal interest rate.

Let's focus on the first case, where liquidity is provided through non-interest bearing securities, akin to traditional money. Having computed in the previous section the optimal supply of liquidity for country H, let's say $\hat{q}$ with $\hat{q} < \bar{q}$, we can use it in (5) to obtain that the corresponding market nominal interest rate, denoted by $\hat{\imath}$, should satisfy

$$1 + \hat{\imath} = \frac{1}{1 - V_q(\hat{q})}.$$

The nominal interest rate is directly tied to the optimal quantity of liquidity in country H and decreasing in it. Using this result into (3), we obtain that the corresponding inflation rate is

$$\hat{\Pi} = \frac{\beta}{1 - V_q(\hat{q})}.$$

The inflation rate is also decreasing with liquidity. In the case of satiation of liquidity, which is the first-best for the world economy, the nominal interest rate is zero, since $V_q(\bar{q}) = 0$, implying a deflation at the rate β, as advocated by Milton Friedman in Friedman (1960). However, a self-interested supplier of liquidity finds optimal to limit the supply below the satiation level. Therefore, the implied nominal interest rate is positive and the inflation rate may also be positive. Specifically, under Assumptions 1 and 2 and $\theta^* = 1$, the corresponding gross inflation rate can be calculated as $\hat{\Pi} = \beta\bar{q}/(\bar{q} - 1)$, since $\hat{q} = \bar{q}/2$, which can be above one for a certain range of values for $\bar{q}$.

The decisions regarding liquidity, interest rates, and inflation are interconnected, particularly when the central bank does not pay interest rate on reserves, and liquidity does not carry an interest rate. Inflation and interest rates cannot be arbitrarily set; they must align with the liquidity policy if it is established first. Alternatively, if the central bank sets an inflation rate target, this will influence the amount of liquidity to be issued. A higher inflation target implies a lower level of liquidity to be supplied in international financial markets. This dynamic can contribute to a shortage of liquidity.

Now, let's consider the implications for the monetary policy of the reserve currency when there is an external shock in the form of increased demand for its currency, represented by θ^*. Figure 2 has shown that a self-oriented hegemon would always reduce the liquidity in country H following a higher external demand. Since q_t falls, the equilibrium interest rate and inflation rate will rise. In a more complex model with tradeables and non-tradeables and price rigidities, the increase in the nominal interest rate will produce a contraction in the non-tradeables sector and disinflation.

Proposition 5 *In a 'paper' currency monetary standard with zero interest rate on central bank's reserves, liquidity, market interest rate and inflation rate are interconnected. An increase in the foreign demand of liquidity rises the market nominal interest rate in the issuer country.*[15]

[15] The latter statement is based on Assumptions 1 and 2.

A fiat-money regime, as described thus far, is susceptible to the same issues that plagued the gold standard: liquidity shortages and restrictive monetary conditions when there is an increase in the global demand for liquidity.

Let's consider the alternative framework in which the central bank pays a positive interest rate on its reserves. We set a simple monetary policy in which the interest rate on reserve is constant, $i_t^R = \hat{\imath}^R$ with $\hat{\imath}^R$ that can be generically non-negative. Given the optimal supply of liquidity discussed above, $\hat{q}$, the ratio between the market and the policy interest rate is given by:

$$\frac{1+i}{1+\hat{\imath}^R} = \frac{1}{1-V_q(\hat{q})}. \tag{32}$$

The key difference is that $\hat{\imath}^R$ and $\hat{q}$ are now independent policy tools, the latter controlled through the choice of q^s. Therefore, if liquidity is set first, then the central bank can control the interest rate i by setting the interest rate on reserves appropriately. This also allows the central bank to control the inflation rate at a desired target independently of the supply of liquidity. Indeed, the inflation rate will be determined using (3) at

$$\Pi = \frac{\beta(1+\hat{\imath}^R)}{1-V_q(\hat{q})}. \tag{33}$$

Given $\hat{q}$, the central bank can set $\hat{\imath}^R$ to achieve a certain target for inflation. The new way of conducting policy by paying an interest rate on reserves allows the central bank to set independently the liquidity policy and the inflation target. This has important implications for the macroeconomic stability of the system.

Proposition 6 *In a 'paper' currency monetary standard with a positive interest rate on central bank's reserves, the market interest rate and the inflation rate can be insulated from liquidity policy and related shocks unless the zero-lower bound on the policy rate is achieved.*

Let's analyze the same scenario as before, where there is an increase in the foreign demand for liquidity, represented by a rise in θ^*. As depicted in Figure 2, this increase will lead to a reduction in q, indicating a decrease in liquidity. However, in this case, any necessary adjustments to the optimal supply of liquidity can be made by modifying the policy rate, while keeping the interest rate on illiquid securities and the inflation rate unchanged. It is important to note that there is a limit to this adjustment due to the zero-lower bound on the policy rate. Once the policy rate reaches that lower bound, an increase in the foreign demand for liquidity will cause the market rate to rise and tighten monetary conditions. This limitation will be further discussed in the next section when describing the 2007-2008 financial crisis, which originated from private money creation.

The key takeaway from this section, in terms of efficiency and stability criteria, is that a self-oriented hegemon may restrict the supply of liquidity, leading to inefficiencies. However, macroeconomic stability can be maintained when the central bank conducts policy by setting the interest rate on reserves, unless external demand shocks are significant enough to push the policy rate to the zero lower bound.

