It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve*

Pierpaolo Benigno[1] and Gauti B. Eggertsson[2]

[1]University of Bern
[2]Brown University and NBER

June 11, 2025

Abstract

We propose a nonlinear "Inverse-L" (Inv-L) New Keynesian (NK) Phillips curve to explain the inflation surge of the 2020s. We measure labor market tightness as the ratio of job vacancies to unemployed workers and introduce the "Beveridge threshold"—a critical level above which both demand and supply shocks have amplified effects on inflation. After presenting robust statistical evidence for the Inv-L NK Phillips curve, we develop an NK model with search-and-matching frictions that provides a theoretical foundation for our empirical findings. Our analysis suggests that it is possible to reduce high inflation when labor market tightness exceeds the Beveridge threshold without triggering a severe recession, provided that inflation expectations remain stable. The necessary adjustment occurs primarily through declining vacancies.

*We are grateful to Luca Benati, Dan Cao, Stéphane Dupraz, Miroslav Gabrovski, Jordi Galí, Jonathon Hazell, Don Kohn, Guido Lorenzoni, Larry Meyer, Pascal Michaillat, Emi Nakamura, Aysegul Sahin, Patrizio Tirelli, Antonella Trigari, Chris Sims, Ivan Werning for helpful discussions and suggestions, to seminar participants at CUNY, UPF-CREI, EFIP, Brown University, CREST, Harvard University, MIT, Trinity College, Dartmouth, Richmond Fed, Università Politecnica delle Marche, to conference participants at NBER Summer Institute Monetary Economics, 5th International Conference on European Studies, the 2023 KDI/HAPI Conference, the Bank of Spain's 6th Annual Research Conference, the LSE Workshop on Macroeconomic Regime Change, Theory, Evidence, and Policy Challenges Ahead, IMF 6th Annual Conference, Central Bank of Iceland-Nortwestern Economic Conference 2024. We thank GaOn Kim, Can Soylu and Cristian Arcidiacono for providing excellent research assistance.

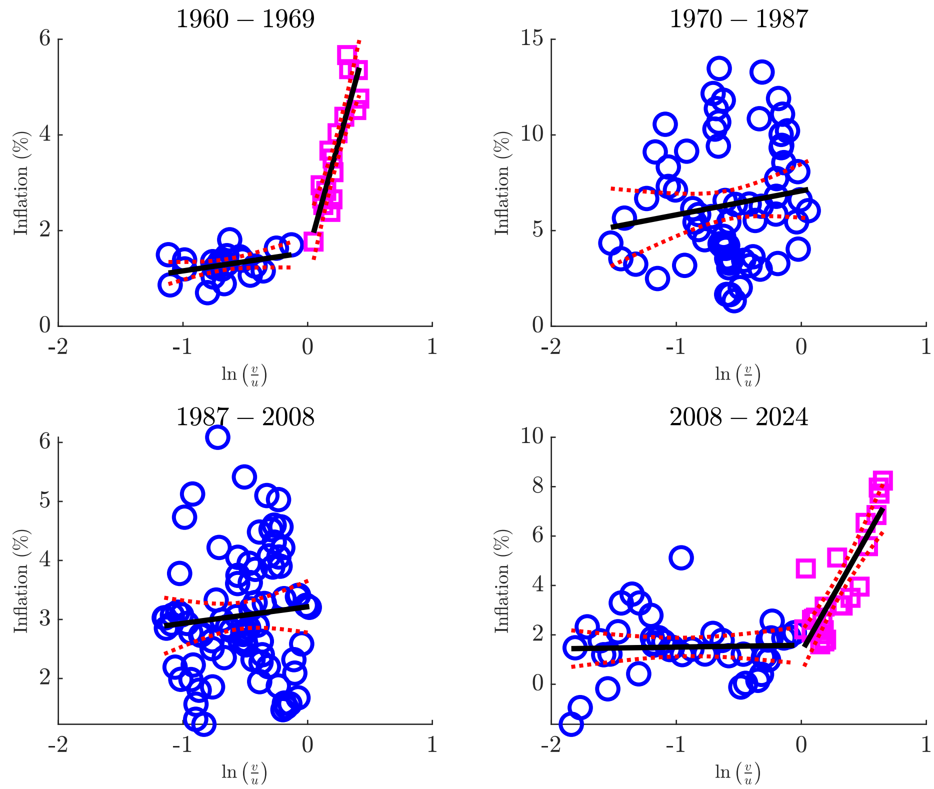


Figure 1: Inflation and Labor Market Tightness in the United States, 1960–2024. The y-axis represents the annualized inflation rate (CPI inflation), while the x-axis represents $\ln\left(\frac{v}{u}\right)$, the natural logarithm of the vacancy-to-unemployment ratio.

1 Introduction

We propose replacing the canonical New Keynesian Phillips curve with an inverse-L (Inv-L) New Keynesian (NK) Phillips curve.[1] The distinctive shape of the Inv-L curve, a slanted backward L, explains the sharp rise in U.S. inflation following the COVID-19 pandemic in early 2021 and its moderation by late 2024. Neither policymakers nor market participants anticipated the force or duration of the surge.[2] This forecast failure exposed a blind spot in modern macroeconomics. We argue that the Inv-L NK Phillips curve accounts for this failure and the observed inflation dynamics, and has significant policy implications.

The conventional NK Phillips curve struggles to explain the surge of the 2020s for three reasons. First, pre-surge estimates indicate a flat, linear curve where economic slack or tightness has little effect on inflation. Second, variations in traditional supply-shock measures predict a negligible impact on core inflation. Third, while conventional wisdom suggests that unanchored inflation expectations played a central role in generating the Great Inflation of the 1970s, expectations remained remarkably stable in the 2020s. This leaves the standard model lacking the usual suspects for the surge.

[1]See Woodford (2003) and Galí (2016) for textbook treatments.
[2]See Figures 17 and 18 in the Appendix

By contrast, our Inv-L NK Phillips curve accounts for the entire surge in inflation — both its onset and moderation in 2024 — by introducing a nonlinearity that amplifies the effects of *both* supply *and* demand shocks during unusually tight labor markets. The proposed curve also sheds light on the six inflationary surges observed over the past 115 years in U.S. history as discussed in Section 2.

Our analysis can be motivated by Figure 1. It draws inspiration from the seminal work of Phillips (1958), who plotted a curve in a scatter diagram relating nominal wage growth to the unemployment rate in the United Kingdom (see Section 2). Our figure, in contrast, shows annualized quarterly inflation along with the vacancy-unemployment ratio (v/u) in the United States from 1960 to 2024, split into four subperiods. One can hardly fail to notice an inverse L shape in the upper left and lower right corners whenever labor market conditions exceed a certain threshold close to $v/u = 1$ $\big(\log(v/u) = 0\big)$. Purple squares, present only in the samples from 1960–1969 and 2008–2024, mark periods of *labor shortage* when v/u crosses this threshold, while blue circles represent normal labor conditions below it.

The threshold defining the kink in the Inv-L Phillips curve has an interesting interpretation. Beveridge (1941) defines $v/u = 1$ as the focal point where the labor market is neither slack nor tight—that is, the number of unemployed workers equals the number of vacant jobs. We label the inflection point the *Beveridge threshold*.

Our first contribution is empirical and aligns closely with the established literature estimating Phillips curves and reflects recent developments showing that the vacancy-unemployment ratio is a better measure of economic activity than the unemployment rate alone.[3]. Our contribution is to identify a piecewise-linear specification that approximates an inverse L with a single threshold. The threshold is determined by maximizing the likelihood estimation resulting in a kink point of approximately 1.[4]

Our empirical results show that crossing the threshold amplifies the impact of supply shocks and v/u itself on inflation. This shift is not only statistically significant but also economically important. For example, the slope of the Phillips curve with respect to v/u increases by a factor of six in our baseline estimation, past the threshold. Once the threshold is crossed, supply shocks significantly spill into core inflation. In contrast, below the threshold, supply shocks are estimated to have a trivial effect on core inflation.

Despite using data from 1960 to 2024, a linear Phillips curve fails to explain the inflation surge in the sample, attributing most of it to residual error terms. In contrast, the empirical estimation of the Inv-L Phillips curve captures the surge effectively, fully explaining the 2022Q1 peak with one-third driven by supply shocks and two-thirds by labor-market tightness.

Our second and primary contribution is a theoretical framework: we develop a new model that incorporates (v/u) into a standard New Keynesian setting, drawing inspiration from the search-and-matching literature.[5] An important implication of this integration is that the resulting Phillips curve

[3] See discussion in Subsection 1.1 and Section 2

[4] Recently, Michaillat and Saez (2022) present a model in which this threshold corresponds to the optimal allocation of labor.

[5] See, e.g., Pissarides (2003) for an overview.

deviates from the traditional formulation by replacing the output gap or unemployment with v/u as the relevant measure of economic activity. Additionally, it incorporates non-linearity through wage setting.

The framework delivers two major theoretical results. First, the INV-L NK Phillips curve remains relatively flat with respect to economic activity until it crosses the Beveridge threshold. After this point, its slope steepens, generating two important effects. Beyond the threshold, supply shocks have a significantly larger impact on inflation. Additionally, inflation becomes more sensitive to v/u; that is, the Phillips curve transitions from flat to steep. The empirical results suggest that the aggregate U.S. time-series data are consistent with our proposed theoretical framework. Another important theoretical result is the formalization of the Beveridge threshold, derived from the model's parameters. We demonstrate that a value of one is just one theoretical possibility; moreover, the Beveridge threshold may vary endogenously over time.

The remainder of this paper is organized as follows. The second part of the introduction clarifies how our work extends and differs from existing contributions, outlining key theoretical assumptions in the process. Section 2 provides historical background on the Phillips curve, situating our proposal within the broader literature and historical data. Readers uninterested in these details can skip this section without disrupting the flow of the paper. Section 3 presents our empirical results. Section 4 outlines our model and defines the Inv-L NK Phillips curve. Section 5 derives a piecewise-linear approximation, identifies the theoretical Beveridge threshold, and demonstrates the role of shocks. Section 6 applies the complete model to policy analysis, examining various historical episodes, including the Great Inflation of the 1970s, the inflation surge of the 2020s, and the "missing deflation" of 2008.

1.1 Additional discussion and related literature

Rather than surveying the extensive Phillips curve literature, we focus on the work most directly related to ours, including several additional details and references in the main text.[6]

Consider first our estimation results. We build on recent research showing $\frac{v}{u}$'s superiority in forecasting inflation over traditional metrics of slack (Furman and Powell (2021), Barnichon, Oliveira, and Shapiro (2021), Domash and Summers (2022), Ball, Leigh, and Mishra (2022), Barnichon and Shapiro (2024)). While these studies use linear specifications — except for Ball, Leigh, and Mishra's (2022) flexible polynomial — we propose a simple nonlinearity captured by a piecewise linear function that becomes steeper once $\frac{v}{u}$ crosses a single threshold.[7] More importantly, the estimation strategy is based on a novel theoretical approach, representing proof of concept.[8]

[6]For overview articles on estimation, see Mavroeidis, Plagborg-Moller, and Stock (2014) and Coibion, Gorodnichenko, and Kamdar (2018). While McLeay and Tenreyro (2019) present several novel results, they also provide a lucid overview of the literature and challenges in identification.

[7]We have a more detailed comparison to Ball et al. (2022) in the main text.

[8]Other papers that consider nonlinearities include Gagnon and Collins (2019). Others have stressed time variation in the slope, such as Benati (2010), Blanchard et al. (2015), Blanchard (2016), and Matheson and Stavrev (2013).

Prior to the 2020s surge, a consensus emerged that the Phillips curve was "flat"—inflation varied little with slack/tightness measures. This view is embedded in key policy models, including the Federal Reserve Board's FRBUS model, which is a critical input for FOMC meetings. Hazell, Herreño, Nakamura, and Steinsson (2022) reinforced this consensus using cross-state U.S. data to show minimal inflation response to unemployment changes.[9] Their finding of a flat Phillips curve aligns with ours, as their 1978-2018 sample never crosses the Beveridge threshold (see also McLeay and Tenreyro (2019) who also use cross-state data). Recent work using similar methods confirms our conclusion that the Phillips curve remains flat until this threshold is breached (Cerrato and Gitti (2023) and Gitti (2023)).[10] Figure 24 in Appendix B, borrowed from Gitti (2023), provides visual support for this pattern: data from 21 U.S. Metropolitan Statistical Areas since 2000 display similar relationship as displayed in Figure 1.

Our theoretical framework is most closely related to the growing literature that integrates NK monetary policy with labor market search and matching, in the spirit of Mortensen and Pissarides (1994). We benchmark our assumptions against Blanchard and Galí (2010a), an influential paper in this large and growing field.[11]

Our key contribution is to demonstrate why the Phillips curve steepens sharply beyond the Beveridge threshold, explaining recent inflation dynamics through a nonlinear relationship with $\frac{v}{u}$ as the measure of slack. This inverse-L (Inv-L) Phillips curve is generated through three key innovations.

First, by distinguishing between new and existing workers, we show that marginal cost reflects new-hire wages rather than average wages. Our framework incorporates unemployment-to-employment transitions, job-to-job moves, and changes in labor-force participation–features often missing from NK search-and-matching models– even if, admittedly, we do so in a relatively reduced form to preserve parsimony.

Second, we address the common problem of wage indeterminacy in search-and-matching models (see Hall (2005)) by introducing an employment agency that helps pin down wages. This tractable mechanism determines wages – if completely flexible – as a function of labor market tightness $\frac{v}{u}$.

Third, following Phillips (1958), we incorporate asymmetric wage-setting dynamics. Existing wages adjust sluggishly but are gradually pulled toward the flexible wage, which serves as an anchor. New-hire wages cannot fall below existing wages–a constraint that binds during slack labor markets. In tight markets, however, new hires can command higher wages than existing workers and track the flexible wage once the labor market crosses the Beveridge threshold.[12]

[9] Their headline result is that a 1 percent decrease in unemployment increases inflation by 0.34 percent.

[10] Cerrato and Gitti (2023) use similar empirical methods to Hazell et al. (2022) but extend the data to include part of the surge. Consistent with our findings, they find that the slope of the Phillips curve increases substantially during the surge. See also Smith, Timmermann, and Wright (2023), who analyze U.S. and E.U. cross-sectional data and detect a kink when the labor market is running hot. Adrjan and Lydon (2023) document a particularly hot labor market for lower-paid workers.

[11] For other examples of this integration, see Walsh (2005), Krause and Lubik (2007), Ravenna and Walsh (2008, 2011), Gertler, Sala, and Trigari (2008), Krause, Lopez-Salido, and Lubik (2008), Galí (2009), Barnichon (2010a), Michaillat (2012, 2014), and Christiano, Eichenbaum, and Trabandt (2016).

[12] See also Benigno and Ricci (2011), Eggertsson, Mehrotra and Robbins (2019), Schmitt-Grohé and Uribe (2016), for alternative models of downward wage rigidities.

These three innovations are key to obtaining the Inv-L NK Phillips curve, highlighting the role of $\frac{v}{u}$ and departing from the NK model that focuses primarily on unemployment or the output gap. Our approach clarifies the theoretical conditions under which $\frac{v}{u}$ provides a more informative measure of inflationary pressures than unemployment alone. Recent studies, including those by Abo-Zeid and Sheng (2024) and Michaillat and Saez (2024a), similarly emphasize the centrality of $\frac{v}{u}$ in understanding inflation dynamics.[13]

Our framework resolves an apparent puzzle: while average wages (e.g., the Employment Cost Index) and the labor share declined during the surge, labor market tightness intensified alongside rising inflationary pressures. This divergence is explained by the fact that marginal costs depend on *new* rather than *average* wages in our model. Beyond the Beveridge threshold, new wages rise substantially above existing wages, which adjust only gradually. This accounts for lower average wages, a declining labor share, and inflationary pressures due to higher marginal costs reflected in surging new-hire wages. Evidence supports this pattern: new wages increased markedly relative to existing ones during the surge, consistent with reported labor shortages and hiring difficulties discussed in Section 4.4.

Our focus on new wages as the measure of marginal cost reconciles our results with Bernanke and Blanchard (2024), but via a different mechanism. While they attribute inflation early in the surge primarily to supply shocks—using average wages as a proxy for marginal costs, consistent with the standard NK model—and emphasize labor market tightness only later, our model suggests that the substantial rise in new wages at the surge's onset indicates labor market pressure was significant throughout, as new wages determine marginal cost.[14]

The pre-surge consensus maintained that oil and commodity shocks had limited, temporary effects on core inflation. Our framework reconciles this view with recent literature emphasizing that once the Beveridge threshold is crossed, both supply and demand shocks become amplified. This mechanism aligns with accounts such as Gagliardone and Gertler (2023), who highlight the interaction between commodity shocks and monetary policy during the surge.

Beaudry, Huo, and Portier (2024, 2025) attribute inflation primarily to rising inflation expectations and supply shocks, arguing that the Phillips curve slope remains unchanged. While their analysis has some empirical support, it largely relies on the University of Michigan Surveys of Consumers' one-year inflation expectations (UMich Inflation Expectations) over special subsamples. We argue that our narrative has stronger backing based on robustness checks across seven alternative expectation measures to our baseline, totaling eight alternative time series estimates of expectations. For our *short sample*, results are robust across all eight metrics, including UMich Inflation Expectations. For the *long sample*, results hold for seven of the eight measures.[15] The exception is the UMich Inflation

[13] Another study highlighting nonlinearities in the Phillips curve is Harding, Linde and Trabandt (2023). A major difference from our work is that instead of generating nonlinearities through labor shortage, they trace them to quasi-kinked demand for goods, as in Kimball (1995).

[14] This reconciles our narrative with theirs: what appears to be a supply shock in standard NK models manifests as labor market pressure when focusing on new wages.

[15] The authors experiment with other data slices yielding similar results to our long sample.

Expectations. The other seven measures include one-year CPI expectations from the U.S. Survey of Professional Forecasters, Cleveland Fed's one-, two-, and five-year expectations, the 12-month Livingston Survey, and a market-based measure backcasted by Groen and Middledorp (2013).

What explains this difference? We show in Section 3.3.3 that the Michigan Survey expectations is the only one out of the eight that is Granger-caused by observed inflation. Specifically, an increase in inflation predicts an increase in the UMich Inflation Expectations, in contrast to the other seven metrics. This arguably contradicts the basic causal structure of the New Keynesian Phillips Curve, where movements in inflation expectations drive observed inflation through firms' price setting. Chodorow-Reich (2024) argues that the Atlanta Fed's Business Inflation Expectations (initiated in 2011) is the most appropriate theoretical construct for firms' pricing decisions. This survey correlates closely with the Survey of Professional Forecasters, which is one of the series that generates robust conclusions in our analysis.[16]

In Figure 1, each point represents the intersection of aggregate demand and supply in our model. This poses a challenge to those advocating a flat Phillips curve. To match it, one must rely on coincidental demand and supply shifts to replicate the inflation-labor market scatterplot, especially once the Beveridge threshold is crossed. In contrast, our interpretation, grounded in both empirical evidence and theoretical analysis, identifies a curve that closely mirrors the theoretical relationship proposed nearly a century ago. This is particularly evident during the 1960s surge, a period marked by limited supply shocks and modest shifts in expectations. Estimating a piecewise linear Phillips curve over this period reveals significant steepening, even when controlling for Michigan Survey inflation expectations.

2 Historical Background

The Phillips curve is one of the most controversial equations in macroeconomics. In this section, we situate our contribution in a broader historical context. Readers primarily interested in our main results may proceed directly to Section 3 without losing the thread of the paper.

Long before Phillips's work, economists had to take a stance on the relationship between aggregate output and prices to make meaningful statements about the national economy. Our analysis is closely related to what Blinder (2022) labels "crude Keynesianism", which relies on the intuitive but fundamental concept of limited production capacity. The early Keynesian literature proposed a simple relationship between prices and output, depicted as an inverted L-shape in the left panel of Figure 2. When output is below potential, prices remain fixed in response to variations in aggregate demand because firms can hire idle workers and some factories remain unused. Once there are no workers left

[16] See further discussion addressing the Cleveland Fed's more recent survey in Chodorow-Reich (2024), started in 2018, which reports overall CPI expectations of CEO's. This series is currently relatively short, so it is difficult to draw strong conclusion. Beudry et al (2025) argue they match most closely the Michigan Consumer Survey Expectation. The Cleveland Fed survey, however, does not contradict our finding since they would fall into the "short sample", which is robust to using UMich Inflation Expectations.

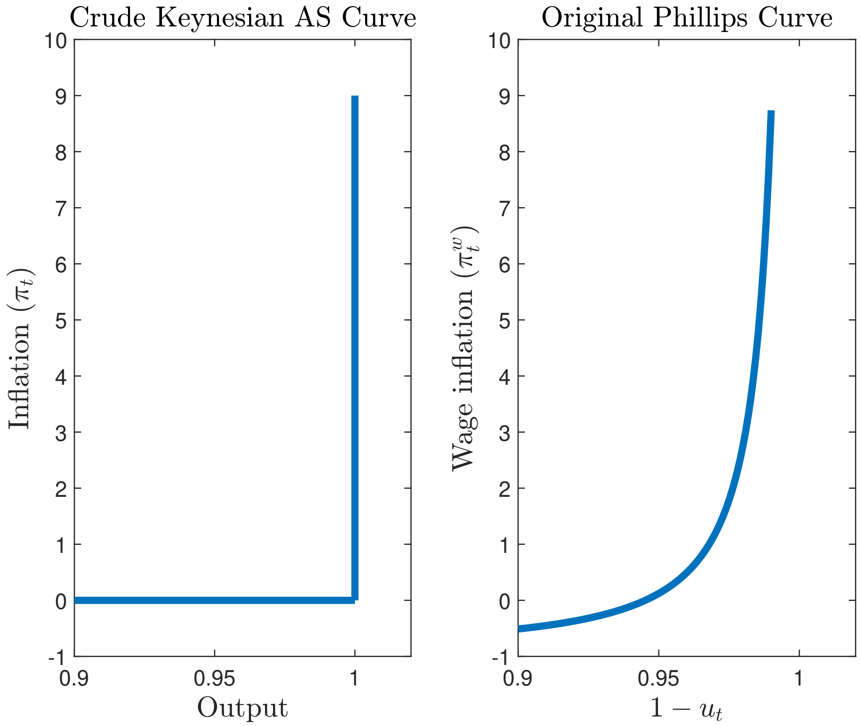


Figure 2: The crude Keynesian Phillips curve versus the original Phillips curve proposed and estimated by Phillips in 1958.

to be hired and factories are at full capacity, production cannot expand further, and the aggregate supply curve is vertical. At this point, the economy enters a neoclassical regime with fully flexible prices. Our Inv-L NK Phillips curve provides theoretical foundations for this old Keynesian proposition but with gently sloping legs rather than a sharp 90-degree angle.

Our framework reconciles key insights from both Keynes and Friedman.[17] Keynes posits that downward wage rigidity explains why increases in nominal spending raise real output and employment. Yet, he also develops a theory of demand-side inflation—similar to the neoclassical account of World War II inflation—which emerges when "the size of the cake is fixed" (Keynes, 1940, p. 4).

Similarly, the view that the economy is fundamentally asymmetric aligns with Friedman's plucking model. In Friedman (1964, 1993), "output is seen as consistently bumping along the ceiling of maximum feasible output, except that it is occasionally plucked down by a cyclical contraction "(Friedman, 1964, p. 17).[18]

Keynesianism took a decisive turn with Phillips's seminal 1958 paper, whose influence is so profound that it rarely requires explicit citation. The term "Phillips curve" has become a standard part of the

[17]See the parallel made in Benigno and Eggertsson (2024b).
[18]See Dupraz, Nakamura, and Steinsson (2019) for a recent attempt to resurrect Friedman's plucking model.

macroeconomic lexicon, much like “Nash equilibrium” is seldom attributed to Nash’s original paper in game theory.

Phillips (1958) emphasized two elements that have largely been lost in subsequent formulations: the relationship between unemployment and *nominal wage* growth (rather than price inflation) and, more importantly, its pronounced nonlinearity. His proposed curve is:

$$\pi_t^w = a + b\left(\frac{1}{u_t}\right)^c,$$

where π_t^w represents wage inflation, u_t is unemployment, and a, b, and c are estimated coefficients. The right panel of Figure 2 reproduces Phillips’s original curve using his 1861-1913 estimates, with $1 - u_t$ on the x-axis to facilitate comparison with crude Keynesianism.[19]

The two key elements of Phillips’s original work that we emphasize were "lost in translation" when the Phillips curve came to America in a classic paper by Samuelson and Solow (1960). This paper played a major role in popularizing the Phillips curve within the economics profession. However, its formulation also explains why the two elements of Phillips’s original work that we highlight faded into the background. First, Samuelson and Solow placed much less emphasis on nonlinearity in their Phillips curve. Second, they replaced wage inflation with the change in the general price level in their famous "Modified Phillips Curve for U.S.," depicted in Figure 2 of Samuelson and Solow (1960, p. 192).[20]

Our Inv-L NK Phillips curve resurrects Phillips’s original insights about labor-market nonlinearity—a concept that surpasses the simple downward rigidity of nominal wages common in the literature. We achieve this by incorporating asymmetric wage setting that depends on labor market tightness within a search-and-matching framework. This resulting asymmetry is central to our Inv-L NK Phillips curve. The return to Phillips’s fundamental nonlinear wage determination inspires our paper’s title.[21]

Turning to more recent history, consider the now-dominant New Keynesian canonical Phillips curve, which our Inv-L NK Phillips curve aims to replace. It is expressed as:

$$\underbrace{\pi_t = \kappa x_t}_{\substack{\text{Keynesian} \\ \text{Phillips Curve}}} \quad + \quad \underbrace{\kappa_\varrho \varrho_t + \beta E_t \pi_{t+1}}_{\substack{\text{New Keynesian} \\ \text{Phillips Curve}}} \tag{1}$$

where π_t denotes inflation, κ and κ_ϱ are coefficients, and x_t measures economic activity (either the output gap, with $\kappa > 0$, or unemployment, with $\kappa < 0$). Coefficient β lies between 0 and 1, E_t

[19] This reproduces Figure 1 from Phillips (1958), using his least squares estimates of $b = 9.636$ and $c = 1.394$, with $a = -0.9$ determined by “trial and error” using U.K. data from 1861 to 1913.

[20] Regarding nonlinearity, Samuelson and Solow remark: “The English data show a quite clearly nonlinear (hyperbolic) relation between wage changes and unemployment, reflecting the much-discussed downward inflexibility. Our American figures do not contradict this, although they do not tell as plain a story as the English” (Samuelson and Solow, 1960, p. 190).

[21] The title also plays off Krugman’s (1998) famous paper on the zero lower bound on interest rates, which helped launch a literature on the liquidity trap—a topic that had been largely forgotten and dismissed as an old Keynesian fairy-tale at the time (Krugman (2021)).

represents the expectation operator, and ϱ_t captures supply shocks.

The first term in equation (1), indicated by the left underbrace, represents the Keynesian Phillips curve. This curve was popularized by Samuelson and Solow (1960). The second term incorporates supply shocks and inflation expectations, reflecting lessons from the Great Inflation of the 1970s and the subsequent Volcker disinflation of the early 1980s. Together, these components form the New Keynesian Phillips curve, a cornerstone of modern monetary policy models.

The Keynesian Phillips curve's collapse as a stable relationship in the 1970s (see Figure 16 in the Appendix) marked a turning point in macroeconomics. This failure gave rise to the rational expectations revolution, and microfounded models became mainstream—partly because they offered an explicit account of expectations formation, which could change rapidly with shifts in policy regime. The breakdown was particularly dramatic because Phelps (1967) and Friedman (1968) had theoretically predicted it: they argued that any government attempt to exploit the inflation-unemployment trade-off would be self-defeating as inflation expectations adjusted to the new policy regime, shifting the relationship as shown in equation (1). When inflation rose in the late 1960s, it appeared the government was attempting to exploit the inflation-output trade-off implied by the Keynesian Phillips curve. The relationship's subsequent collapse in the 1970s, exactly as predicted, lent considerable credibility to Friedman and Phelps's prophecy.

The Keynesian Phillips curve's demise in the 1970s is now primarily attributed to two factors: unanchored inflation expectations and supply disruptions. The inclusion of expected inflation in equation (1), now central to all modern Phillips curve formulations, explains why central banks prioritize anchoring inflation expectations—a rise in expected inflation acts much like a negative supply shock.

Our work makes two key contributions to the NK formulation in equation (1). First, we emphasize the nonlinearity and steepness of the curve under labor market tightness, where the parameters κ and κ_ϱ increase once the Beveridge threshold is crossed. This feature is crucial for explaining the recent inflationary surge and reconciling the analysis with historical U.S. inflation dynamics. Second, we depart from the NK Phillips curve by replacing unemployment with the vacancy-to-unemployment ratio (v/u) as our measure of slack. This choice proves particularly insightful as historical data show that inflation's sensitivity increases when v/u crosses the Beveridge threshold.

U.S. economic data document five instances where $\theta \equiv (v/u)$ substantially crossed a Beveridge threshold of one: World War I, World War II, the Korean War, the Johnson tax cuts coupled with Vietnam War spending, and the recent labor shortages of the 2020s. Each crossing coincided with an inflation surge, as shown in Figures 3 and 4. However, price controls during the first three episodes complicate empirical analysis, leading us to focus on the period from 1960 to 2024 in the next Section.[22]

Notably, the Great Inflation of the 1970s stands as the only major U.S. inflation surge in the past 115 years not associated with crossing the Beveridge threshold. This aligns with the conventional

[22]Rockoff (1981) also discusses Nixon's price controls from August 1971 to April 1974, shown in Figure 4. These controls are less problematic for our analysis since they occurred without a labor shortage, and our focus is on nonlinearity when $v/u > 1$.

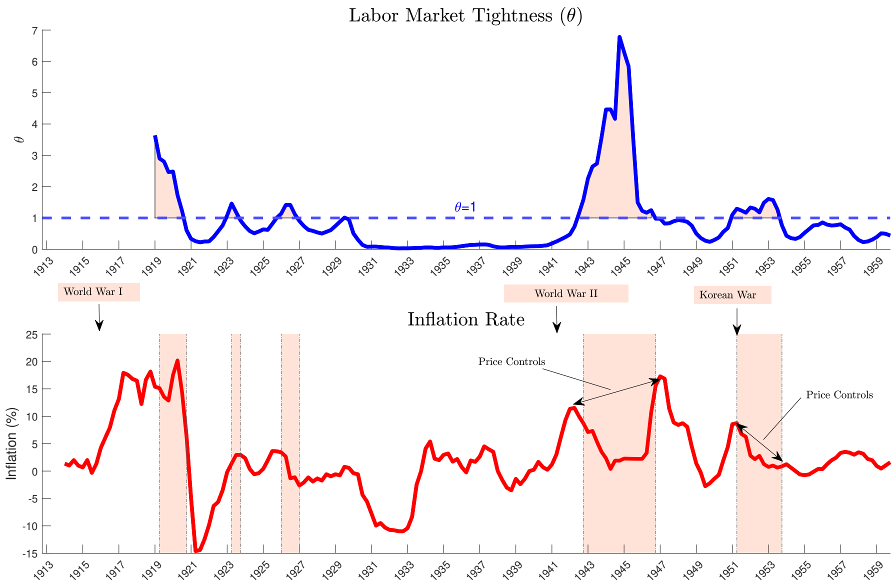


Figure 3: Top panel: CPI inflation rate at annual rates. Bottom panel: (θ) vacancy-to-unemployed ratio. Period 1913 Q1 – 1959 Q4. Source: Petrosky-Nadeau and Zhang (2021) and BLS.

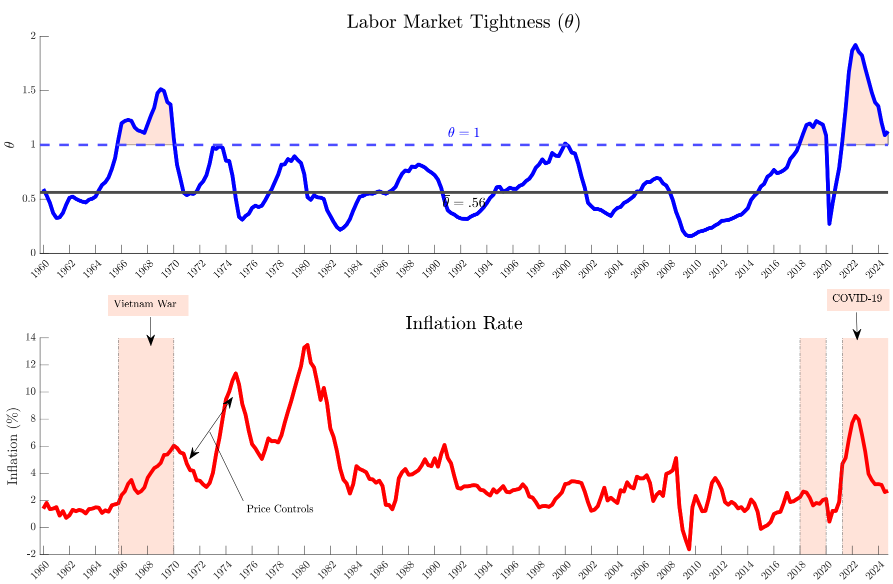


Figure 4: Top panel: (θ) vacancy-to-unemployed ratio. Bottom panel: CPI inflation rate at annual rates. Period 1960 Q1 – 2024 Q4. Source: Petrosky-Nadeau and Zhang (2021), Barnichon (2010), and BLS.

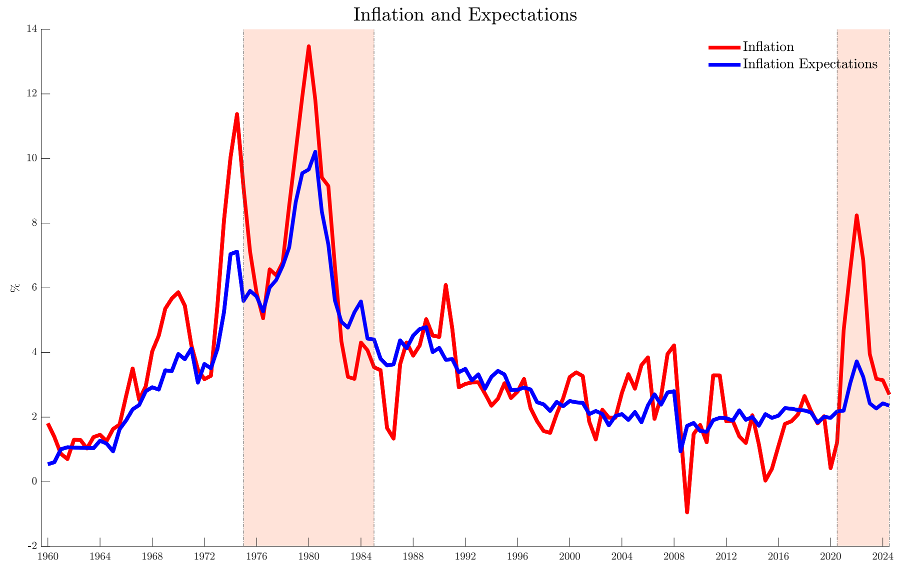


Figure 5: Inflation: CPI inflation rate at annual rates. 12-month Livingston inflation expectations.

wisdom attributing a significant role to unanchored inflation expectations. As evidenced by the Livingston survey data in Figure 5, expectations closely followed current inflation during this period. In contrast, inflation expectations remained remarkably stable during the 2020s surge, particularly at longer horizons—a distinction that underscores the importance of focusing on labor market nonlinearities rather than expectation dynamics, as discussed in detail in the next Section.