6 Private liquidity

The analysis in the preceding sections has mainly focused on a scenario in which the government acts as the sole provider of liquidity, aiming to maximize the welfare of its residents. However, these limitations are not realistic when considering both model-based and historical perspectives. From a modeling standpoint, we have demonstrated that by restricting the supply of liquidity in both a gold-standard and a 'paper' currency monetary system, the government retains certain rents. These profitable opportunities can serve as an incentive for private intermediaries to enter the liquidity market.

Moreover, an examination of historical evidence reveals that the government's supply of liquidity has often been constrained, influenced by fiscal capacity. Liquidity, also known as safe assets, has taken various forms of private liquidity over time. These have included banknotes and bills of exchange in the eighteenth century, deposits in the nineteenth and twentieth centuries, and money market mutual funds in the twenty-first century.

However, historical records also highlight instances of liquidity crises associated with these instruments, necessitating government intervention as a lender of last resort to address heightened demand for liquidity in adverse circumstances. A notable example is the 2007-2008 financial crisis, during which the private sector created mortgage-backed securities perceived as safe assets for liquidity purposes, leading to government intervention when the liquidity crisis occurred.

One fundamental reason behind these failures is that private liquidity lacks the inherent safety and backing enjoyed by government money, which is supported by the central bank. In contrast, for private liquidity to be deemed safe, it must have appropriate backing. In this context, our focus lies on liquidity provided by intermediaries, where the backing can be provided through assets and/or equity.

To introduce private supply of liquidity, we amend the preferences of the households in both countries by assuming that the utility from liquidity is of the form

$$V\left(q_t + \phi_t d_t\right)$$

in which now q_t denotes government liquidity, described early, and d_t is private liquidity, all in real terms. For simplicity, in what follows, we set $\theta_t = \theta_t^* = 1$. The above specification also allows for private and public liquidity not to be perfect substitute. We model this through the variable ϕ_t, with $0 \leq \phi_t \leq 1$, which can be time varying. The lower ϕ_t, the lower the contribution to utility provided by private liquidity is with respect to public liquidity. Likewise, we assume that the utility from liquidity in the rest of the world is given by $V\left(q_t^* + \phi_t d_t^*\right)$, where for the sake of simplicity we use the same variable ϕ_t to denote the degree of substitution between private and public liquidity in foreign utility. In the analysis that follows, a decrease in ϕ_t aims to capture the worsening in the quality of private assets that triggered the 2007-2008 financial crisis.

The optimization problem of households has now to account for the demand of private liquidity that takes the form

$$V_q\left(q_t + \phi_t d_t\right) = \frac{1}{\phi_t}\frac{i_t - i_t^d}{1 + i_t}, \tag{34}$$

in which i_t^d is the interest rate on private liquid securities, which might be different from the interest rate on government liquidity i_t^R. The other first-order conditions of the household's problem are given in Section 1, with the qualification of the new argument of the function $V(\cdot)$ and the assumption $\theta_t = 1$. Combining equation (5) with (34), we can write

$$\frac{1 + i_t^d}{1 + i_t^R} = \frac{1 - \phi_t V_q\left(q_t + \phi_t d_t\right)}{1 - V_q\left(q_t + \phi_t d_t\right)}$$

showing that private and public liquidity have the same interest rate if both are used and $\phi_t = 1$, otherwise the interest rate on private liquidity will be higher than that on public liquidity, reflecting the worse liquidity properties of private securities.

Likewise in country F, the demand of private securities is implicitly given by

$$V_q\left(q_t^* + \phi_t d_t^*\right) = \frac{1}{\phi_t}\frac{i_t - i_t^d}{1 + i_t}. \tag{35}$$

6.1 Creation of private liquidity by domestic intermediaries

When examining the supply side, we explore different ways of creating liquidity. To begin, we consider a scenario where private liquidity is generated by intermediaries located in country H. These intermediaries have the ability to invest in illiquid securities issued by households, which are free from default risk.[16] This scenario represents an ideal setting where the transformation of illiquid securities into liquid ones occurs domestically, without experiencing any currency mismatches.

Intermediaries live for two periods in an overlapping way. Intermediaries entering at time t have the following budget constraint

$$B_t^f + \delta_t D_t = D_t, \tag{36}$$

since they issue liquid securities D, in units of the reserve currency, to invest in private illiquid securities, denoted by B^f, in units of the same currency. We are assuming

[16] These are securities labelled with B in the household's budget constraint (2).

that there are frictions in the creation of liquidity for which there is a proportional cost to the securities issued, with $0 \leq \delta_t < \phi_t$ at all times.[17]

Intermediaries' next-period profits are given by

$$\Psi_{t+1} = (1 + i_t)B_t^f - (1 + i_t^d)D_t$$

and therefore, using (36),

$$\Psi_{t+1} = (1 + i_t)(1 - \delta_t)D_t - (1 + i_t^d)D_t.$$

Assuming free entry in the market of private intermediation, profits are driven to zero, which determines the spread between lending and borrowing rates in equilibrium

$$\frac{(1 + i_t)}{(1 + i_t^d)} = \frac{1}{1 - \delta_t}. \tag{37}$$

This spread is given by the cost of intermediation δ_t.[18] Absent this cost, free entry drives to zero any spread between the two money-market rates.