Our proposed Inv-L NK Phillips curve retains the critical feature of the NK Phillips curve: the importance of inflation expectations due to forward-looking price setting. This aspect is not present in "crude Keynesianism" or in much of the early literature on the Phillips curve leading up to the 1970s. Unanchored inflation expectations are crucial in our framework to reconcile it with the 1970s Great Inflation. This period is the only inflationary episode in the last 115 years in the U.S. not associated with the labor market crossing the Beveridge threshold, as we discuss further in Benigno and Eggertsson (2024a).

Our follow-up paper further develops a connection that we only briefly explore here: the relationship between the Phillips curve and another cornerstone of macroeconomics, the Beveridge curve. In the U.S., both the Phillips and Beveridge curves steepen at the same point, the Beveridge threshold. The nature of the disinflationary process after a surge, whether it leads to a soft or hard landing in terms of unemployment, depends on whether the surge was driven by labor market tightness (v/u) above or below the Beveridge threshold.[23]

[23] Blanchard (1989) provides an early and insightful study of the interaction between these two curves. Recent contributions to this area include Barnichon and Shapiro (2024) and Michaillat and Saez (2024b).

3 Empirical Results

Our strategy is to stay as close as possible to the existing literature on Phillips curve estimation, which has a long history. Our innovation is simple: we introduce a dummy variable that interacts with supply disturbances and labor market tightness. The model in Section 4 directly implies these extensions.

Although Figure 1 hints at the possibility of a nonlinear Phillips curve, each point on a scatter plot represents an equilibrium outcome viewed through the prism of a general equilibrium model. In the model we present, for instance, each data point is determined by the intersection of the Phillips Curve (aggregate supply) and the aggregate demand. Aggregate demand is influenced by the spending decisions of households and the government, as well as by monetary policy. Consequently, from a theoretical perspective, one may not have strong reasons to believe that the scatter plot, viewed in isolation, provides compelling evidence one way or the other.

The identification problem inherent in a scatter plot of this kind has been well understood for over a century, dating back at least to Lenoir (1913).[24] If only the supply curve shifts, the data traces out demand. Conversely, if only demand shifts, the data traces out aggregate supply. Viewed in this light, the data for the 1960s, displayed in the upper-left panel of Figure 1, is interesting. According to the measures discussed below, the factors influencing the Phillips curve's shift (such as various proxies for inflation expectations and supply disturbances) remained relatively stable during this period. Meanwhile, shifts in aggregate demand during the 1960s are well documented (see, for example, Blinder, 2022). The figure speaks for itself.

The literature on Phillips curve estimation and identification is extensive, and we refer readers to the survey articles cited in Footnote 6 for comprehensive reviews. A key insight from McLeay and Tenreyro (2019, Section 5) is that the Phillips curve relationship can be empirically recovered when supply shocks and inflation expectations are properly controlled for.

Our main empirical specification is an ordinary least-squares regression with constant coefficients, allowing for piecewise nonlinearity through a dummy variable. We conduct two additional estimations: first using lagged values as instruments, a common alternative approach, and second allowing time-varying coefficients estimated via the Kalman filter (detailed in Appendix B). Recall that we define $\theta \equiv \frac{v}{u}$ and the Beveridge threshold as θ^*. All three approaches support our two central conclusions: (i) the Phillips curve exhibits statistically and economically significant nonlinearity when the Beveridge threshold is crossed ($\theta > \theta^*$), and (ii) supply shocks are amplified beyond this threshold.

The empirical implementation requires several decisions regarding which data best represent the variables in our theoretical framework, developed in the next section. We examine alternative specifications in Section 3.3.3, with additional details provided in the Appendix. First, however, we explain our use of v/u rather than unemployment alone as our measure of economic slack, following recent literature cited in the Introduction.

[24] For the historical context of the identification problem in macroeconomic models, see Christ (1994).

3.1 Why is it useful to use vacancy-to-unemployed ratio as measure of economic slack

The inflation surge in the 2020s began in March 2021 as the U.S. economy was recovering from the recession triggered by COVID-19. During this period, both the core Personal Consumption Expenditures (PCE) index and the Consumer Price Index (CPI) exceeded the Federal Reserve's 2 percent target for year-on-year inflation. The core PCE peaked at 5.6 percent in February 2022. The Federal Reserve began increasing the Federal Funds rate in March 2022, coinciding with the decline of core PCE. The CPI peaked at 9 percent in June 2022, partially fueled by the war in Ukraine, which inflated oil prices excluded from the core PCE.

One reason the Federal Reserve delayed rate increases for a full year after inflation exceeded its target was that traditional measures of slack, such as unemployment, remained well above pre-pandemic levels. For instance, in March 2021, the unemployment rate stood at 6 percent, while the Federal Reserve Open Market Committee considered maximum employment to be 4 percent. Another widely used metric, the prime-age-employment-to-population ratio, was 76.9 in March 2021, considerably below the pre-pandemic ratio of 80.5. It was not until these metrics returned to their pre-pandemic levels in March 2022 that the Federal Reserve began raising rates. [25]

However, despite traditional measures of slack indicating little reason for inflationary pressures from the labor market, the ratio of vacancy to unemployed was flashing red. Many firms were desperately seeking to hire more workers, with numerous establishments partially closing and displaying signs using "labor shortage" as the reason for the temporary closure (see for example anecdotal evidence from the U.S. in Figure 20). The vacancy-to-unemployed ratio exceeded 1 in May 2021, reaching its peak in March 2022 when the Federal Reserve initiated rate hikes. By that time, the v/u ratio had surpassed 2, signifying the tightest labor market since World War II.

A number of recent empirical studies have opted to use v/u instead of more traditional measures of labor market tightness, such as unemployment, to explain the inflation surge in the 2020s. The v/u ratio has proven superior to unemployment alone across multiple performance metrics. Furman and Powell (2021) find that even in the pre-pandemic period (2000–2019), v/u was the best predictor for core CPI, although it did not outperform alternatives like unemployment by a significant margin.[26]

As we will see in the theoretical Section, the v/u ratio becomes more informative than u alone under two conditions: time variation in either (i) the employment match efficiency or (ii) the job separation rate. Both factors shift the Beveridge curve we derive in our model, and we show both were quantitatively important during the 2020s inflation surge.

[25] For a more detailed discussion of the Federal Reserve's policy decisions during this period, see Eggertsson and Kohn (2023).

[26] Figure 25 in the Appendix shows that in the decade leading up to the inflation surge, the v/u ratio signaled largely the same information as unemployment, consistent with Furman and Powell's (2021) finding that the gains from using v/u instead of u to predict core CPI were relatively modest. Figure 25 is based on a regression proposed by Kalantzis (2023).

3.2 Empirical results using a benchmark regression

In the benchmark empirical framework, we consider the following ordinary least squares regression:

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t)\ln\theta_t + (\beta_\varrho + \beta_{\varrho_d} D_t)\varrho_t + \beta_{\pi^e}\pi_t^e + \varepsilon_t, \tag{2}$$

where β_c, β_π, β_θ, β_{θ_d}, β_ϱ, β_{ϱ_d}, β_{π^e} are parameters, and ε_t is a zero-mean normally-distributed error. D_t is a dummy variable that takes value one if $\ln(\theta_t) \equiv \ln\frac{v}{u} \geq 0$. $\pi_t \equiv \ln\frac{P_t}{P_{t-1}}$ is inflation, π_{t-1} is its one-quarter lag, $\ln\theta_t$ is the logarithm of the vacancy-to-unemployed ratio, ϱ_t is a supply shock, and π_t^e is inflation expectations.

In formulating (2), we follow as closely as possible the recent literature. Our empirical contribution is simple: accounting for nonlinearity in a parsimonious way. The nonlinearity takes a special form, as can be seen from equation (2), i.e., it is piecewise linear in logs. When $\theta > 1$, the slope of the regression can differ from that under normal circumstances.

Our piecewise-linear specification differs from, but complements, other recent approaches to Phillips curve nonlinearity. Ball, Leigh, and Mishra (2022) is closely related to our work. While our approach focuses on regime-specific nonlinearity (determined by whether $\theta_t > \theta^*$), their specification allows for continuous nonlinearity through squared and cubed terms of θ_t, which they find to be statistically significant. Figure 1 motivates our piecewise-linear specification, suggesting the relationship in logs can be well approximated by two linear segments linked at a single threshold point. Moreover, this specification aligns naturally with the theoretical model we develop in the next section.

Table 1 presents estimates from an OLS regression using U.S. quarterly data from 1960 Q1 to 2024 Q4. The dependent variable is core CPI inflation (excluding food and energy prices), with all inflation rates expressed in annualized terms relative to a 2 percent baseline.[27] While we have already discussed our key explanatory variable θ (in logarithmic units), the next subsection details our choice of other variables. Given the lack of consensus in the literature, we demonstrate robustness to alternative specifications, including identifying the Beveridge threshold by maximizing the likelihood function of the model.

The first main result from Table 1 is that the Phillips curve exhibits significant nonlinearity in θ (at the 1% level). Columns (3) and (4) show this for the full sample and 2008–2024 subsample, respectively. The slope of the curve when $\theta > 1$ is the sum of the second and third rows, whereas for $\theta < 1$ it is given by the second row alone.[28] While the point estimate for $\theta < 1$ has the expected sign, we cannot reject that the Phillips curve is flat in this region, in the full sample, consistent with the crude Keynesian model (see panel (a) of Figure 2 in Section 2). When we constrain the slope to be constant, as shown in columns (1) and (2), the coefficient is larger and statistically significant, suggesting this significance is driven by periods where $\theta > 1$. The estimated slope increases by a factor of about 6

[27] Estimates are invariant to this adjustment, except for the constant. The choice of 2 percent as baseline, matching the Federal Reserve's current inflation target, aids in interpreting the constant term.

[28] For example, in column (4), the slope when $\theta > 1$ is $0.5185 + 5.4627 = 5.9812$.

in the full sample and 8 in the short sample, when comparing the constant-slope specification to the slope in periods where $\theta > 1$.

Table 1: Phillips Curve Estimates

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.3696^{***} (0.0947)	0.272 (0.2445)	0.2547^{***} (0.093)	-0.1469 (0.1926)
$\ln\theta$	0.6722^{***} (0.1758)	0.7235^{**} (0.3642)	0.2315 (0.1973)	0.5185^{*} (0.3057)
$\theta \geq 1$			3.7753^{***} (0.8353)	5.4627^{***} (0.8929)
Supply shock (ϱ)	0.0378^{**} (0.0192)	0.0187 (0.0395)	0.0447^{**} (0.0205)	-0.0096 (0.0229)
$\theta \geq 1$			0.1038 (0.0995)	0.2743^{**} (0.1205)
Inflation expectations	0.6612^{***} (0.1064)	0.7608 (0.6038)	0.8104^{***} (0.1011)	0.5182 (0.4554)
Constant	0.5522^{***} (0.1513)	0.9027^{**} (0.3892)	0.1922 (0.1636)	0.3906 (0.3366)
R^2 adjusted	0.8134	0.5063	0.8263	0.6617
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

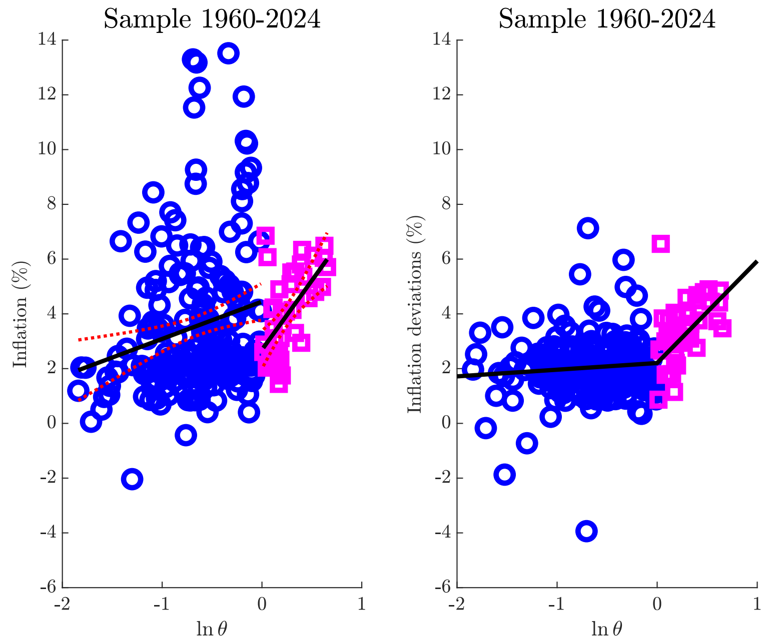


Figure 6: Left panel: scatter plot of the raw data of inflation and $\ln\theta$ used in the regression (3) of Table 1, sample 1960 Q1 - 2024 Q4. Right panel: scatter plot of 'inflation deviations' and $\ln\theta$, sample 1960 Q1 - 2024 Q4. 'Inflation deviations': inflation in deviations from its lagged value, the supply shock and inflation expectations using regression (3) of Table 1.

The second major result is that supply shocks have a larger effect when $\theta > 1$. For the full sample, the point estimate suggests this effect is roughly twice as large, although statistical significance is lacking. This imprecision reflects limited identification power: the full sample contains only two periods where $\theta > 1$ (the 1960s and recent period), and the 1960s had virtually no supply shocks (see Figure 22 in the Appendix). However, in the 2008-2024 sample, we observe a large and statistically significant effect of supply shocks when $\theta > 1$. In contrast, when $\theta < 1$, supply shocks have economically and statistically negligible effects.

Beyond our two main results, Table 1 reveals additional insights. The coefficient on lagged inflation is statistically significant in the full sample but not in the 2008-2024 period, when $\theta > 1$ for much of the sample. Section 5 shows this pattern emerges naturally from our theoretical model.

A second insight from Table 1 concerns inflation expectations. While the coefficient is statistically significant in the full sample, it becomes smaller and insignificant in the 2008–2024 period. This reduced significance reflects limited variation in our measure of expected inflation during this period, though we explore alternative measures in Section 3.3.3.

Figure 6 offers a visual interpretation of our regression results. The left panel combines all data points from Figure 1 into a single scatter plot. The right panel shows inflation after removing the effects of all explanatory variables from equation (2) except $\ln\theta$ and the constant. The solid line traces out the estimated Inverted-L New Keynesian Phillips Curve in $(\ln\theta_t, \pi_t)$ space, showing the relationship in the absence of shocks.

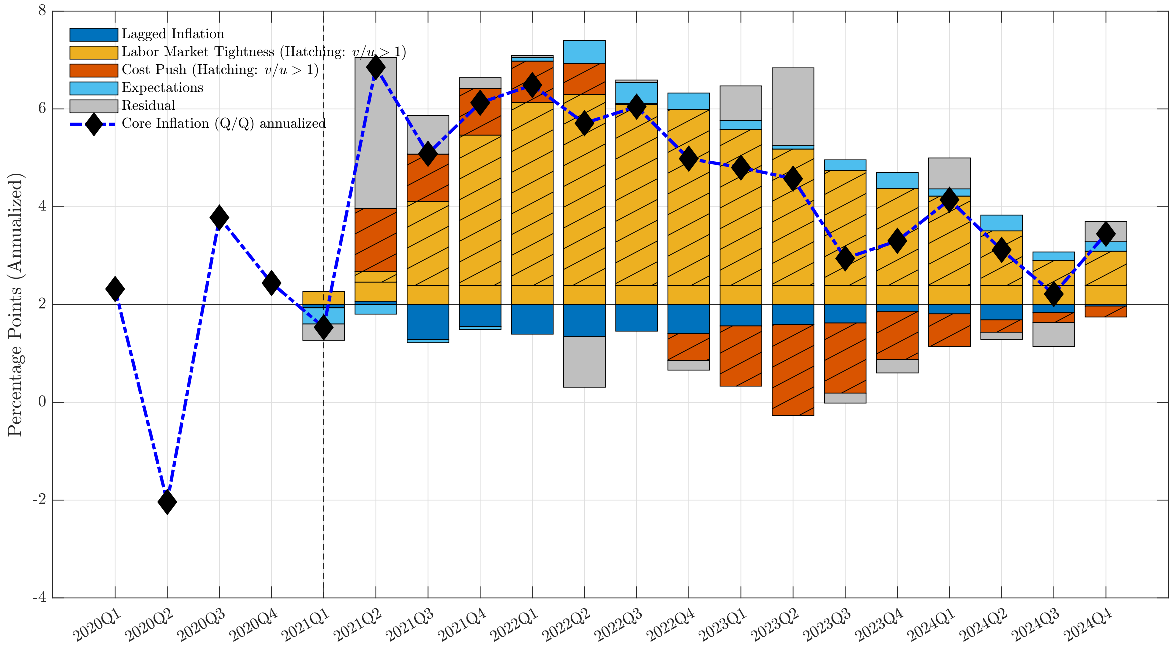


Figure 7: Decomposition of the regression (4) of Table 1, sample 2008 Q3 - 2024 Q4, among the various regressors of equation (2). For the variable $\ln \theta$, hatching corresponds to the contribution of the variable for the portion of θ that exceeds the unitary value. For the supply shock ϱ, hatching corresponds to the contributions of the variable when $\theta > 1$. Core inflation and all the components are plotted at annualized quarterly rates.

Figure 7 helps to interpret the quantitative contributions of individual regressors in equation (2), estimated over the sample period from 2008 Q3 to 2024 Q4. This figure shows actual data, represented as quarter-on-quarter (annualized) percentage increases in core CPI inflation, plotted with a dashed blue line with diamonds. For each period, core inflation is decomposed into the contributions from various regressors.

First, consider the role of labor tightness on the flat part of the Phillips curve. As shown by the solid yellow bars, an increase in labor tightness up to the unitary value accounts for a modest inflation increase of about 0.4 percent. However, the majority of the inflation surge is explained by the labor tightness that arises due to the change in the slope of the Phillips curve once $\theta > 1$ and the contribution of θ exceeding the unitary value.

Similarly, cost-push shocks, once interacted with $\theta > 1$, played a significant role in driving the surge, particularly at the beginning. Towards the end of the period, however, these shocks exerted downward pressure on inflation, which was entirely offset by the tight labor market, sustaining inflation rates above the two percent target.

Supply chain disruptions were widely cited as a key driver of early inflation dynamics. A substantial literature emphasizes how bottlenecks and supply chain disruptions contributed significantly to the inflation surge, particularly in its early stages.[29] Figure 21 provides examples of anecdotal evidence

[29]See e.g. Blanchard and Bernanke (2024) and references therein.

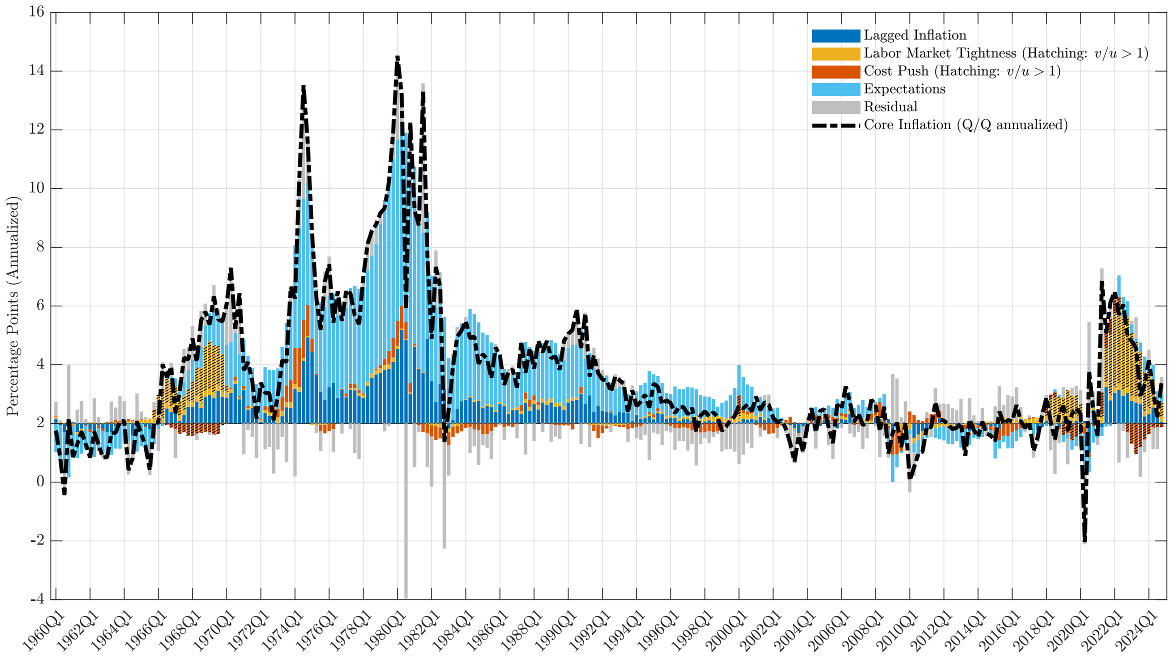


Figure 8: Decomposition of the regression (3) of Table 1, sample 1960 Q1 - 2024 Q4, among the various regressors of equation (2). For the variable $\ln \theta$, hatching corresponds to the contribution of the variable for the portion of θ that exceeds the unitary value. For the supply shock ϱ, hatching corresponds to the contributions of the variable when $\theta > 1$. Core inflation and all the components are plotted at annualized quarterly the rates.

of supply chain disruptions from this period. While our supply shock metric builds on pre-2020 methodologies and does not specifically account for these disruptions, this limitation may explain a key finding: As shown in Figure 7, the model's largest unexplained components—represented by the gray bars—occur in the first two quarters of the surge. This timing suggests that these residuals may capture the unusual supply-side disruptions. Nevertheless, we maintain our pre-2020 measurement approach to avoid the risk of ex-post overfitting.

The constant of the regression suggests an interesting economic interpretation. It can be related to the value for θ at which inflation is equal to 2% in the absence of shocks, which we denote by $\bar{\theta}$. We obtain $\bar{\theta} = 0.4359$ for the full sample and 0.4708 for the 2008-2024 sub-sample.[30]

An interesting finding from the full sample estimation is that unlike the recent inflation surge, the Great Inflation was driven primarily by unanchored expectations, which aligns with conventional wisdom as shown in Figure 8, the full sample analog of Figure 7. More significantly, a point we think is underappreciated in the literature, this unanchoring—the "original sin"—started during the labor shortage of the late 1960s, which then may have prepared the groundwork for the expectation driven inflation surge in 1970's. This historical experience contrasts sharply with current data, which can

[30] To see this, first observe that the mean of the supply shock is close to zero, and all inflation measures are considered as deviations from the 2% target rate. However, the variable $\ln \theta$, is not measured as a deviation from its average value. This implies that we can support it through equation $\ln \bar{\theta} = -\beta_c/\beta_\theta$, obtaining $\bar{\theta} = 0.4359$ and $\bar{\theta} = 0.4708$, for the third and fourth specifications of Table 1, respectively.

explain why a repeat of the 1970s was not observed, at least so far.

Our follow-up paper by Benigno and Eggertsson (2024a) provides a detailed analysis of Figure 8, comparing the two periods of labor shortages and their effects on inflation expectations, while also contrasting them with the four other major inflation surges in the past 115 years of U.S. economic history discussed in Section 2. In that paper we also show how the model account for the brief period prior the pandemic when the Beverdige threshold was crossed, without significant inflation pressures.

Our estimation offers insight into the forecasting failures of policymakers and markets discussed in the introduction. As shown by the solid yellow bars in Figure 7, the contribution of labor market tightness to inflation was negligible without accounting for the change in slope. This estimated flatness is of similar order of magnitude to pre-surge literature estimates. Similarly, the effect of supply shocks on core inflation was also minimal without accounting for the change in slope. Our thesis is that these forecast failures stemmed from failing to recognize the slope changes. Forecasters may have been lulled into complacency by the apparent stability of long-term inflation expectations, which was interpreted as precluding a repeat of the Great Inflation of the 1970s, together with the belief in a flat Phillips curve.

3.3 Choice of variables in benchmark regression, and robustness check using common alternatives

In the following subsections, we document the variables used in our benchmark regression in areas where the literature lacks full consensus. We then assess the robustness of our results to commonly suggested alternatives, concluding with the widely used estimation strategy that employs lagged values as instruments. Appendix B discusses another alternative, which allows for time-varying parameters estimated via a Kalman filter.

The two central empirical results are generally robust, to variable selection or different estimation strategies, even if point estimates may vary: (i) the Phillips-curve slope increases significantly when $\theta > \theta^*$, and (ii) under these conditions, supply shocks have a more substantial impact on inflation. The robustness of these conclusions is particularly encouraging due to the well-known sensitivity of Phillips-curve estimates to specification choices and variable selection.[31]

3.3.1 Choosing the value for the Beveridge threshold: justifications and robustness checks

We first consider the Beveridge threshold that maximises the likelihood of our model. The maximum likelihood procedure supports our benchmark choice of the unitary value for the Beveridge threshold, which we choose for presentational purposes, and historical context. The likelihood function is

[31]See e.g. the survey by Mavroeidis, Plagborg-Moller, and Stock (2014) previously cited, which carefully documents this.

relatively flat between $\theta^* = 0.9812$ (our maximum likelihood estimate) and $\theta^* = 1$ for the 2008-2024 sample. Figure 26 use the exact value $\theta^* = 0.9812$ resulting in minor differences. We consider this flat profile, along with historical precedent and simplicity, a reasonable justification for our benchmark used in the estimation. For the full sample, 1 remains a local maximum, but another local maximum is at a lower value of θ, shown in Table 2 of Appendix A. The main findings remain unchanged in the latter part of the sample, with the estimated coefficient being similar. However, for the full sample, using the lower value of θ reveals that the effect of supply shocks becomes statistically significant when $\theta > \theta^*$, while the slope coefficient on θ declines.

Our theoretical framework, developed in the next section, suggests that this threshold may vary between regions, countries, and time. This can be especially important when modeling different U.S. regions. Understanding the determinants of the Beveridge threshold remains an important topic for future research.

3.3.2 Measuring supply shocks and robustness checks

Our finding of non-linearity when once the Beveridge threshold is crossed, i.e., $\theta > \theta^*$, is robust to various traditional measures of supply shocks. The literature uses several approaches: 'headline shocks,' defined as the difference between headline and core inflation using either CPI or PCE (Ball et al., 2022), and the gap between import price and GDP deflator changes (Blanchard et al., 2015). Figure 22 in the Appendix presents these three measures, highlighting an important point: traditional metrics suggest supply shocks were not particularly large during the recent inflation surge. In our benchmark regression, we take an agnostic stance by using the first principal component of these series as our measure for supply shock. Tables 3 and 4 confirm our main findings hold using these measures separately, with point estimates modestly higher using the Blanchard et al. (2015) proxy but lower using CPI headline shocks alone.

3.3.3 Measuring inflation expectations and robustness checks

Our results are robust to different measures of inflation expectations with some caveats. We follow the recent literature by using direct measures of expectations in the estimation. The canonical New Keynesian Phillips curve (Equation 1 in Section 2) specifies expectations of one quarter in advance as the relevant metric for firms pricing decisions, i.e, $\pi_t = E_t \pi_{t+1}$. There is no data on one-quarter firms inflating expectation. We consider wide variety of expectations used in the literature, to explore robustness relative to the benchmark considered below.

The perspective we adopt for the construction of expectations follows Bernanke and Blanchard (2024). They posit that one-quarter-ahead inflation expectations at time t are determined by[32]

[32]This is obtained by combining Equations 5 and 6 in their paper.

$$\pi_t^e = E_t \pi_{t+1} = h \cdot \underbrace{\pi_t}_{\text{Current inflation}} + (1-h) \cdot \underbrace{\pi_t^{me}}_{\text{Measured short, medium or long-term inflation expectations}} \tag{3}$$

where h represents the weight of current inflation in forming inflation expectations for the next quarter and π_t^{me} is some measure of inflation expectations. The value of h does not affect the estimation results. Instead, it influence the structural interpretation of the estimated coefficients which is relatively modest.[33]

Following Bernanke and Blanchard (2024) our benchmark analysis uses Federal Reserve Bank of Cleveland measure of inflation expectations to proxy π_t^{me}. An advantage of this measure is that it combines various survey evidence along with various market-based data such as Treasury yields, inflation data, and inflation swaps. We choose a 2-year horizon expectation, which balances Bernanke's (2007) view that longer-term expectations are more important for firms' price-setting behavior than short-term ones, with the common practice of using one-year expectations. The Phillips curve is derived from the best pricing decisions of the firms, and therefore the firms' expectations are of primary relevance.[34] Figure 27 in the Appendix presents this series.

Our key findings are robust to alternative measures of inflation expectations (π_t^{me}), with one exception discussed below. These robustness checks are important since the literature is yet to settle on the most reasonable measure to use. The most common inflation survey in the literature is the survey of professional forecasters. As shown in Table 5 our two central results are robust to this alternative, with even the point estimates being close to the benchmark.

Bernanke (2007) argues that longer-term expectations, of a 5-10 year horizon, are of greatest relevance for firms' pricing decisions on the basis of the track record of the Board of Governors in forecasting inflation, which relies heavily on them. Bernanke (2007) cites studies showing the Board forecast outperforms several statistical benchmarks. We consider 5-year expectations from the Federal Reserve Bank of Cleveland, but the 10 year forecast is almost identical (with correlation of 99.89). Table 6 shows that the main results are robust to using this measure, with the point estimate and standard deviation close in value for each coefficient. In Table 7 we report a metric which may capture Bernanke's argument better that long-term expectations and the credibility of the inflation target instead of temporary variations in the inflation rate. We consider 5-year-5-year forward measure from Groen and Middeldorp (2013).[35] The results are robust for the last part of the sample, when market based metrics can be applied. For the full sample, which relies on back-casting, however, while the

[33]For $h > 0$, the estimated coefficients β_{π^e} and β_θ and the other's differ from their structural counterparts $\tilde{\beta}_{\pi^e}$ and $\tilde{\beta}_\theta$ etc according to $\beta_{\pi^e} = \frac{(1-h)\tilde{\beta}_{\pi^e}}{1-h\tilde{\beta}_{\pi^e}}$. All other parameters in the regression are multiplied by the factor $\frac{1}{1-h\tilde{\beta}_{\pi^e}}$ to obtain a structural interpretation, e.g., $\beta_\theta = \frac{\tilde{\beta}_\theta}{1-h\tilde{\beta}_{\pi^e}}$.

[34]The Cleveland Fed measure of inflation expectations only goes back to 1982 Q1. We use the interpolated Livingston Survey's inflation expectations to patch it backward. Note that the Livingston Survey's 12-month CPI forecast effectively represents a 14-month horizon (Carlson, 1977).

[35]The Groen-Middeldorp measure uses market data for recent periods but relies on backcasting for earlier periods such as the 1970s.

point estimates are robust, the standard errors are large and the estimate of the steep part of the slope looses statistical significance.

We consider various more short-run alternative, such as 1-year inflation expectations from the Survey of Professional Forecasters and the Cleveland Fed (the latter is shown in Table 9). Table 10, furthermore, shows that the results are also robust to using the 12-month Livingston survey inflation expectations, which are available for the entire sample of analysis, though at a semi-annual frequency.[36]

Our result is also also robust to using the University of Michigan's 12-month-ahead consumer inflation expectations but only if we exlude the Great Inflation and its decline. As shown in Table 8, the two central findings of our benchmark empirical analysis continue to hold for the recent period (2008-2024), with results consistent with the benchmark regression, even though the point estimate for the slope of the steep part of the Phillips curve is smaller. In the full sample, however, the results are not robust: the slope of the Phillips curve becomes smaller once the Beveridge threshold is crossed, a result that does not seem to have sensible economic meaning.

This lack of robustness in the expectation measures is specific to part of the Michigan survey data. We have already noted that the results are robust using the short sample. They remain robust when considering the period from 1992 Q1 to 2024 Q4, which follows the Great Inflation of the 1970s and its aftermath. Additionally, the results are stable when examining the sample before the Great Inflation, from 1960 Q1 to 1969 Q4, which includes data points from both a tight and loose labor market.

One interpretation of this finding is that consumer inflation expectations from the Michigan survey are more backward-looking than other measures, making them less suitable as a proxy for the forward-looking expectations that drive firms' pricing decisions. Additionally, the period from 1970 to 1992, when inflation expectations were unusually unstable, may render the results unreliable.

Future inflation expectations drive firms' price adjustments according the the New Keynesian Phillips curve. When firms reset prices, they must forecast future inflation and market conditions, as empirical evidence documented by Nakamura and Steinsson (2008) suggests that prices typically remain unchanged for several quarters. For the Michigan survey of consumer inflation expectations, we conjecture that causality may run in the opposite direction: currently observed inflation influences consumers' inflation expectations more significantly for this measure than for others, rather than expectations determining current inflation through firms' price-setting behavior. We provide suggestive evidence for this conjecture below.

Different measures of inflation expectations show varying degrees of dependence on observed inflation, as indicated by their correlations when forecasts are made.[37] Table 15 in the Appendix presents these correlations for our baseline and alternative measures, focusing on the sample from 2008 Q3

[36]Beaudry, Hou, and Portier (2025) confirm the robustness of our findings when using professional forecasters' inflation expectations, except when an additional dummy is added to the constant in regression (3) for $\theta > 1$. This dummy does not have any particular meaning when estimating a piecewise linear function with a threshold of 0 for $\ln \theta$.

[37]It is not obvious what the appropriate data counterpart is to observed inflation when inflation expectations are formed. In the Granger-causality test, we proxy observed inflation by realized inflation at time $t-1$ for expectations formed at time t, which ensures that observed inflation is available to people as they make a forecast in period t.

to 2024 Q4.[38] The 12-month-ahead Michigan Survey and the Survey of Professional Forecasters exhibit the strongest correlations with observed inflation, with coefficients above 0.7. In contrast, the 5-year-5-year inflation expectations show a negative correlation, while our baseline measure has a moderate 0.6 correlation. This pattern becomes even more pronounced during the 2020 Q4–2024 Q4 inflation episode, where the Michigan Survey's correlation rises to 0.82 while other measures, including our baseline at 0.18, show much weaker correlations. These correlations provide some support to our conjecture that the Michigan survey expectations are more heavily influenced by the most recent data.