Introducing a market of private liquidity supplied by competing intermediaries has strong implications for market rates and the equilibrium allocation. In what follows we focus on an equilibrium in which private and public liquidity coexist. Combine (34) and (37) to obtain

$$V_q\left(q_t + \phi_t d_t\right) = \frac{\delta_t}{\phi_t} \tag{38}$$

showing that the marginal utility of liquidity is determined by the ratio between the respective friction in the supply and demand market of private liquidity. It also follows that

$$V_{q^*}\left(q_t^* + \phi_t d_t^*\right) = \frac{\delta_t}{\phi_t}. \tag{39}$$

Moreover, using equation (5), we obtain that

$$\frac{(1 + i_t)}{(1 + i_t^R)} = \frac{\phi_t}{\phi_t - \delta_t}. \tag{40}$$

Demand and supply frictions in private liquidity determine the equilibrium level of liquidity in each country and spreads. The key observation is that the spread between illiquid securities and the policy rate in (40) would be zero when there are no frictions in the supply of such securities. Frictions in demand only become relevant when there are supply frictions.

[17] See Woodford (1995) for a similar framework in a closed-economy model.

[18] Note that the cost of intermediation results in a mark-up of the lending rate over the deposit rate. A similar outcome could be achieved by relaxing the assumption of perfect competition to allow for a monopolistic market.

Proposition 7 *In a liquidity market of domestic intermediaries that transform illiquid securities denominated in the reserve currency into liquid securities in the same currency, when there are no intermediation costs, i.e.* $\delta_t = 0$*, private liquidity achieves the full satiation equilibrium and spreads in money markets are zero. This poses no challenge to the central bank's control over prices and market interest rates.*

The results presented in the Proposition can be easily understood by examining equations (38)–(40). Notably, these results are independent of the degree of substitution between public and private money. Unrestricted competition in the supply of appropriately backed private liquidity has the capability to achieve efficient liquidity allocation on a global scale. This outcome remains unaffected by the availability of government liquidity, which can even be zero.

There are two important features of these results that deserve attention. Firstly, a crucial driver of the outcome is the ability to back liquidity through investments in default-free assets denominated in the same currency. The existence of these securities, which should not be taken for granted, empowers intermediaries to convert illiquid securities into liquid ones without relying on equity. The presence of equity would further reinforce the result allowing intermediaries to invest also in risky securities.[19] Secondly, the presence of unrestricted competition leads to the elimination of any rent in the liquidity supply, resulting in a zero spread between liquid and illiquid securities. The reduction of rents in the liquidity market is made possible because intermediaries increase the supply of liquidity beyond the point of satiation. These two features can be seen as a modern formalization of the "real-bills doctrine" first emphasized by Smith (1776), which highlights the efficiency gains associated with private financial competition through the backing of real bills, representing safe private indebtedness, as discussed by Sargent (2011). The requirement for intermediaries to hold only safe yet illiquid securities can be further relaxed when intermediaries have the ability to raise equity at market prices to absorb potential losses. In this scenario, competition for the supply of safe and liquid securities would effectively incentivize intermediaries to provide the necessary backing for such securities.

An efficient private supply of liquidity does not pose any challenge to the monetary policy of the issuer country in achieving macroeconomic stability. This is evident in the perfect control it maintains over the inflation rate and money market rates, as demonstrated in (32) and (33). In fact, it even bolsters the stability of the system by insulating inflation and interest rates from the destabilizing fluctuations in external liquidity demand. Key is setting monetary policy by paying an interest rate on reserves.

Deviations from these assumptions yield interesting results. As indicated by the equations mentioned above, even in the presence of default-free illiquid securities, frictions in intermediation activity result in a supply of liquidity below the satiation

[19] See Benigno and Robatto (2019) for a similar result in a model in which liquidity is created by intermediaries investing in risky assets and raising equity.

point. This is done to provide intermediaries with resources to compensate for the financial frictions they encounter, which also explains the existence of a positive spread between illiquid and liquid securities, given by (37). Spreads and liquidity levels are now also a function of the variable ϕ_t capturing the substitutability between public and private liquidity arising from demand.

We run now some experiments to study macroeconomic stability under this regime. Suppose there is an increase in the intermediation costs, i.e. a rise in δ_t, then this implies an increase in the spread between illiquid and liquid securities, as shown in (37), and that with respect to the policy rate, as shown in (40). The marginal utility of liquidity increases in both countries consistently with a drop in liquidity.

Consider now a fall in ϕ_t, an experiment that captures a deterioration in the perceived quality of the private securities for liquidity, this yields to an increase in the spread between the rate on illiquid assets and the policy rate, as shown in (40). In this case, however, the spread between the rates on illiquid and liquid securities does not change while, again, the level of liquidity drops in both countries. There are also implications for the inflation rate and the exchange rate, which are respectively determined by

$$\Pi_{t+1} = \beta(1 + i_t^R)\Delta_t,$$

and

$$\frac{S_{t+1}}{S_t} = \frac{(1 + i_t^R)}{(1 + i_t^*)}\Delta_t,$$

having used (3), (4), (5) and defined

$$\Delta_t \equiv \frac{\phi_t}{\phi_t - \delta_t}.$$

The variable (Δ) captures demand and supply distortions in the creation of private securities. To close the model, assume for simplicity that policy is set through simple Taylor rules of the type

$$(1 + i_t^R) = \frac{1}{\beta}(1 + \bar{\imath}_t^R)\Pi_t^{\gamma} \qquad (1 + i_t^*) = \frac{1}{\beta}(1 + \bar{\imath}_t^*)\Pi_t^{*\gamma} \tag{41}$$

for some parameter γ satisfying the Taylor's principle, i.e. $\gamma > 1$, and some non-negative sequences $\bar{\imath}_t^R$ and $\bar{\imath}_t^*$. Recall that the policy rate in country F is i_t^*. Using these policy rules into the above equations and solving them forward, we obtain that

$$\ln \Pi_t = -\frac{1}{\gamma}\sum_{T=t}^{\infty}\left(\frac{1}{\gamma}\right)^{T-t}\left(\ln \Delta_T + \ln(1 + \bar{\imath}_T^R)\right)$$

and

$$\ln S_t - \ln S_{t-1} = -\frac{1}{\gamma}\sum_{T=t}^{\infty}\left(\frac{1}{\gamma}\right)^{T-t}\left(\ln \Delta_T + \ln(1 + \bar{\imath}_T^R) - \ln(1 + \bar{\imath}_T^*)\right).$$

An increase in Δ_t, which can be driven by either higher intermediation costs (δ_t) or a decline in the quality of private assets (ϕ_t), has significant implications. It leads to the appreciation of country H's currency and a decrease in prices. In a more complex model, a rise in the interest rate on illiquid securities can further trigger a contraction in economic activity, accompanied by deflationary pressures.