The 12-month-ahead inflation expectations from the Michigan survey stands out as the only measure where observed inflation Granger-causes inflation expectations. This means that if you observe high inflation, you will expect high inflation tomorrow.[39] To establish this, we examine whether a rise in observed inflation in a given quarter predicts a rise in inflation expectations (i.e. Granger causality). As shown in Table 16, only the Michigan survey exhibits Granger causality from observed inflation to inflation expectations among all inflation expectation measures. In contrast, our benchmark measure and other alternatives show no such relationship.

The causality from expectations to inflation, however, runs in the opposite direction for most measures. An increase in inflation expectations Granger-causes observed inflation for all measures except the 5-year-5-year forward and 5-year Cleveland inflation expectations. The immunity of these long-term expectations to observed inflation aligns with conventional wisdom regarding the Federal Reserve's credibility during the recent inflation surge, where long-term expectations remained well anchored. This finding is particularly encouraging for our two key empirical results, as both long-term expectation measures yield similar overall outcomes to our benchmark estimation, as shown in Tables 6 and 7.

Our results are also robust to recently developed measures of firm expectations, which exist only for part of the most recent sample, using proxy series that are most closely related to them. [40]

3.4 Additional robustness checks: Alternative estimation strategies, changing the dependent variable, and replacing lagged inflation with real wages

Our two key empirical findings remain robust to alternative estimation approaches and definitions of variables. Using lagged variables as instruments—a common approach in Phillips curve estimation—

[38]An advantage of restricting our attention to the last of the four sub-periods shown in Figure 1 is that we have direct measures for all expectation variables and do not need to rely on any extrapolations or back-casting.

[39]More precisely observed inflation gives statistically significant information to predict expected inflation.Please refer to Footnote 37 for a precise definition of data counterparts to observed and inflation expectations.

[40]Arguably the most relevant expectations are those that directly track firms expectation. The longest such series available is the Federal Reserve of Atlanta Business Expectations which goes back to 2011. Chodorow-Reich (2024) shows this series closely matches the Survey of Professional Forecasters and argues for its applicability. We have already shown our result are robust to this series. A more recent survey is Cleveland's Survey of Firms Inflation Expectations, which goes back to 2018. While it may be too early to draw strong conclusion, early evidence suggest it tracks closely the Michigan survey. As we have shown the results are robust to using the Michigan Survey expectations for the short-sample, which is the only subperiod which includes this metric for firm expectations.

confirms our results. This is shown in Table 11, where we instrument both inflation lags and the natural logarithm of the vacancy-to-unemployment ratio with their first lags.[41]

An alternative to our baseline regression allows for only one breakpoint. An extreme alternative is to permit the parameters to change in every period. The time-varying parameters can be estimated using a Kalman filter. Our two central empirical results are robust to this approach. Following Blanchard, Cerutti, and Summers (2015), we apply this method to the period from 2008 Q3 to 2024 Q4. Inflation demonstrates heightened sensitivity to both labor market tightness and supply shocks during the surge, which is consistent with our two central results. The detailed results and discussion are presented in Appendix B.

Our findings remain robust when using core PCE instead of core CPI as the dependent variable (Table 12) and when measuring labor market tightness in levels rather than logarithms (Table 13). We prefer the benchmark that utilizes the natural logarithm of $\theta = v/u$, as it is implied by the theoretical model we derive.[42]

We deviate from our theoretical model by introducing lagged inflation into the Phillips curve. This approach allows us to align closely with existing literature. However, the model presented in the next section implies that lagged wages should enter the equation instead. Table 14 confirms our findings using this alternative.

3.5 The importance of new wages to the literature finding small role for labor market tightness

A central feature of the Inverted-L New Keynesian framework is that marginal cost is determined by the wages of new hires rather than by average wages, a key property we will demonstrate in the next section. Moreover, we will show that v/u serves as a sufficient statistic for new wages.

During labor shortages ($\theta \geq \theta^*$), the wages of new hires can significantly deviate from those of existing workers within the theoretical framework. This discrepancy can contribute to inflationary pressures, even if average real wages remain flat or decline.

Bernanke and Blanchard (2024), which we cite extensively, suggest that "if the Phillips curve is truly nonlinear, the nonlinearity should appear in the wage Phillips curve, as much of the influence of labor market slack on price inflation results from its effect on labor market conditions and wages."[43] They establish this in their wage equation using average wages to measure firms' marginal costs.

[41] The limitation of this approach due to weak instrument problem is reviewed in Mavroeidis, Plagborg-Moller, and Stock (2014)

[42] Additionally, using natural logarithms has the advantage that it is irrelevant whether v or u appears in the denominator.

[43] Figure 23 shows that a piecewise linear Phillips curve steepening at the Beveridge threshold fits the data in the sample 1960–1969 and 2008–2024 in a scatter plot of wage inflation, using average hourly earnings of production and nonsupervisory employees, against the log of the vacancy-to-unemployed ratio.

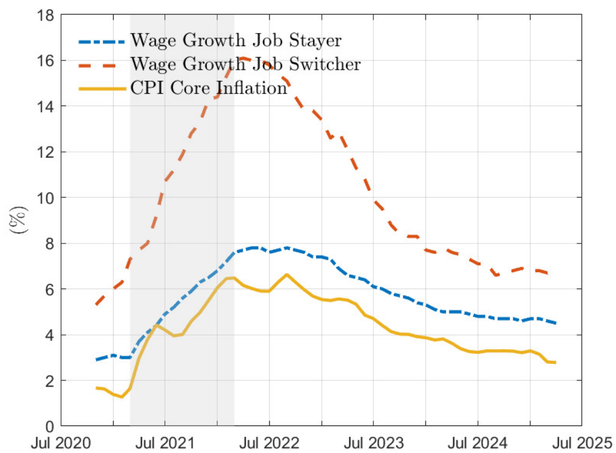


Figure 9: Measure of new wages relative to existing using income of workers to move from one job to another (job-to-job transitions) relative to the income of those that stay in their existing job. Source: ADP Pay Insight derived from payroll record of about 1/5 of US private sector employees

They incorporate a nonlinear term into their equation for average wages and find no evidence of nonlinearity.

However, as we argue here and formalize in our model, new wages are a better way to approximate marginal costs in these circumstances. To increase production, firms need to hire *new* workers.

The most compelling evidence for the rise in real wages at the onset of the surge comes from Automatic Data Processing Inc. This payroll processing company tracks approximately 17 million workers. A commonly used proxy for "new wages" is the wages of workers who switch jobs. ADP Pay Insights monitors all components of compensation over time, including bonuses, for both workers who remain with the same firm and those who transition to another, as long as they stay within the ADP universe.

Figure 9 shows the evolution of nominal wage growth for workers who switch jobs compared to those who remain in their positions. The shaded region indicates the inflation surge, defined as occurring from March 2020 to March 2021.

Figure 28 shows data from the Wage Growth Tracker of the Federal Reserve Bank of Atlanta, based on the Current Population Survey (CPS) a consistent pattern. This survey has two possible factors that may bias the results downward: it does not account for workers who change jobs due to relocation and excludes bonuses or other irregular payments, calculating only wages per hour. These factors may introduce a downward bias.

Both measures, however, may suffer from the issue that a new worker switches jobs because he has found a more productive match with an alternative firm. Figure 29, based on Crump et al. (2024), shows that there is a highly significant increase in the posted wage per vacancy during the period

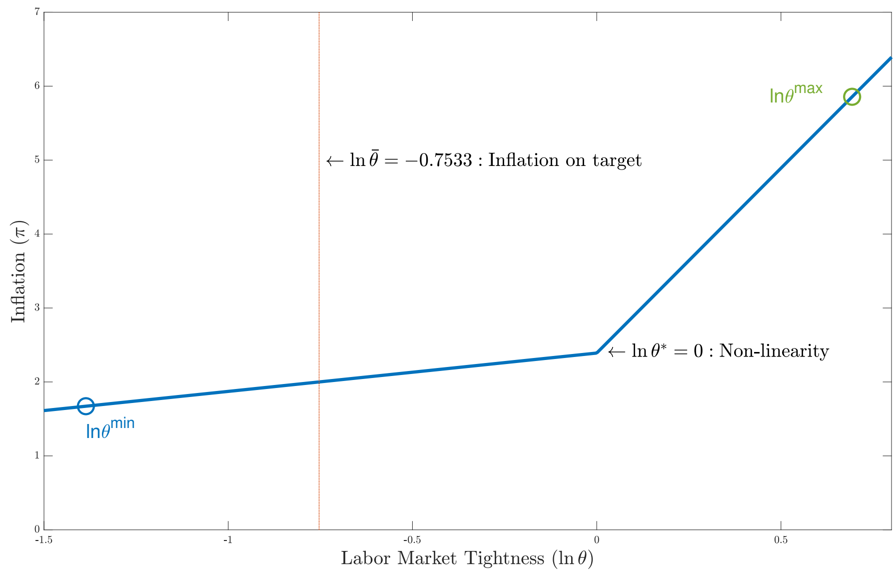


Figure 10: Inv-L NK Phillips Curve as a function of labor market tightness using the estimates of Table 1, column (4).

surrounding the inflation surge. This increase is particularly pronounced for the wages of relatively low-skilled workers.

4 The model

4.1 Roadmap

In this section, we outline the main theoretical result to help readers anticipate the rationale behind the empirical specification presented in Section 3. We then use the underlying general equilibrium model to account for the inflation surge, the Great Inflation of the 1970's and the missing deflation post 2008.

A simplified variation of the Inv-L NK Phillips curve we derive is:

$$\pi_t - \pi = \begin{cases} \kappa^{tight}\hat{\theta}_t + \kappa_{\varrho}^{tight}\hat{\varrho}_t + \beta E_t(\pi_{t+1} - \pi) & \text{labor shortage } \hat{\theta}_t > \hat{\theta}_t^* \\ \\ \kappa\hat{\theta}_t + \kappa_{\varrho}\hat{\varrho}_t + \kappa_{\beta}E_t(\pi_{t+1} - \pi) & \text{normal } \hat{\theta}_t \leq \hat{\theta}_t^* \end{cases} \tag{4}$$

where $\hat{\varrho}_t$ captures all model shocks, discussed in detail in this section.[44]

Figure 10 plots this relationship using regression results from column (4) of Table 1, showing both the Phillips curve slope and the point where θ (labor market tightness) reaches the inflation target. The

[44]Variables with a hat denote log-deviations from their respective steady states; π is the inflation target.

key finding is that $\kappa^{tight} > \kappa$ after crossing the Beveridge threshold. The red vertical reference line demonstrates that beyond a specific tightness level, a tighter labor market drives inflation above the 2% target. The curve is flat around this target until the Beveridge threshold is reached.

When θ reaches θ^* (where $\ln\theta^* = 0$), inflation rises to 2.4%. However, in labor shortage territory—above the Beveridge threshold—doubling θ from $\theta^* = 1$ to 2 increases inflation from 2.4% to 6%.

When there is labor shortage, the response of inflation to supply shocks, $\hat{\varrho}_t$, is also more pronounced, i.e. $\kappa_\varrho^{tight} > \kappa_\varrho$, consistent with the empirical evidence. The effect of inflation expectations, however, is not as clear-cut.

The remainder of this section summarizes the non-linear microfoundations of Equation (4) and presents the complete set of equilibrium conditions of the model.

4.2 Households and hiring technology

There is a continuum of representative households of measure one. The members have different disutilities of working. No decision is made about each member's hours of work (intensive margin), but the household decides how many members work (extensive margin). In other words, household j chooses the labor market participation rate. The utility at time t of household j is given by preferences as in Greenwood, Hercowitz and Huffman (1988):[45]

$$\sum_{T=t}^{\infty} \beta^{T-t} \frac{1}{1-\sigma} \left(C_T(j) - \chi_T \int_0^{F_T(j)} f^\omega \, df + \Psi_T \right)^{1-\sigma} \xi_T \tag{5}$$

where $C_t(j)$ is consumption of household j, $F_t(j)$ is the number of members who participate in the labor market, while β is the household discount factor satisfying $0 < \beta < 1$. Each household member is indexed by f and has fixed disutility f^ω from labor force participation, with $\omega > 0$.[46] The variable χ_t is an exogenous shock to labor force participation, and $\sigma > 0$ is a parameter. The variables Ψ_t and ξ_t are treated as exogenous by the household and affect all households in the same way; Ψ_t is introduced to simplify the Euler Equation for consumption as clarified in Section 6, while ξ_t is an intertemporal disturbance that moves the natural rate of interest.

Household members are ordered by their disutility from working. For example, it may be more costly to have an aging grandmother in the labor force than a prime age woman. Integrating the disutility of labor force participation yields

$$\int_0^{F_t(j)} f^\omega df = \frac{F_t(j)^{1+\omega}}{1+\omega}. \tag{6}$$

The household decides labor force participation. Not all of the labor force is employed, however, owing to frictions in the labor market. We will describe the technology for finding a job shortly.

[45] The use of GHH preferences allows us to abstract from wealth effects in labor force participation, which simplifies the algebra.

[46] We borrow this modeling device from Galí (2009), with the difference that he uses it to characterize hours worked on the intensive rather than the extensive margin.

In every period t the supply of labor of household j is divided between the employed household members that participate in production and the unemployed

$$F_t(j) = N_t(j) + U_t(j), \tag{7}$$

where $N_t(j)$ is workers employed and $U_t(j)$ is the unemployed workers at time t that do not contribute to production in equilibrium.

The standard search-and-matching literature implies that transition dynamics to steady state are quick, typically measured in months (see e.g. Pissarides, 2003).[47] This is frequently used to justify analyzing the Beveridge curve over time as if in steady state, but subject to shocks, ignoring transition dynamics. Our empirical model uses data on a quarterly basis. Following Pissarides's observation, we make two simplifying assumptions that allow us to abstract from transition dynamics by eliminating state variables, implying, for example, a Beveridge curve which holds period by period, while subject to shocks, without needing to impose steady state.

The first simplifying assumption is that in each period, $(1 - z_t)$ of the labor force (F_t) is attached to a firm, where z_t is exogenous process that satisfies $0 \leq z_t \leq 1$. Implicitly, we think of this as employment based on existing firm-employee relationships from period $t - 1$. In this paper, however, we do not explicitly model when these relationships formed or keep track of the employment history at the level of each employee.

We define the $(1 - z_t)F_t$ workers attached to firms as *existing* workers, while the $z_t F_t$ workers who are not attached but are able to secure employment in period t are defined as *new hires*. While perfectly substitutable in production, what differentiates the two groups of workers is that the attached workers receive existing nominal wages prevailing in the market W_t^{ex}, while newly hired workers receive W_t^{new}. These two wages may or may not be the same, see Section 4.5.

The total number of employed workers of household j at time t can thus be decomposed into

$$N_t(j) = N_t^{ex}(j) + N_t^{new}(j), \tag{8}$$

where $N_t^{ex}(j)$ represents workers from existing work relationships given by

$$N_t^{ex}(j) = (1 - z_t)F_t(j), \tag{9}$$

while $N_t^{new}(j)$ is new hires in period t. The workers who form new employment relationship are drawn from the pool of unattached workers $z_t F_t(j)$.

The second key assumption is that we posit a hiring technology, in which the total number of hires,

[47] Pissarides observes that "only a small fraction of any disturbance survives after a couple of quarters." Pissarides concludes that transition dynamics are very quick and, for most empirical work (e.g., tracing the Beveridge curve), "we can treat the labor market as if it is always at or very near its steady state." The bottom-line of his calibration is that half the adjustment is completed in a single month. Approximately 95% of the adjustment is done within about 4 months.

H_t, relative to unemployed workers in equilibrium at time t, U_t, is:

$$\frac{H_t}{U_t} = m_t \left(\frac{V_t}{U_t}\right)^{1-\eta}, \tag{10}$$

where m_t is an exogenous process representing hiring efficiency (analogous to matching efficiency in standard search-and-matching models), $0 < \eta < 1$ is an elasticity parameter, and U_t is equilibrium unemployment in period t — the portion of the labor force (F_t) not participating in production. These variables are not indexed by j as they do not depend on the choices of each atomistic household.[48]

To understand the nature of the equilibrium, it is helpful to observe that $0 \leq U_t \leq z_t F_t$ must hold. Within the group of unattached workers (totaling $z_t F_t$), some may find employment while others are unemployed in equilibrium. The key distinction between unattached workers who secure employment and attached workers is their compensation: the former get paid W_t^{new}, while the latter W_t^{ex} which may or may not co-incide.

What is the interpretation of the total number of new hires, H_t? In standard search-and-matching models, new hires come from the pool of unemployed workers — a state variable determined at time $t-1$. Hence, H_t represents the portion of the unemployed workers from period $t-1$ that find jobs at time t. While our model is consistent with this interpretation in steady state, our preferred interpretation is broader.

The only restriction on new hires is that $0 \leq H_t \leq z_t F_t$. The number of unattached workers, $z_t F_t$, does not need to co-incide with the number of unemployed from period $t-1$ as in the standard model. Intuitively, the hiring technology stipulates that the number of total new hires, relative to job seekers unable to secure employment at time t, depends upon the exogenous matching technology parameter, m_t, and the ratio of number vacancies and people that end up unemployed in equilibrium at time t. Hence our hiring technology is subtly different from the standard matching function, typically interpreted as suggesting that U_t represents number of people that are searching for jobs.

Theoretically newly hired workers in the model may belong to a number of sub-groups aside from those that where unemployed at time $t-1$. Consider a worker employed in period $t-1$, but is unattached in period t and successfully lands a new job. This worker is part of $z_t F_t$ that is hired, showing up as a job-to-job transition in the data. People who enter the labor force and get hired without any prior connection to firms are another sub-group. A third sub-group are those counted as unemployed workers at $t-1$ who are not attached to firms in period t. A fourth group are workers that are recalled, people hired by firms that had previously fired them or temporarily laid off. Whether a worker that is recalled belongs to attached or unattached workers (who are hired) depends on if they are paid the wages of existing workers, or new workers. [49]

[48] This second assumption is a bit more subtle than the first. In typical search and matching models, U_t is a state variable representing unemployment from the prior period, with some unemployed workers finding jobs in period t. In our model, U_t is equilibrium unemployment at time t, determined *in equilibrium* by the number of people unable to secure jobs given the hiring technology (10). Rather than being a state variable, U_t is determined jointly with other endogenous variables at each time t, similar to the household employment choice in the neoclassical model.

[49] In line with our broader interpretation of newly hired, H_t, Blanchard and Summers (2022) specify conditions under

Regardless of interpretation, the key assumption is that the probability of unattached members belonging to $z_t F_t$ being hired is given by the hiring technology (10).

Given the hiring technology, the hiring probability of an unattached worker at time t is

$$f(\theta_t) = \frac{H_t}{z_t F_t} = u_t \frac{m_t \theta_t^{1-\eta}}{z_t}. \tag{11}$$

where $u_t \equiv U_t / F_t$ is the unemployment rate.

The probability of unattached workers representing a new hire is taken as given by the household since it depends on aggregate variables. The number of new employment matches of the members of household j is then

$$N_t^{new}(j) = H_t(j) = f(\theta_t) z_t F_t(j) = u_t m_t \theta_t^{1-\eta} F_t(j). \tag{12}$$

Using this, the budget constraint of the household can be written as:

$$B_t(j) + P_t C_t(j) + T_t = (1 + i_{t-1}) B_{t-1}(j) + W_t^{ex}(1 - z_t) F_t(j) + W_t^{new} u_t m_t \theta_t^{1-\eta} F_t(j) + Z_t^F + Z_t^E + P_t q_t \bar{O}_t, \tag{13}$$

where $B_t(j)$ is a risk-free nominal bond denominated in units of currency at time t carrying the nominal interest rate i_t, P_t is the price index associated with consumption basket $C_t(j)$, T_t are lump-sum taxes, and Z_t^F and Z_t^E are the firms' and the employment agencies' profits divided equally across households. Finally, $P_t q_t \bar{O}_t$ represents the revenues that households receive by selling an intermediate input, denoted by $\bar{O}$, to firms at the exogenous real price q_t. A natural interpretation of this endowment is oil.[50]

Substituting equation (6) into the household utility, the household maximizes (5) subject to (13) by its choice of $C_t(j), B_t(j), F_t(j)$ taking as given all the variables not indexed by j in the maximization problem.

The necessary and sufficient conditions for the household maximization problem are straightforward to derive and are summarized below. Since all households behave the same in equilibrium we suppress the superscript j going forward.

The household's optimal labor-force participation is:

$$F_t = \left(\frac{(1 - z_t) w_t^{ex} + u_t m_t \theta_t^{1-\eta} w_t^{new}}{\chi_t} \right)^{\frac{1}{\omega}}, \tag{14}$$

which U_t is a reasonable proxy for a broader pool of workers that transition into new jobs. They stress the proxy is valid if the number of each pool of workers is relatively constant over time and argue that the composition effects were not quantitatively important during the period of central interest, i.e., the inflation surge of 2020s. It is worth pointing out that in principle one would not need such restriction to apply, as long as one is willing to accept the hiring technology (10) as technological constraint. A alternative specification of the hiring function is to replace U_t with a measure meant to capture search efforts of all groups looking for jobs. We leave this extension to future work.

[50]Since the price is treated as exogenous, the household supplies the input perfectly elastically.

which says that participation is increasing both in tightness and in the real wage, defined as $w_t^i \equiv W_t^i / P_t$ where $i = new, ex$, but can be negatively affected by the shock χ_t. The optimal consumption decision is:

$$X_t^{-\sigma} = \beta(1+i_t) E_t \left\{ X_{t+1}^{-\sigma} \Pi_{t+1}^{-1} \frac{\xi_{t+1}}{\xi_t} \right\}, \tag{15}$$

in which β is the utility discount factor, $\Pi_t \equiv P_t / P_{t-1}$ and

$$X_t \equiv C_t - \chi_t \frac{F_t^{1+\omega}}{1+\omega} + \Psi_t. \tag{16}$$

Finally, a necessary condition for optimality is that the household's intertemporal budget constraint holds with equality.[51]

We assume that C_t is a consumption basket given by a Dixit-Stiglitz aggregator of the form

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right]^{\frac{\epsilon_t}{\epsilon_t - 1}},$$

where i indexes consumption of a good of variety i, and $\epsilon_t > 1$ is the elasticity of substitution among the differentiated goods, which is a stochastic variable. The household's optimal choice of good of variety i at time t implies

$$c_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\epsilon_t} C_t,$$

where $p_t(i)$ is the price of variety i and

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\epsilon_t} di \right]^{\frac{1}{1-\epsilon_t}}.$$

4.3 Firms

There is a continuum of firms of measure one, each firm i producing a good of variety i. The source of demand is household and government consumption, C_t and G_t, plus the cost, $\gamma_t^c V_t$, to the employment agency of posting vacancies measured in units of the consumption good, where $\gamma_t^c > 0$ is stochastic. We assume that the government spending bundle and the vacancy cost, $\gamma_t^c V_t$, take the same form as the Dixit-Stiglitz household consumption basket. The aggregate resource constraint is

$$Y_t = C_t + G_t + \gamma_t^c V_t \tag{17}$$

A representative firm i faces the following demand, $y_t(i)$, for its output, given the assumption we have made

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\epsilon_t} Y_t, \tag{18}$$

[51] Or equivalently we can state a transversality condition; see e.g. Woodford (2003) for a discussion.

where $p_t(i)$ is the price of goods of variety i. Firms use labor and intermediate input to produce goods according to the technology

$$y_t(i) = A_t N_t(i)^\alpha O_t(i)^{1-\alpha}, \tag{19}$$

with $0 < \alpha < 1$ where A_t is productivity, $N_t(i) = N_t^{ex}(i) + N_t^{new}(i)$ is the labor employed by firm i, while $O_t(i)$ is an intermediate input of production (e.g. oil).

The firms' discounted value of current and expected future profits are:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \left\{ p_T(i) y_T(i) - W_T^{ex} N_T^{ex}(i) - (1+\gamma_t^b) W_T^{new} N_T^{new}(i) - P_T q_T O_T(i) - \frac{\varsigma}{2} \left(\left(\frac{p_T(i)}{p_{T-1}(i)} \right) \frac{1}{\Pi} - 1 \right)^2 P_T Y_T \right\} \tag{20}$$

where $Q_{t,T} \equiv \beta^{T-t}(X_T^{-\sigma}/P_T)/(X_t^{-\sigma}/P_t)$ is the stochastic discount factor the household uses at time t to value future nominal income at time T. As in the price-adjustment model of Rotemberg (1982), ς is a parameter measuring the cost of adjusting prices with respect to the inflation target Π.[52] The term γ_t^b represents the fee on the wage bill of new workers that the firm has to pay to the employment agency to hire them. As discussed in next section, it can alternatively be interpreted as the cost per new worker paid by a human resource department within the firm, with the cost measured in units of the consumption good.

The maximization of the firm is constrained by the limit that the firm cannot hire more existing workers at the existing wage rate than the number of workers attached to the firm, i.e.

$$N_t^{ex}(i) \leq (1 - z_t) F_t \tag{21}$$

where the right hand side of the inequality represents all the existing workers. Firm can hold onto the workers that are attached to it as existing workers and is free to fire them. To add workers beyond existing workers, however, it must add new workers. While existing workers can be fired, new workers can only be added, i.e.

$$N_t^{new}(i) \geq 0. \tag{22}$$

Finally, the total labor employed by the firms is the sum of new and existing workers, i.e.

$$N_t(i) = N_t^{ex}(i) + N_t^{new}(i) \tag{23}$$

The problem of the firm can be stated as choosing all variables sub-scripted with i to maximize (20) subject to (18), (19), (21), (22) and (23). The first order conditions are shown in Appendix E. We consider an equilibrium in which constraint (21) is binding while (22) is not, therefore

$$N_t(i) > N_t^E. \tag{24}$$

[52] Calvo's price-setting model leads to the same AS equation in a first-order approximation. However, we use Rotemberg's assumption in order to simplify the presentation.

As we show in the Appendix, a sufficient condition for the firm to always choose to first retain its existing workforce before hiring new workers is that

$$(1+\gamma_t^b)W_t^{new} > W_t^{ex}. \tag{25}$$

This condition arises from the observation that, given the perfect substitutability of new and existing workers, the firm opts to first hire from its existing workforce whenever the cost of hiring a new worker for production exceeds that of the existing worker. The important implication of distinguishing between new and existing workers is that it clarifies that, provided $N_t(i) > N_t^E$, the marginal cost of production for the firm depends only on the wages of the new workers as well as the hiring cost.

Since all firms face the same problem, there is a symmetric equilibrium in which $p_t(i) = P_t$ and $y_t(i) = Y_t$. The aggregate Phillips curve is derived directly by combining the first-order conditions detailed in Appendix E:

$$\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi} = \frac{\epsilon_t - 1}{\varsigma}\left(\frac{\mu_t}{A_t}\left(\frac{(1+\gamma_t^b)w_t^{new}}{\alpha}\right)^{\alpha}\left(\frac{q_t}{1-\alpha}\right)^{1-\alpha} - 1\right) + \beta E_t\left\{\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{\Pi_{t+1}}{\Pi}-1\right)\frac{\Pi_{t+1}}{\Pi}\right\}, \tag{26}$$

where we have defined $\mu_t \equiv \epsilon_t/(\epsilon_t - 1)$ and w_t^{new} is the real wage paid to new workers. Since all firms behave in the same way aggregate output is

$$Y_t = A_t N_t^{\alpha} \bar{O}_t^{1-\alpha}, \tag{27}$$

where we have used that in equilibrium the market for the intermediate input endowment is given by $O_t = \bar{O}_t$, for some exogenous process $\bar{O}_t$. Finally, note that

$$N_t = [(1-z) + u_t m_t \theta_t^{(1-\eta)}]F_t, \tag{28}$$

using (8) and (12), and

$$N_t = [1-u_t]F_t, \tag{29}$$

by definition of the unemployment rate as $u_t \equiv \frac{U_t}{F_t}$.

4.4 Wage determination

The central idea in Phillips (1958) is that the relationship between nominal wage growth and labor market tightness is non-linear.[53] He argues that even if demand for labor is low workers are unwilling

[53] Our assumption relies, like Phillips's observation, on downward rigid nominal wages. Currently, extensive empirical literature has documented that wages are rigidly downwards. The idea dates back at least to Malthus, who noted that "it rarely happens that the nominal price of labor universally falls," Malthus (1798). Bewley (1999) interviewed corporate executives documenting their reluctance to cut nominal wages. More recently, substantial nominal wage rigidity has been studied in U.S. administrative data by Fallick, Lettau and Wascher (2011), in worker surveys by Barattieri, Basu and Gottschalk (2014), and in cross-country data by Schmitt-Grohé and Uribe (2016). See also Fortin (2015) on Canadian data.

to take jobs at wages below the "prevailing rates." This implies that despite low demand for workers, and high unemployment "wage rates fall only very slowly." Yet the converse is not true according to Phillips. Workers are perfectly happy to accept wages that are *higher* than the prevailing wage rate. For this reason when labor markets are sufficiently tight "we should expect employers to bid wage rates up quite rapidly."

An implication of Phillips idea is that in periods of labor market tightness, the spread between new and existing wages should increase. In the literature, new wages are typically approximated using, as their data counterpart, the wage a person receives upon taking a new job as we discussed in Section 3.5 and illustrated in Figures 9 along with Figures 28 and 29 in Appendix C using alternative sources. These figures indicate that during the inflationary surge, the wage growth of job switchers substantially exceeded that of job stayers. For simplicity, we abstract from the spread under normal circumstances, since our focus is on the increase in new wages during the surge as a key mechanism through which the shocks triggering the surge were transmitted.

To formalize Phillips idea we assume that the wage of a worker hired at time t is:

$$W_t^{new} = \max\{W_t{}^{ex}, P_t w_t^{flex}\} \tag{30}$$

where w_t^{flex} is the *flexible real wage*, i.e. the wage rate that clears the market in the absence of any constraints, while $W_t{}^{ex}$ is wages of workers that are attached to existing firm relationships.

To see how this captures Phillips idea, consider a weak labor market. Then the prevailing wage rate, i.e. $W_t{}^{ex}$, is greater than the wage rate that clears the market, $P_t w_t^{flex}$. The max operator says that newly hired workera are unwilling to work for a lower wage rate than prevailing at the firm for existing workers, so that if $W_t{}^{ex} > P_t w_t^{flex}$ then $W_t^{new} = W_t{}^{ex}$. Yet, since workers are perfectly happy to work for *higher* wages than the existing workforce, then if the labor market is sufficiently tight, and $W_t{}^{ex} < P_t w_t^{flex}$, then $W_t^{new} = P_t w_t^{flex}$.[54] We can restate equation (30) in real terms

$$w_t^{new} = \max\{w_t{}^{ex}, w_t^{flex}\}. \tag{31}$$

As is well known in search and matching literature the wage rate is not in general determined, since each employment relationship generates a surplus. How the surplus is divided between the employer and firms can be done in several different ways. The most common in the labor-search literature is Nash bargaining. In contrast, monetary models with price stickiness that incorporate search-and-matching, often assume that the real wage are exogenous and only a function of the shocks. This is a useful polar case we consider in Section .[55] One of the theoretical contributions of this paper is to move away from this common assumption in favor of Phillips idea of asymmetric response of wages to tightness and slack.

[54] Consistent with the derivation of the AS equation (26) the inequality $(1+\gamma_t^b{}_t)W_t^{new} > W_t^{ex}$ is always satisfied.

[55] See e.g. Blanchard and Gali (2010a) and related literature cited in the Introduction

To do this, we propose a simple model of employment agencies that have access to the hiring technology described in equation (10) and screen workers for employment eligibility. This provides the foundation for the flexible wage rate. Once we derive the flexible wage rate, we then show how the wages of existing employees evolve as a function of it. There is good reason to start with the flexible wage: it serves as an anchor for the wages of existing workers. As observed in the data, when the nominal wages of new employees rise above those of existing employees, the nominal wages of existing employees also increase, though to a lesser extent. If inflation is sufficiently high, real wages for existing workers may even decline—along with the real average wage—while the real value of new wages rises rapidly. This is indeed what was observed at the onset of the inflation surge in the 2020s.

4.5 Employment agencies

In this section we derive the flexible wage rate which will apply to new workers in tight labor markets in the general model. It will also serve as a anchor towards which existing wages are pulled in the general setting. Using our derivation of this wage, we can in principle close the model assuming it applies to all workers.

If wages are flexible we derive a relationship of the form:

$$w_t^{\text{flex}} = g(\theta_t), \tag{32}$$

where $g' > 0$ and $g'' < 0$.

There are many ways to establish microfoundations that generate a function of this form. Indeed, it seems that any reasonable model with full wage flexibility should imply that the real wage increases as the labor market becomes more tight.[56]

We start deriving equation (32) in the next subsection and then move to the other polar extreme: rigid wages in Section 4.5.2, common in the monetary search-and-matching literature – an alternative benchmark. Section 4.5.3 proposes a generalized model by introducing a flexible form for wage rigidities, capturing Phillips idea of asymmetric response of wages to variations in economic activity,

An alternative interpretation to a hiring agency is that it corresponds to a human resource department within each firm.[57] We introduce hiring agencies seperately from the firms pricing problem to simplify the presentation because the two problems can be solved independently.

[56]See the analysis of Blanchard (1989).

[57]Unlike some other work, e.g., Michaillat (2014), this is because we write the costs and benefits in terms of the final goods instead of labor. If the cost of hiring would be written in terms of labor costs it would interact with the pricing problem of the firms. In contrast, in our model the optimal pricing decision of the firm, and the optimal vacancy posting by the employment agency do not depend on each other directly.

4.5.1 Flexible wages

There is a continuum of employment agencies of measure one corresponding to the number of firms. The employment agencies find workers suitable for employment by firms. Since each agency is small, they take as given the wage rate and the rate of employment matches per vacancy posted. Using the hiring technology, equation (10), the number of matches per vacancy posted for flexible wages are:

$$n(\theta_t) = \frac{H_t}{V_t} = \frac{m_t(U_t)^{\eta}(V_t)^{1-\eta}}{V_t} = m_t(\theta_t)^{-\eta}. \tag{33}$$

Consider the problem of a representative agency j. It charges a one time fee γ_t^b to the firm that is proportional to the salary of a new worker it screens for employment, while incurring a cost γ_t^c for every vacancy it posts.