These shocks capture situations of financial stress in the creation of private liquid securities, similar to what was observed during the 2007-2008 financial crisis. Under the same policy rate, they result in higher interest rates on illiquid securities and a decrease in overall liquidity, exerting a deflationary or disinflationary impact. This framework justifies the reactions of central banks during the financial crisis, such as lowering the policy rate up to the zero lower bound and intervening to alleviate stress in credit and liquidity markets, even substituting private liquidity with public liquidity.

First, the equations above demonstrate that in order to compensate for the increase in Δ_t, the policy rate i_t^R of the liquidity issuer should decrease. In the case of a substantial shock, the zero lower bound on the nominal interest rate may become a constraint on this adjustment, making a decrease in prices inevitable and amplifying the recessionary impact.

Second, the government can reduce spreads in money markets by substituting private liquidity with public liquidity and by alleviating stress conditions in credit and financial markets. This helps in mitigating the adverse effects of financial stress and contributes to stabilizing the overall liquidity and credit environment. Additionally, it's important to note that the decline in utility resulting from reduced liquidity represents a tangible cost. This reduction serves as a proxy for output costs, reflecting issues such as the deterioration of collateral quality.

Consistent with these findings, we have observed the Federal Reserve lowering its policy rate to the zero-lower bound and intervening with various liquidity facilities. Moreover, during the acute phase of the 2008 crisis, the deposit insurance limit was increased in several countries, and other forms of government guarantees were introduced. In the U.S., the insurance limit was increased from $100,000 to $250,000. Moreover, the Federal Deposit Insurance Corporation (FDIC) set up the Temporary Liquidity Guarantee Program with the objective of bringing stability to financial markets and the banking industry. The program provided a full guarantee of non-interest-bearing transaction accounts and of the senior unsecured debt issued by a participating entity for about a year. Taken together, these two measures dramatically increased the fraction of the liabilities of U.S. financial institutions that were guaranteed by the government.

6.2 Creation of private liquidity by foreign intermediaries

We describe now an alternative framework in which the supply markets for private liquidity are segmented. Intermediaries in country H follows the modelling done in the previous section. We focus here on the intermediaries supplying liquidity to country F. These intermediaries create liquid securities in the reserve currency by investing in government securities, the liquid securities issued in country H and the domestic illiquid bonds. They have the following balance sheet at time t

$$\frac{D_t^*}{S_t} = \frac{Q_t^f}{S_t} + A_t^f, \tag{42}$$

in which D_t^* denotes the liquid securities that they issue, which are denominated in the reserve currency, Q_t^f are the holdings of government bonds of country H and A_t^f of those of country F. Next-period profits Ψ_{t+1}^* are given by

$$\Psi_{t+1}^* = v(1+i_t^R)\frac{Q_t^f}{S_{t+1}} + (1+i_t^*)A_t^f - (1+\tilde{\imath}_t^d)\frac{D_t^*}{S_{t+1}}$$

in which v, with $0 < v \leq 1$, captures frictions to trade internationally the government securities of country H. We can think of some intermediary that profits in supplying the government securities across borders. Differently, there are no such costs of intermediation in investing in government securities of country F. Finally $\tilde{\imath}_t^d$ denotes the interest rate on the liquid securities issued by intermediaries that might be different from that of country H. Intermediaries maximize discounted nominal profits given by

$$\frac{\Psi_{t+1}^*}{1+i_t^*} = v\frac{(1+i_t^R)}{(1+i_t)}\frac{Q_t^f}{S_t} + A_t^f - \frac{(1+\tilde{\imath}_t^d)}{(1+i_t)}\frac{D_t^*}{S_t}$$

in which we have used UIP.[20] Maximization of profits is constrained by the balance sheet (42). By inspection, it can be seen that it would be optimal to set $Q_t^f = 0$ whenever $v < 1$. We assume, however, that regulation requires intermediaries to hold some foreign assets to back the liquidity issued in foreign currency, i.e. $Q_t^f \geq \rho D_t^*$ with $0 < \rho \leq 1$. At optimum, intermediaries will choose to satisfy the constraint with equality, therefore we can write their profits as

$$\frac{\Psi_{t+1}^*}{1+i_t^*} = \left[\left(v\frac{(1+i_t^R)}{(1+i_t)} - 1\right)\rho + 1 - \frac{(1+\tilde{\imath}_t^d)}{(1+i_t)}\right]\frac{D_t^*}{S_t},$$

having used (42) to substitute for A_t^f.