The number of matches it generates is then given by $n(\theta_t)V_t(j) = m_t(\theta_t)^{-\eta}V_t(j)$. The revenues and the cost of the employment agency are measured in units of the final good. Profits are:

$$\max_{V_t(j)} \left(\underbrace{\gamma_t^b w_t^{flex} m_t \theta_t^{-\eta}}_{\text{Marginal benefit}} V_t(j) - \underbrace{\gamma_t^c}_{\text{Marginal cost}} V_t(j) \right). \tag{34}$$

In an interior solution, as long as the marginal benefit is greater or equal to the marginal cost, the agency posts vacancies. This leads to a decrease in wages for new hires, as well as an increase in labor market tightness, θ_t, which reduces the number of new hires per vacancy posted. Equilibrium is reached when marginal benefits equal marginal costs, i.e.:

$$\underbrace{\gamma_t^b w_t^{flex} m_t \theta_t^{-\eta}}_{\text{Marginal benefit}} = \underbrace{\gamma_t^c}_{\text{Marginal cost}}, \tag{35}$$

so that the flexible wage rate is

$$w_t^{flex} = \frac{\gamma_t^c}{\gamma_t^b} \frac{1}{m_t} \theta_t^{\eta}. \tag{36}$$

Suppose for the time being that the wages of existing and new workers are determined by the flexible wage:

$$w_t^{ex} = w_t^{new} = w_t^{flex}. \tag{37}$$

This is *the* fundamental assumption we relax next. In the meantime, assuming no constraint on how much wages can decline, this equation closes the model once we add the goods market equilibrium:[58]

$$Y_t = C_t + G_t + \gamma_t^c u_t \theta_t F_t, \tag{38}$$

and a specification of the monetary policy rule

$$i_t = \psi(\Pi_t, \xi_t, A_t, G_t, ...) \tag{39}$$

for some function $\psi(\cdot)$. A full specification of the equilibrium if the wage of workers are flexible can now be stated formally.[59]

4.5.2 Exogenous real wage rigidities

When all wages are fully flexible we learned that the hiring agencies post vacancies until wages fall, and tightness increases sufficiently, so that marginal cost is equal to marginal benefit of posting a vacancy. We now consider the possibility that the wages of existing workers is greater than the flexible wage rate, i.e.,

$$w_t^{ex} > w_t^{flex}.$$

We follow Phillips' suggestion, formalized in equation (31), that new workers refuse to work for a wage that is below the existing wage rate. This bound implies that the wage of new workers is equal to the existing wage:

$$w_t^{new} = w_t^{ex}. \tag{40}$$

Before moving to the more general analysis in the next section, consider the case in which the existing wage rate is exogenous, as in Blanchard and Galí (2010a):

$$w_t^{ex} = w(\Omega_t) \tag{41}$$

where the vector Ω_t includes all exogenous shocks.

By fixing the wage rate, the equilibrium can be defined as before (see footnote 59), by replacing equation (36) with $w_t^{\text{ex}} = w(\Omega_t)$ and (40). It should be noted that the employment agency problem does not appear explicitly in any of the equations defining the equilibrium. Yet, it is useful to be clear about its role in the background.

Denote the equilibrium labor demanded by firms in equilibrium by $\tilde{N}_t^d$. Associated with this level of employment, denote the equilibrium aggregate vacancies by $\tilde{V}_t$.

[58]We have used $Y_t = C_t + G_t + \gamma_t^c V_t$, noticing that $V_t = F_t(V_t/U_t)(U_t/F_t) = u_t\theta_t F_t$. Moreover we abstract from the resources taken by the quadratic cost of price setting. See Eggertsson and Singh (2019) for various ways in which this has been justified.

[59]An equilibrium with flexible wages and prices is defined by a collection of stochastic processes for $\{i_t, X_t, \Pi_t, Y_t, C_t, F_t, \theta_t, N_t, U_t, w_t^{ex}, w_t^{new}, w_t^{flex}\}_{t=0}^{\infty}$ that satisfy (14), (15), (16), (26), (27), (28), (29), (36), (37), (38), (39) and $u_t = U_t/F_t$ given exogenous processes for $\{A_t, G_t, \xi_t, \chi_t, \Psi_t, m_t, q_t, \gamma_t^c, \gamma_t^b, \epsilon_t, \bar{O}_t\}_{t=0}^{\infty}$.

This number of vacancies represents the optimal vacancy creation by the employment agencies assuming that the number of matches per job vacancy is given by:

$$n(\theta_t) = \begin{cases} m_t(\theta_t)^{-\eta} & \text{if } V_t(j) \leq \tilde{V}_t \\ 0 & \text{if } V_t(j) > \tilde{V}_t \end{cases} \tag{42}$$

This says that once the hiring agencies have satisfied the labor demanded by the firms, $\tilde{N}_t^d$, then posting additional vacancies generates no additional hires. Firms have already satisfied their need for labor at the exogenous wage and are not looking to hire additional workers. Before this logic did not apply, since more vacancy posting lowered wages, thus triggering an increase in labor demand. This process continued until the marginal cost and benefits of the hiring agency were equalized. With wages fixed, this is no longer the case. This implies it becomes optimal for the hiring agency to post vacancies elastically to satisfy the firm's demand for new hires at the exogenous level of wages, i.e., up to $\bar{V}_t$, while posting no more vacancies beyond that, since the firms have hired all the workers they desire.[60]

Hiring agencies are not made explicit in typical New Keynesian models integrating search and matching frictions with exogenous wage. It is thus implicitly assumed that vacancies respond fully elastically to labor demand. The agency problem formulated here provides a simple narrative for an assumption commonly maintained.

The assumption of real wages being exogenous is helpful for many questions. Our interest here, however, is in understanding the role of labor market tightness on inflation. This is why we move beyond the commonly assumed fixed wage rate.

4.5.3 Endogenizing a wage norm: bringing back the Phillips curve

As discussed in Section 2, the key idea introduced by Phillips was the nonlinearity in the wage Phillips curve: while new workers are reluctant to accept wages below existing levels, they are quite willing to accept higher wages, leading to a rapid acceleration in wages in a very tight labor market. But what can we reasonably assume about the behavior of existing workers? There is considerable evidence suggesting that the wages of existing workers are downward rigid. Yet, some degree of adjustment is observed in the data.

In this analysis, we adopt a pragmatic approach. We assume that wages of existing workers evolve according to a relatively flexible specification. This approach nests the fully flexible wage at one extreme and the exogenous wage, as commonly seen in the existing literature just reviewed, at the other:

$$W_t^{ex} = (W_{t-1}^{ex}(\Pi_{t+1}^e)^{\delta})^{\lambda}(P_t w_t^{flex})^{1-\lambda}\phi_t. \tag{43}$$

[60] In this equilibrium the hiring agency is making a profit on its last vacancy posting.

To understand this specification, consider the special case in which $\lambda = 1$, $\delta = 0$ and $\phi_t = 1$. Then $W_t^{ex} = W_{t-1}^{ex}$. Wages of existing workers stay constant at their previous period nominal value, capturing Keynes' idea that wages are nominally downward rigid in the absence of shocks. Consider $\lambda < 1$. Wages of existing workers are now anchored by labor-market conditions implied by the flexible wage rate, towards which they are gradually pulled, with perfect wage flexibility being as $\lambda \to 0$.

We have already introduced Phillips idea in equation (30) that new workers will not accept lower wages than existing wages, but are happy to accept higher ones. This means that in a slack labor market, the wages of new workers are *equivalent to the wage rate of existing workers*, $W_t^{new} = W_t^{ex}$, but pulled down towards the flexible wage rate at a speed that depends upon how close λ is to zero. For high enough λ, the wage rate falls slowly, as suggested by Phillips, even if the unemployment rate is high. Conversely, in a tight labor market when $W_t^{flex} > W_t^{ex}$, both existing and new wages are pulled upwards by w_t^{flex}, which is determined by market forces, but the new wages are pulled up faster, the higher is the value of $\lambda \geq 0$.

We introduce an additional feature by including the variable Π_{t+1}^e, relevant if $\delta > 0$. This captures the idea that inflation expectations affect wage-setting behaviour.[61] Finally ϕ_t is exogenous, which means that the wage norm can also depend on exogenous shocks and allows for flexibility in determining the steady state.

Using equations (31) and (43), we can write the wages of new hires in real term:

$$w_t^{new} = \begin{cases} w_t^{flex} & \text{for } \theta_t > \theta_t^* \\ w_t^{ex} = \left(w_{t-1}^{ex} \frac{(\Pi_{t+1}^e)^\delta}{\Pi_t} \right)^\lambda (w_t^{flex})^{1-\lambda} \phi_t & \text{for } \theta_t \leq \theta_t^*. \end{cases} \tag{44}$$

In a tight labor market ($\theta_t > \theta_t^*$), the wages of new hires are set at a flexible rate, determined by the employment agency's optimizing behavior, which equates the marginal cost with the marginal benefit of posting vacancies. Moreover, the flexible wage rate plays a crucial role in determining the wages of existing workers, serving as an anchor that gradually pulls them toward the market-clearing level.

On the contrary, in a loose labor market ($\theta_t \leq \theta_t^*$), if the wages of new hires are constrained by the wages of existing workers, households refuse to accept salaries below the prevailing wage rate of existing workers. Wages fall only gradually toward the flexible wage rate, despite high unemployment.

What remains to finalize the model is determining the Beveridge threshold θ^*, which indicates when labor scarcity prevails and the wages of new hires are set flexibly. This threshold is implied by equating $w_t^{\text{flex}} = w_t^{\text{ex}}$, yielding:

[61] These expectations can, for example, be anchored by the inflation target of the central bank. Alternatively, this variable can allow us to model "price-wage" spiral commonly thought to have played a role in the 1970s, even if we will leave this to future research for lack of space.

$$\theta_t^* = \frac{\gamma_t^b}{\gamma_t^c} m_t \left(w_{t-1}^{ex} \frac{(\Pi_{t+1}^e)^\delta}{\Pi_t} \right)^{\frac{1}{\eta}} (\phi_t)^{\frac{1}{\lambda\eta}}. \tag{45}$$

As suggested by the formula, θ_t^* can vary over time and depend on institutional features that may differ across regions and countries, captured in reduced form by ϕ_t. We defer discussion of the determinants of θ_t^* to Section 5 where we address the role of other shocks in the Phillips curve. As shown in the Appendix, the threshold in steady state can be derived as a function of structural parameters and does not need to be 1.

We can now formally define equilibrium in the model with nominal wage rigidities.[62]

5 The Inv-L NK Phillips Curve

The model we have just sketched out can be presented closer to the spirit of Phillips' original suggestion (depicted in Figure 2) than later incarnations of his work. The central result is that this modified curve now takes an inverse-L shape, as in Figure 2, in a first order approximation around the steady state. Two main economic propositions underlie the inverse-L shape. The first is simply that while, given sufficient time and price incentives, most factors of production can typically be increased in one way or another, one factor will always be in limited supply over any reasonable time horizon: the number of people who can work. Second, it has long been recognized that more than other prices the price of labor (wages) falls slowly even when it is in excess supply (high unemployment). Together, these observations imply that over some range, an increase in demand increases production without significant inflation pressures, as more people are drawn into the labor force. Firms increase their production while their marginal costs only modestly increase. However, given the first proposition, this process is bound to hit a wall once the labor force is fully employed. At this point, the Beveridge threshold is crossed so that higher demand expresses itself mostly in higher inflation.

Our first step toward characterizing the Phillips curve is a log-linear approximation of the standard New Keynesian Phillips curve given by (26), which can be expressed as:

$$\pi_t - \pi = \frac{1-\epsilon}{\varsigma}\alpha(\underbrace{d_\gamma \hat{\gamma}_t^b + \hat{w}_t^{new}}_{\text{Marginal Cost of Labor}}) + \frac{1-\epsilon}{\varsigma}(\underbrace{\hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t}_{\text{Cost Push Shocks}}) + \beta E_t(\pi_{t+1} - \pi) \tag{46}$$

where a hat denotes the log-deviation of a variable with respect to the steady state, $\pi_t \equiv \ln \Pi_t$, $\pi \equiv \ln \Pi$ and the parameter d_γ is defined in Appendix E. The first part of this expression, highlighted by the first curly bracket, underscores that the primary driving force behind inflation is the marginal cost of hiring a new worker. This cost is composed of two terms, the wage bill of the new workers,

[62] An equilibrium is defined by a collection of stochastic processes for $\{i_t, X_t, \Pi_t, Y_t, C_t, F_t, \theta_t, N_t, U_t,, w_t^{ex}, w_t^{new}, w_t^{flex}, \theta_t^*\}_{t=0}^{\infty}$ that satisfy (14), (15), (16), (26), (27), (28), (29), (36), (37), (38), (39), (44), (45) and $u_t = U_t/F_t$ given exogenous processes $\{A_t, G_t, \xi_t, \chi_t, \Psi_t, m_t, q_t, \epsilon_t, \bar{O}_t, \phi_t\}_{t=0}^{\infty}$.

$\hat{w}_t^{new}$, as well as the cost of hiring new workers, i.e., $\hat{\gamma}_t^b$. The second term represents what the literature typically identifies as cost push or supply shocks, while the last term represents inflation expectation as is standard.

An important implication of this characterization is that it demonstrates that, while marginal costs have an effect on inflation, as in the standard New Keynesian model, they are expressed differently. In the canonical New Keynesian model, marginal costs refer to wages of all employees, usually measured by time series such as the aggregate labor share or the Employment Cost Index (ECI), a BLS survey of employers' payrolls that measures total compensation. In contrast, our model suggests that the relevant marginal costs are instead approximated by the cost of adding new workers to the workforce, whose wages may or may not correspond to the wages of existing workers, depending on the state of the labor market.[63] This cost, in turn, is summarized by the wages of new hires as well hiring costs, which might for example include bonuses.[64]

The non-linearity of the Phillips curve arises because the wage rate of new hires depends on whether the labor market is tight. Let us define $\hat{w}_t^{new} \equiv \ln w_t^{new} - \ln \bar{w}$ and $\hat{w}_t^{flex} \equiv \ln w_t^{flex} - \ln \bar{w}^{flex}$. Then we rewrite equation (31) in logs as

$$\hat{w}_t^{new} = \max(\hat{w}_t^{ex}, -c_w + \hat{w}_t^{flex}) \tag{47}$$

where $c_w = \ln \bar{w}/\bar{w}^{flex}$. As discussed in Appendix E, we consider a steady state with a real wage given by $\bar{w}$ with the steady-state flexible real wage below that value, i.e., $\bar{w}^{flex} < \bar{w}$. The parameter c_w is therefore positive. We approximating around $\bar{\theta}$, which is below the kink point $\bar{\theta}^*$, to characterize normal times conditions when the labor market is slack. This is consistent with the empirical evidence of Figure 4.

Since the expression for w_t^{flex}, w_t^{ex} and θ^* are all linear in logs (see equations (36), (44) and (45)), the following expression, expressed in natural logs, is exact and involves no approximation error:

$$\hat{w}_t^{new} = \begin{cases} -c_w + \hat{w}_t^{flex} = -c_w + \eta\hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t & \hat{\theta}_t > \hat{\theta}_t^* \\ \hat{w}_t^{ex} = \lambda(\hat{w}_{t-1}^{ex} - (\pi_t - \pi) + \delta E_t(\pi_{t+1} - \pi)) + (1-\lambda)(\hat{w}_t^{flex}) & \hat{\theta}_t \leq \hat{\theta}_t^*, \end{cases} \tag{48}$$

assuming $\pi_{t+1}^e = E_t \pi_{t+1}$.

[63] Under normal circumstances, i.e. in the absence of labor shortage, the two move together, consistent with the work of Gertler, Huckfeldt and Trigari (2020). It is during periods of labor shortage that the distinction between the wages of new and existing hires becomes critical in the model, consistently with the evidence shown in Figure 28.

[64] Bernanke and Blanchard (2024), for example, suggest that labor market tightness did not, at the very beginning of the inflation surge of 2022, play a fundamental role. Their result, however, depends upon the assumption that they measure marginal costs via ECI, in contrast to our model. Perhaps more importantly, our framework highlights the interaction between labor tightness and supply shock. One advantage of our approach is that we can explain why supply shocks had such an outsized effect during this period. Moreover, our model is consistent with the lack of disinflation post-2008.

Substituting equation (48) into the Phillips curve (46), it can be written as:

$$\pi_t - \pi = \begin{cases} -c + \kappa^{tight}\hat{\theta}_t + \kappa_\nu^{tight}(\hat{\nu}_t + \hat{\vartheta}_t^{tight}) + \beta E_t(\pi_{t+1} - \pi) & \hat{\theta}_t > \hat{\theta}_t^* \\ \kappa_w \hat{w}_{t-1} + \kappa \underbrace{\hat{\theta}_t}_{\text{tightness}} + \kappa_\nu(\underbrace{\hat{\nu}_t}_{\text{cost-push}} + \underbrace{\hat{\vartheta}_t}_{\text{matching}}) + \kappa_\beta E_t(\pi_{t+1} - \pi) & \hat{\theta}_t \leq \hat{\theta}_t^*, \end{cases} \tag{49}$$

where the coefficients satisfy $\kappa^{tight} > \kappa > 0$, $\kappa_\nu^{tight} > \kappa_\nu > 0$, $\kappa_w > 0$ and $c > 0$. It is ambiguous if κ_β is greater or less than β. These results can be confirmed by the analytical expression of the coefficients, which are given in Appendix E.

It is worth commenting briefly the accuracy of the log-linear approximation of the model. The approximation of the New Keynesian Phillips curve shown in equation (49) takes the same form as in the standard model, with the only difference being that w_t^{new} appears in this expression instead of average wages. As discussed above, however, our expression for w_t^{new} involves no approximation error, as it is linear in logs, and thus, while log-linear approximations always involve approximation errors, there is little ground for believing it to be more significant in this application than in the canonical model. This may seem surprising, given that we are approximating a piecewise linear Phillips curve. The key insight is that the kink arises from the max operator (47), and the expression within it involves no approximation, as the variables are linear in logs.