Free competition eradicates all rents to zero and determines the rates on liquid private securities at

$$\frac{(1+\tilde{\imath}_t^d)}{(1+i_t^R)} = (1-\rho)\frac{(1+i_t)}{(1+i_t^R)} + v\rho. \tag{43}$$

The interest rate on private securities, i_t^d, is influenced by intermediation frictions. It is insightful to examine some simplified scenarios. When regulations impose stringent

[20]Note that UIP holds because households in country H can still invest freely in securities denominated in country F.

requirements for intermediaries to hold sufficient liquid securities as backing for the issued ones ($\rho = 1$), the rate on privately issued liquidity is proportional to the policy rate i_t^R, as expressed by $(1+\tilde{\imath}_t^d) = v(1+i_t^R)$. In the absence of frictions in international trading of government liquid securities, the two rates are identical. However, if the regulations are less strict, $(1+\tilde{\imath}_t^d)$ is affected by the interest rate on illiquid securities issued in country H and the spread with the policy rate. Specifically, in times of stress in country H's money markets, causing an increase in the spread between illiquid and liquid securities, these effects can spill over to the rest of the world, resulting in a higher rate on private liquidity securities in country F.

Proposition 8 *In a liquidity market of foreign intermediaries characterized by imperfection in the transformation of liquidity, turbulences in the money market in the reserve-currency country spill over to the rest of the world.*

Consider the demand of liquidity in country F originating from a utility function that includes as argument only d_t^*, i.e. $V(d_t^*)$. It is given by

$$V_q(d_t^*) = 1 - \frac{(1+\tilde{\imath}_t^d)}{(1+i_t)}.$$

We can use equation (43) to write

$$V_q(d_t^*) = \left(1 - v\frac{(1+i_t^R)}{(1+i_t)}\right)\rho.$$

In certain situations, achieving satiation of liquidity in country F is possible. A first case is in an unregulated market ($\rho = 0$), where there are no restrictions on liquidity backing. However, a complication arises in this case in a more complex model with uncertainty, as currency mismatch on intermediaries' balance sheets can result in losses that must be absorbed by either debt holders or equity holders. Another case of achieving full satiation is when there are no frictions in trading internationally liquid securities ($v = 1$), and liquidity is fully satisfied in the issuing country ($i_t^R = i_t$).

However, beyond those particular cases, there are in general spillover effects across countries due to securities trading. One interesting spillover effect we have discussed is when the spread in the money market increases in country H. This generally leads to higher money market rates abroad and a decrease in liquidity. A liquidity crisis in the reserve-currency country can spread internationally, as was evident during the 2007-2008 episode and recent turmoil in U.S. financial markets. In this instance, the liquidity operations conducted by the Federal Reserve, including swap facilities with foreign central banks, have played a crucial role in mitigating the adverse consequences of dollar shortages in international markets

7 Conclusion

The challenge of establishing a stable and efficient international monetary system appears to have no clear solution. Looking at the provision of liquidity, this study has argued that it cannot be left to the will of a monopolist, that even were benevolent has no incentive in providing the efficient supply. The reason is the improvement in the country's current account.[21] The system becomes prone to instabilities due to shifts in the external demand of liquidity. These vulnerabilities can be quite evident under certain monetary regime, like it was the Gold Standard. The innovation of a monetary policy framework in which policy is conducted setting the interest rate on reserves, which determine also the interest rate on liquidity, can help to insulate the issuer country from external demand shocks, unless the zero-lower bound is reached.

The residual rents remaining in the market due to a government monopolistic supplier can be eliminated by the competition among private intermediaries seeking to benefit from lower borrowing costs in the liquidity market. Ideally, when these intermediaries adequately back the supply of safe liquidity, competition enables the efficient provision of liquidity. This backing can even be achieved through risky securities by allowing equity to absorb potential losses. An efficient private supply of liquidity does not pose a challenge to monetary policy's control of inflation and money markets.

However, deviations from this ideal framework reveal sources of inefficiency and instability, necessitating government and central bank intervention. This discussion highlights a multi-task role for the central bank, encompassing liquidity provision, macroeconomic stability, and acting as a lender of last resort for intermediaries and often, the treasury. These interconnections create environments characterized by nuanced financial and fiscal dominance.[22]

In this somewhat gloomy scenario, there is a glimmer of hope stemming from recent developments in the currency market, thanks to the emergence of cryptocurrencies and their accompanying blockchain technology. These innovations introduce three significant changes that could potentially transform the international monetary system.

Firstly, we now have currencies that can transcend national boundaries, effectively disconnecting the traditional link between a nation, its government, and its currency. This cross-border adaptability is a notable departure from the past.

The second innovation lies in the potential advantages that come with the competition in supplying these currencies, something governments typically lack an incentive to pursue, as they typically enforce their own national currency. In an ideal world, we might foresee a future where individuals can choose their monetary system, deciding

[21] Limits on the supply of government liquidity can also stem from constraints on its backing, determined by taxes and assets held by the central bank.

[22] See Brunnermeier (2015) and Benigno et al. (2021) for a discussion of financial and fiscal dominance during the last fifteen years.

which units they wish to transact with, settle contracts, or handle debit and credit operations. As envisioned by Hayek in 1976, there is no reason to doubt that this could lead to an enhancement in currency quality, driven by users seeking the best currency in terms of its macroeconomic stability.

The third novelty is the blockchain technology, which introduces a means of certifying and verifying information through cryptographic guarantees—a highly objective approach compared to the traditional paper-based, subjective guarantees, see Chainlink (2022). This technology has the potential to evolve to enable cryptographic-guaranteed assessments of securities, offering users trustworthy information about the quality of financial securities, that is more accurate, accessible and auditable than current alternatives. This would replace, for example, the reliance on subjective evaluations from rating agencies, potentially reducing the sources of instability outlined in this work. In the context of the model discussed in this work, this innovation has the potential to substantially reduce the barriers that hinder the smooth interaction between the supply and demand for private safe securities. Using the notation of the model, as the parameter ϕ_t approaches 1 and δ approaches 0 due to this innovation, the efficient supply of liquidity could be reached.

With these three innovations in mind, it is conceivable that in the near future, competition for superior currencies could lead to the emergence of automated monetary policies solely focused on maintaining currency value stability, without the interferences arising from fiscal and financial dominance. Simultaneously, cryptographic truth could accurately price the risk associated with securities denominated in the currency's units, allowing private entities to compete in efficiently providing liquidity with a trustworthy backing.

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8 Appendix

In this Appendix, we consider the optimal policy problem when the government in country H cares about the world aggregate welfare given by the sum of domestic and foreign welfare:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ C_t + C_t^* + L\left(g_t\right) + \theta_t V\left(q_t\right) + \theta_t^* V\left(q_t^*\right) \right\}.$$

We also consider the general case in which the government issuing the reserve currency faces a cost for raising taxes. This cost is paid in units of the consumption good. Therefore $Y_t = C_t + C_t + \psi(\tau_t)$, in which the function $\psi(\cdot)$ is such that $\psi(\cdot) = 0$ for $\tau_t \leq k$; $\psi'(\cdot) > 0$ and $\psi''(\cdot) > 0$ for $\tau_t > k$, in which τ_t are real taxes given by $\tau_t = T_t/P_t$ and k is a non-negative threshold above which raising taxes becomes costly. We can then write the above objective function as:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ Y_t - \psi(\tau_t) + L\left(g_t\right) + \theta_t V\left(q_t\right) + \theta_t^* V\left(q_t^*\right) \right\}. \tag{44}$$

The government issuing the reserve currency is subject to constraints (18) in which $\psi(\tau_t)$ should account for an expenditure needed to raise taxes according to the functional form discussed above. We can use in it (7) iterated forward to replace $P_{g,t_0}/P_{t_0}$ with $\sum_{t=t_0}^{\infty} \beta^{t-t_0} L_g(g_t)$ and (6) to replace $\frac{i_t - i_t^R}{1+i_t}$ with $\theta_t V_q(q_t)$. Moreover, we note that $q_t^s = q_t + q_t^*$. We can then write (18) as:

$$K_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \tau_t - \psi(\tau_t) + \theta_t V_q(q_t)(q_t + q_t^*) - L_g(g_t)(g_t^c - g_{t_0-1}^c) \right\} \tag{45}$$

in which

$$K_{t_0} \equiv \frac{(1 + i_{t_0-1}^R) Q_{t_0-1}^s}{P_{t_0}}.$$

Note, moreover, that the following constraints apply

$$\theta_t V_q(q_t) = \theta_t^* V_q(q_t^*) \tag{46}$$

$$G = g_t + g_t^c, \tag{47}$$

$$g_t \geq 0, \tag{48}$$

$$g_t^c \geq 0. \tag{49}$$

Constraint (46) is implied by (6) and (11).

The optimal policy problem is to maximize (44) under the constraints (45), (46), (47), (48), (49) and the additional constraint $K_{t_0} = \bar{K}_{t_0}$. The latter constraint, which characterizes the optimization problem from a "timeless perspective", makes the optimization problem stationary and time consistent. We can write the following Lagrangian:

$$\begin{aligned}\mathcal{L}_{t_0} &= \sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{Y_t-\psi(\tau_t)+L\left(g_t\right)+\theta_t V\left(q_t\right)+\theta_t^* V\left(q_t^*\right)+\right.\\ &\quad+\lambda\left(\tau_t-\psi(\tau_t)+\theta_t V_q(q_t)(q_t+q_t^*)-L_g(g_t)(g_t^c-g_{t_0-1}^c)\right)+\\ &\quad\left.+\gamma_t(\theta_t V_q(q_t)-\theta_t^* V_q(q_t^*))+\vartheta_t(G-g_t-g_t^c)+\mu_t g_t+\mu_t^c g_t^c\right\},\end{aligned}$$

in which λ is the Lagrange multiplier attached to constraint (45), γ_t the multipliers attached to constraints (46), ϑ_t the multipliers to constraints (47), μ_t and μ_t^c are non-negative multipliers associated with (48), (49), with $\mu_t g_t = \mu_t^c g_t^c = 0$. The first-order conditions with respect to τ_t, q_t, q_t^* g_t, g_t^c are, respectively, given by:

$$\lambda=\frac{\psi^{'}(\tau_t)}{1-\psi^{'}(\tau_t)}, \tag{50}$$

$$\theta_t V_q\left(q_t\right)+\lambda(\theta_t V_q(q_t)+\theta_t V_{qq}(q_t)q_t^s)+\gamma_t\theta_t V_{qq}(q_t)=0, \tag{51}$$

$$\theta_t^* V_q\left(q_t^*\right)+\lambda\theta_t V_q(q_t)-\gamma_t\theta_t^* V_{qq}(q_t^*)=0, \tag{52}$$

$$L_g\left(g_t\right)-L_{gg}(g_t)(g_t^c-g_{t_0-1}^c)-\vartheta_t+\mu_t=0, \tag{53}$$

$$-L_g(g_t)-\vartheta_t+\mu_t^g=0. \tag{54}$$

The first observation is related to the optimal choice of the gold allocation. Since $L_g(g_t)$ in constraint (45) is positive, to raise resources it is optimal to lower g_t^c below $g_{t_0-1}^c$. However, lowering g_t^c, increases g_t that raises utility. Therefore it is optimal to set $g_t^c = 0$ at all times. It follows that $\mu_t^g > 0$ and $\mu_t = 0$, whereas ϑ_t is determined by (53).

Consider now the case in which $\psi^{'}(\tau_t) = 0$ or, alternatively, in which the optimal real taxes are below k, then from (50) it follows $\lambda = 0$. Using this into (51) and (52), and considering that $V_{qq}(q_t)$ and $V_{qq}(q_t^*)$ are non-positive while $V_q\left(q_t\right)$ and $V_q\left(q_t^*\right)$ are non-negative, and they are all zero only simultaneously, then it should necessarily be the case that $V_q\left(q_t\right) = V_q\left(q_t^*\right) = 0$. This result establishes the optimality of the full satiation equilibrium when taxes are non-distortionary or the optimal level of taxes is below the threshold k. This proves Proposition 1.

Consider the case of distortionary taxes. The conditions (50)–(52) imply two solutions. One in which $V_q\left(q_t\right) = V_q\left(q_t^*\right) = 0$, and therefore $q_t = q_t^* = \bar{q}$. The tax rate $\bar{\tau}$ in this solution is then given by the constraint (45)

$$2\frac{1-\beta}{\beta}\bar{q}=\bar{\tau}-\psi(\bar{\tau})+L_g(G)g_{t_0-1}^c \tag{55}$$

in which we have used the result that in a full satiation equilibrium $1 + i^R = \Pi/\beta$.

The second solution implies a level of liquidity below satiation in both countries, i.e. $\tilde{q}_t < \bar{q}$ and $\tilde{q}_t^* < \bar{q}$. In this case, we can combine (51) and (52) to eliminate γ_t and get

$$\begin{aligned} 0 &= \theta_t V_q(\tilde{q}_t) + \lambda(\theta_t V_q(\tilde{q}_t) + \theta_t V_{qq}(\tilde{q}_t)\tilde{q}_t^s) \\ &\quad + \frac{\theta_t V_{qq}(\tilde{q}_t)}{\theta_t^* V_{qq}(\tilde{q}_t^*)}\left(\theta_t^* V_q(\tilde{q}_t^*) + \lambda\theta_t V_q(\tilde{q}_t)\right). \end{aligned}$$

Using constraint (46) to replace $\theta_t^* V_q(\tilde{q}_t^*)$ with $\theta_t V_q(\tilde{q}_t)$, we can write

$$-\frac{V_q(\tilde{q}_t)}{V_{qq}(\tilde{q}_t)\tilde{q}_t} = \frac{\lambda}{1+\lambda}\frac{1}{(1+\Gamma(\tilde{q}_t, \tilde{q}_t^*, \theta_t/\theta_t^*))}\frac{\tilde{q}_t^s}{\tilde{q}_t},$$

in which we have defined the function

$$\Gamma(\tilde{q}_t, \tilde{q}_t^*, \theta_t/\theta_t^*) \equiv \frac{\theta_t V_{qq}(\tilde{q}_t)}{\theta_t^* V_{qq}(\tilde{q}_t^*)}.$$

Therefore, using (50), we can obtain the relationship between the elasticity of the utility derived from liquidity and the marginal cost of taxation

$$-\frac{V_q(\tilde{q}_t)}{V_{qq}(\tilde{q}_t)\tilde{q}_t} = \Phi(\tilde{q}_t, \tilde{q}_t^*, \theta_t/\theta_t^*)\psi'(\tilde{\tau}) \tag{56}$$

in which

$$\Phi(\tilde{q}_t, \tilde{q}_t^*, \theta_t/\theta_t^*) \equiv \frac{\tilde{q}_t + \tilde{q}_t^*}{\tilde{q}_t(1+\Gamma(\tilde{q}_t, \tilde{q}_t^*, \theta_t/\theta_t^*))}.$$

Constraint (56), together with (46) and

$$\frac{(1-\theta_{t_0-1}V_q(\tilde{q}_{t_0-1}))(\tilde{q}_{t_0-1}+\tilde{q}_{t_0-1}^*)}{\beta} = \sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\tilde{\tau} - \psi(\tilde{\tau}) + \theta_t V_q(\tilde{q}_t)(\tilde{q}_t + \tilde{q}_t^*) + L_g(G)g_{t_0-1}^c\right\} \tag{57}$$

determine the sequences $\{\tilde{q}_t, \tilde{q}_t^*\}_{t=t_0}^{\infty}$ and the constant tax $\tilde{\tau}$, given initial conditions $\tilde{q}_{t_0-1}$, $\tilde{q}_{t_0-1}^*$, $g_{t_0-1}^c$, θ_{t_0-1} and the sequences $\{\theta_t, \theta_t^*\}_{t=t_0}^{\infty}$. Equation (57) is derived from (45), in which we have used (3) and (5) to substitute for $1+i_{t_0-1}^R$.

The analysis simplifies in the special case in which $\theta_t = \theta_t^*$ for which (46) implies that $\tilde{q}_t = \tilde{q}_t^*$. Equation (56) then implies a constant liquidity policy and the equivalence between the elasticity of the utility with respect to liquidity and the marginal cost of taxation:

$$-\frac{V_q(\tilde{q})}{V_{qq}(\tilde{q})\tilde{q}} = \psi'(\tilde{\tau}). \tag{58}$$

In a stationary solution, with $\theta_t = \theta_t^* = \theta = 1$ and $\tilde{q}_{t_0-1} = \tilde{q}_{t_0-1}^* = \tilde{q}_{t_0}$, (57) simplifies further to

$$2\frac{(1-\beta)\tilde{q}}{\beta} = \tilde{\tau} - \psi(\tilde{\tau}) + 2\frac{V_q(\tilde{q})\tilde{q}}{\beta} + L_g(G)g_{t_0-1}^c. \tag{59}$$

To compare whether the solution $(\bar{\tau},\bar{q})$ is welfare dominated by $(\tilde{\tau},\tilde{q})$, we consider special assumption on the utility from real liquidity as in Assumption 2

$$V(q)=\begin{cases}\ln\left(\frac{q_t}{\bar{q}}\right)-\frac{q_t}{\bar{q}} & q_t<\bar{q}\\ -1 & q_t\geq\bar{q}\end{cases}$$

and that the taxation cost is of the form

$$\psi(\tau)=\frac{\tau^2}{2}.$$

Moreover we also assume that $g^c_{t_0-1}=0$.

Under these assumptions, (58) and (59) can be written as

$$\tilde{q}=\bar{q}(1-\tilde{\tau}),$$

$$2\frac{(1-\beta)\tilde{q}}{\beta}=\tilde{\tau}\left(1-\frac{\tilde{\tau}}{2}\right)+\frac{2}{\beta}\left(1-\frac{\tilde{q}}{\bar{q}}\right),$$

which can be combined to get

$$\frac{(1-\beta)}{\beta}\bar{q}(1-\tilde{\tau})=\frac{\tilde{\tau}}{2}\left(1-\frac{\tilde{\tau}}{2}\right)+\frac{1}{\beta}\tilde{\tau}. \tag{60}$$

Note, moreover, that (55) implies:

$$\frac{1-\beta}{\beta}\bar{q}=\frac{\bar{\tau}}{2}\left(1-\frac{\bar{\tau}}{2}\right), \tag{61}$$

in which the right-hand side is increasing in $\bar{\tau}$ and maximum at $\bar{\tau}=1$ reaching the value 1/4. We further combine (60) and (61) to obtain:

$$\frac{\bar{\tau}}{2}\left(1-\frac{\bar{\tau}}{2}\right)(1-\tilde{\tau})=\frac{\tilde{\tau}}{2}\left(1-\frac{\tilde{\tau}}{2}\right)+\frac{1}{\beta}\tilde{\tau}, \tag{62}$$

which implies

$$\frac{\bar{\tau}}{2}\left(1-\frac{\bar{\tau}}{2}\right)(1-\tilde{\tau})\geq\tilde{\tau},$$

since $\frac{\tilde{\tau}}{2}\left(1-\frac{\tilde{\tau}}{2}\right)$ is non-negative and $\beta<1$.

Therefore

$$\tilde{\tau}\leq\frac{\frac{\bar{\tau}}{2}\left(1-\frac{\bar{\tau}}{2}\right)}{1+\frac{\bar{\tau}}{2}\left(1-\frac{\bar{\tau}}{2}\right)}. \tag{63}$$

Consider the utility flow in the solution with full satiation of liquidity:

$$\bar{U}=-\frac{\bar{\tau}^2}{2}-2,$$

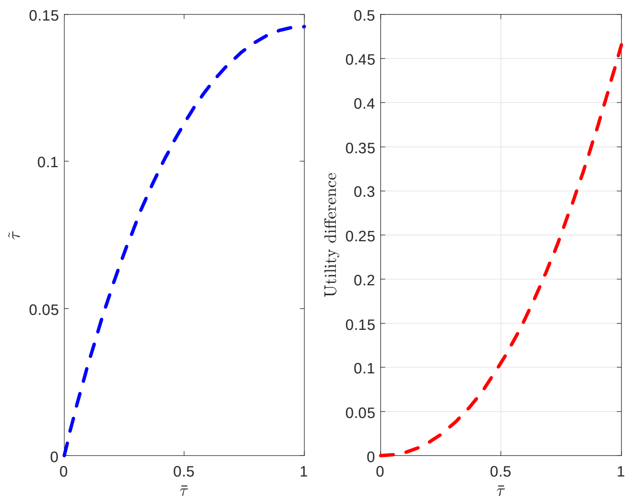

Figure 3: Left panel: $\tilde{\tau}$ as a function of $\bar{\tau}$. Right panel: difference in utility between the optimum liquidity and the full satiation solution as a function of $\bar{\tau}$.

disregarding output and the utility derived from gold. Instead, the utility in the equilibrium with lower liquidity is by

$$\tilde{U} = -\frac{\tilde{\tau}^2}{2} + 2\ln(1-\tilde{\tau}) - 2(1-\tilde{\tau}),$$

disregarding the same terms. Their difference is:

$$\tilde{U} - \bar{U} = \frac{\bar{\tau}^2}{2} - \frac{\tilde{\tau}^2}{2} + 2\ln(1-\tilde{\tau}) + 2\tilde{\tau},$$

which is non-negative for any $\bar{\tau} \in [0.1]$, given that $\tilde{\tau}$ is determined by (62). This result is shown numerically in Figure 3 for $\beta = 1$, and therefore holds for any $0 < \beta < 1$ given that $\tilde{\tau}$ decreases with β, as shown by (62). Figure 3, in the left panel, shows the relationship between $\tilde{\tau}$ and $\bar{\tau}$, determined by (62) when $\beta = 1$, and the utility difference, $\tilde{U} - \bar{U}$, as a function of $\bar{\tau}$. Two results follow: $\tilde{\tau}$ is substantially lower than $\bar{\tau}$ because of the resources obtained through the liquidity premium in the below-satiation liquidity equilibrium; utility is always higher in the below-satiation equilibrium compared to the full-satiation solution.