5.1 Four central predictions of the INV-L New Keynesian Phillips Curve and a key result

The theoretical results shown in equation (49) highlight four major theoretical predictions which are born out in the empirical Section 3.

First, $\kappa^{tight} > \kappa$: This says that when the labor market is sufficiently tight, i.e. $\hat{\theta}_t > \hat{\theta}_t^*$, then inflation responds much more strongly to labor market tightness than under normal circumstances. This prediction is a central focus of our empirical analysis in Section 2.

Second: $\kappa_\nu^{tight} > \kappa_\nu$. This says that that cost-push and markup shocks have a greater impact on inflation when the labor market is tight $\hat{\theta}_t > \hat{\theta}_t^*$, which is another key result of our empirical analysis.

Third, while real wages enter the Phillips curve as a lagged variable when $\hat{\theta}_t < \hat{\theta}_t^*$, the curve becomes perfectly forward-looking when $\hat{\theta}_t \geq \hat{\theta}_t^*$. This prediction is also supported by our empirical analysis, although it is of less central importance.

Finally, a key result of our analysis is that we can replace the empirical measure of new wages with the ratio of vacancies to unemployment, i.e., θ. This is valuable for connecting to an emerging empirical literature that argues v/u is a better measure of slack than u_t. From the perspective of our theoretical framework, this substitution is also of central importance: θ_t is a sufficient statistic for some key exogenous shocks, which would otherwise appear explicitly if the model were expressed solely in terms of output or unemployment—such as labor force participation shocks, χ_t.[65]

5.2 The determinant of the Beveridge threshold and the interpretation of shocks

Using logs, we can rewrite Equation (45) in deviations from the steady state and solve for the Beveridge threshold:

$$\hat{\theta}_t^* = \eta^{-1}\hat{w}_{t-1}^{ex} + \eta^{-1}\delta E_t\pi_{t+1} - \eta^{-1}\pi_t + \lambda^{-1}\eta^{-1}\hat{\phi}_t + \hat{m}_t + \hat{\gamma}_t^b - \hat{\gamma}_t^c \tag{50}$$

The expression for the Beveridge threshold in Equation (50) is intuitive. It is best understood by revisiting Equation (48), which defines the Beveridge threshold. As θ_t increases beyond θ_t^*, new wages exceed existing ones (i.e., $w_t^{new} = w_t^{flex} > w_t^{ex}$), and the slope of the Phillips curve changes accordingly. The critical condition is thus $w_t^{flex} > w_t^{ex}$. The more likely this inequality holds for a given θ_t, the lower the threshold θ_t^*.

The Beveridge threshold decreases if existing real wages from the previous period are below their steady-state level or if their real value declines due to current inflation (as wages are fixed in nominal terms). Similarly, a negative exogenous shock to existing wages, represented by $\hat{\phi}_t$, will also lower the threshold.

The final three terms in Equation (50)—$\hat{m}_t$, $\hat{\gamma}_t^b$, and $\hat{\gamma}_t^c$—directly influence the flexible wage rate for a given θ_t, thus affecting the Beveridge threshold. For example, lower matching efficiency ($\hat{m}_t < 0$) reduces the threshold.

The traditional "cost-push" and supply are denoted by $\hat{v}_t$ and defined as

$$\underbrace{\hat{v}_t}_{\text{Cost push/supply shocks}} \equiv \underbrace{\hat{\mu}_t}_{\text{Markups}} - \underbrace{\hat{A}_t}_{\text{Productivity}} + (1-\alpha)\underbrace{\hat{q}_t}_{\text{Oil price}}. \tag{51}$$

The shocks to which the hiring agency is sensitive—specifically, shocks to the matching technology and shocks to the marginal cost of posting vacancies, as well as the marginal benefit of successfully placing a worker—are summarized by $\hat{\vartheta}_t$

$$\underbrace{\hat{\vartheta}_t}_{\text{Matching shocks}} \equiv \alpha(d_\gamma - (1-\lambda))\underbrace{\hat{\gamma}_t^b}_{\text{Firm hiring cost}} + \alpha(1-\lambda)\underbrace{\hat{\gamma}_t^c}_{\text{Vacancy cost}} - \alpha(1-\lambda)\underbrace{\hat{m}_t}_{\text{Matching efficiency}} + \alpha\underbrace{\hat{\phi}_t}_{\text{Wage shock}}, \tag{52}$$

[65] Moreover, while data on $\frac{v}{u}$ is readily available over long time periods, there is no consensus on the best measure of the wages of newly hired workers, and the available evidence does not yet cover as long a time span as that available for v/u. This is also ground for future research.

in which $d_\gamma \equiv \gamma^b/(1+\gamma^b)$.

Unlike the cost-push shock $\hat{\nu}_t$, which remains the same regardless of the labor market's tightness, the shocks to matching technology take a different form when the labor market is tight; in this case, they are summarized by

$$\hat{\vartheta}_t^{tight} = \alpha(\hat{\gamma}_t^c - (1-d_\gamma)\hat{\gamma}_t^b - \hat{m}_t).$$

The reason is that these shocks operate through the determination of flexible wages. In normal times, the flexible wage receives a weight $1-\lambda$, while during periods of labor tightness, they are transmitted directly to wages.

Let us now turn to the policy implications.

6 The policy framework for the Inv-L NK Phillips curve

In this section we show that our general framework can be applied to explain three significant episodes central to U.S. monetary policy analysis: the inflation surge of the 2020s, the "missing disinflation" following the financial crisis of 2008, and the Great Inflation of the 1970s. We utilize the results from our regression analysis to parameterize the model, thereby evaluating the extent to which it can rationalize, at least in broad strokes, the observed patterns in U.S. data. In our follow-up paper, Benigno and Eggertsson (2024), we explore in better detail the other inflation surges observed in US data over the past 115 years in comparison.

We have demonstrated that the Phillips curve can be expressed in terms of $\hat{\theta}_t$, aligning the model's predictions with our findings and recent empirical work, indicating this measure of economic tightness is a better predictor of inflation than other common proxies of aggregate demand. Additionally, there is a direct connection between θ and output. Since most policy discussions typically revolves around output and inflation, it is advantageous to cast the model in these more conventional terms that follows.

To streamline the model and focus more closely on the key points, we make the simplifying assumption that in equilibrium $X_t = C_t = Y_t - G_t$.[66] This implies that the Euler Equation for consumption takes the traditional form as in the New Keynesian literature:

$$\hat{Y}_t - \hat{G}_t = E_t\hat{Y}_{t+1} - E_t\hat{G}_{t+1} - \sigma^{-1}(\hat{\imath}_t - E_t(\pi_{t+1} - \pi^*) - \hat{r}_t^e) \quad (53)$$

where $\hat{r}_t^e \equiv \hat{\xi}_t - E_t\hat{\xi}_{t+1}$ is the part of the natural rate of interest that is generated by shocks to preferences and π^* is the inflation target of the central bank which the model is approximated around. Here

[66] To obtain this result, we assume that in equilibrium $\Psi_t = F_t^{1+\omega}/(1+\omega)$.

$\hat{G}_t$ represents direct government consumption. In what follows, we interpret it to correspond more broadly to general "fiscal expansion" including spending due to tax rebates, as, e.g., the COVID-19 epidemic.[67]

To characterize the AS equation, we simplify the wage mechanism of the new hires to

$$w_t^{new} = \begin{cases} w_t^{flex} & \text{if } \theta_t > \theta_t^* \\ \\ \bar{w}^{\lambda}(\Pi_t^{-1})^{\lambda}(\Pi_{t+1}^{e})^{\delta\lambda}(w_t^{flex})^{1-\lambda} & \text{if } \theta_t \leq \theta_t^* \end{cases}$$

in which $\bar{w}$ represents the steady-state real wage. This implies that no lagged variables appear in the Phillips curve when the labor market is slack – a feature that, while empirically relevant for the full sample in our estimation, is not critical for the policy analysis. Making the model perfectly forward-looking enables a tighter analytic characterization, which can be illustrated graphically.

Denote by Y_t^* the level of output when the Phillips curve becomes steeper, i.e. when $\theta_t > \theta_t^*$. The Appendix shows that the Phillips curve is then given by:

$$\pi_t - \pi^* = \begin{cases} -\tilde{c} + \tilde{\kappa}^{tight}\left(\hat{Y}_t + \frac{\alpha}{\omega}\hat{\chi}_t\right) + \tilde{\kappa}_{\nu}^{tight}\hat{\nu}_t + \tilde{\kappa}_{\beta}^{tight}E_t(\pi_{t+1} - \pi^*) & \text{if } \hat{Y}_t > \hat{Y}_t^* \\ \\ \tilde{\kappa}\left(\hat{Y}_t + \frac{\alpha}{\omega}\hat{\chi}_t\right) + \tilde{\kappa}_{\nu}\hat{\nu}_t + \tilde{\kappa}_{\beta}E_t(\pi_{t+1} - \pi^*) & \text{if } \hat{Y}_t \leq \hat{Y}_t^*, \end{cases} \tag{54}$$

where the coefficients, which are detailed in Appendix E, satisfy $\tilde{\kappa}^{tight} > \kappa > 0$, $\tilde{\kappa}_{\nu}^{tight} > \tilde{\kappa}_{\nu} > 0$ if $\lambda > 0$. The relationship between $\tilde{\kappa}_{\beta}^{tight}$ and $\tilde{\kappa}_{\beta}$ is again ambiguous, while $\tilde{c} > 0$. To simplify the analysis, we set $\tilde{\kappa}_{\beta}^{tight} = \tilde{\kappa}_{\beta} = 1$, eliminating any long-run trade-off between inflation and output, though this is not crucial to our main results.[68] Relative to our earlier exposition we see the relevance of a new shock, which was previously embodied in $\hat{\theta}_t$. This is the shock to labor participation, $\hat{\chi}_t$. An increase in $\hat{\chi}_t$ represents higher dis-utility of working, which for a given output leads to higher inflationary pressures.

We close the model with a simple policy rule:

$$\hat{\imath}_t = \rho\hat{r}_t^e + \phi_{\pi}(\pi_t - \pi^*) + e_t \tag{55}$$

where $\phi_{\pi} > 1$ is a reaction coefficient of inflation deviating from the target, denoted by π^*. Recall that $\hat{r}_t^e$ contains movements in aggregate demand explained by the demand disturbance ξ_t. We allow

[67]During the COVID-19 epidemic, fiscal expansion largely took the form of direct cash transfers to households. In our model, such transfers are neutral due to Ricardian equivalence. However, it is well known how to extend this framework to incorporate borrowing-constrained agents (see, e.g., Eggertsson and Krugman, 2012). A recent survey by Eggertsson and Sergeyev (2025) makes the connection to our model explicit, showing that the structure remains unchanged except that taxes now appear alongside G_t.

[68]By assuming that $\tilde{\kappa}_{\beta}^{tight} = \tilde{\kappa}_{\beta} = 1$ then in the steady state, where all variables take a constant value, the Phillips curve implies that inflation cancels out and output is at steady state.

Table A: Parameters based on existing literature

Calibrated parameters	Symbol	Value
Intertemporal rate of substitution	σ	0.5
Taylor rule coefficient	ϕ_π	1.5
Matching technology parameter	η	0.4
Steady state unemployment	u	0.04
Fraction of labor force attached to existing firms	$1-z$	0.9267
Share of intermediate input (oil) in the production function	$1-\alpha$	0.1
Inverse of the elasticity of labor-force supply	ω	1
Response of existing wages to flexible wages	λ	0.5

the central bank to offset these movements in proportion to ρ, with $0 \leq \rho \leq 1$, so that if $\rho = 1$, they are fully offset. We leave open the question of whether the central bank responds to supply shocks or demand shocks, such as government spending, beyond their effects on inflation. The central bank's response to these variables can be incorporated into the monetary policy shock e_t, which we leave unspecified.

The following discussion is highly stylized. However, the results from the regression in Section 2 can be directly mapped to the parameters of the Phillips curve.

We consider three major economic events where inflation dynamics have been central: 1) The inflation surge of the 2020s which was largely unexpected; 2) The Great Inflation of the 1970s; 3) The 2008 "missing disinflation." For episodes 1) and 3) we use the estimates of Table 1 regarding the sample 2008 Q3-2024 Q4, for 2) we use the entire sample.

Table A shows the values of the parameters we take as given. The model is written in quarterly frequencies to match our data observations. Most values are typical relative to the literature, such as σ and ϕ_π. The matching parameter η is taken from Blanchard, Domash, and Summers (2022). The steady-state unemployment rate is 4 percent, approximating the current estimate by the Federal Reserve of the level of unemployment consistent with inflation being on target (see, e.g., Eggertsson and Kohn (2023)). We use labor market data, specifically the latest value of the time series z_t shown in Figure 30, to calibrate $1-z$, the fraction of the labor force attached to existing jobs. This will be further discussed in Section 6.1.1. One interpretation of the intermediate input in the production function is that it represents oil. The value we use is higher than the typical estimates of the share of oil in output (e.g., Blanchard and Gali (2010b) report it as 3 percent of output). We assume 10 percent to allow for a broader interpretation of intermediate goods, considering the significant impact of tradable goods prices during the inflation surge of the 2020s. We also assume $\omega = 1$, a standard value in the monetary literature,[69] and set λ at 0.5, which means that existing wages are pulled equally towards the wage rate that would prevail if wages were flexible and towards past wages.

[69]Even though this value is typically chosen in models where the labor choice of households is on the intensive margin as opposed to the extensive margin.

Table B: Coefficients in the Phillips curve (54) based on the estimation and the calibrated parameters

Coefficients in the Phillips curve (54)	1960–2024	2008–2024
$\tilde{\kappa}$	0.0029	0.0065
$\tilde{\kappa}_v$	0.0448	-0.0096
$\tilde{\kappa}^{\text{tight}}$	0.0495	0.0747
$\tilde{\kappa}_v^{\text{tight}}$	0.1517	0.2735

Given the parameters in Table A, we can use the expressions in the Appendix, together with empirical analysis, to back out the implied values for the key parameters of the Phillips curve (54), which are summarized in Table B.[70].

Under a special assumption about the shocks, the model is boiled down to a pair of equations that can then be plotted as a simple AS-AD diagram. The assumption is as follows: in period 0, the exogenous variables unexpectedly take on certain values. In each subsequent period, the shocks revert back to the steady state with a probability of $1-\tau$. Once back to the steady state, the shocks remain there forever. The time period when $0 \leq t < t_L$ is called the short run and is indexed by the subscript S. The time period when $t \geq t_L$ is called the long run and is indexed by L. The major convenience of this assumption is that it implies that in the long run, provided $\phi_\pi > 1$, there is a unique bounded solution for inflation, output, and the interest rate given by

$$\pi_L - \pi^* = \hat{Y}_L = \hat{\imath}_L = 0. \tag{56}$$

This solution is obtained by solving jointly equation (53)-(55) making the assumption that $\hat{Y}_L \leq \hat{Y}^*$.

Let us suppose that in the short-run, then inflation expectation are given by:

$$E_t\pi_{t+1} = \tau E_{t,S}\pi_{t+1} + (1-\tau)E_{t,L}\pi_{t+1} = \tau E_{t,S}\pi_{t+1} + (1-\tau)\pi_L^e \tag{57}$$

where $E_{t,S}$ is expectation at time t conditional on the shock remaining unchanged, while $E_{t,L}$ denotes the expectation at time t conditional on the shock reverting back to steady state. Here we allow for the possibility that people's expectation of long-run inflation, π_L^e, may be different from π^* we just derived. We do this to capture the possibility that expectations can become unanchored from the central bank's inflation target. This is especially relevant for understanding the 1970s.

Expectations for output in the short-run are:

$$E_t\hat{Y}_{t+1} = \tau E_{t,S}\hat{Y}_{t+1} + (1-\tau)E_{t,L}\hat{Y}_{t+1} = \tau E_{t,S}\hat{Y}_{t+1,S}. \tag{58}$$

[70] See Appendix E for the formulas to compute the $\tilde{\kappa}'s$

Since the Phillips curve has no long-run trade-offs, even if expectations of long-run inflation become unanchored in the short run, long-run output is expected to stabilize at zero, i.e., $\hat{Y}_L = 0$. Substituting expectations into equations (53) and (55) once again leads to a unique bounded solution for $\hat{Y}_S$, $\hat{\pi}_S$, and $\hat{\imath}_S$, which are no longer zero but instead depend on the value of the shocks and π_L^e. To illustrate this solution graphically, we can substitute (55) into (53) to obtain the aggregate demand (AD) curve.

The AD curve is:[71]

$$\hat{Y}_S = \underbrace{\hat{D}_S}_{\text{Shocks to AD}} - \sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}(\pi_S - \pi^*) + \sigma^{-1}(\pi_L^e - \pi^*) \tag{59}$$

This implies that demand is decreasing in inflation because the central bank increases its interest rate in response by more than one-to-one, i.e., $\phi_\pi > 1$. The term $\hat{D}_S$ represents demand shocks, which can be summarized as:

$$\hat{D}_S = \underbrace{\hat{G}_S}_{\text{Fiscal policy shock}} - \frac{\sigma^{-1}}{1-\tau} \times \underbrace{e_S}_{\text{Monetary Policy Shock}} + \frac{\sigma^{-1}}{1-\tau} \times \underbrace{(1-\rho)\hat{r}_t^e}_{\text{Other demand shocks not offset by MP}} \tag{60}$$

The short-run AS curve can similarly be written as:

$$\pi_S - \pi^* = \begin{cases} -\frac{\tilde{c}}{1-\tau} + \frac{\tilde{\kappa}^{tight}}{1-\tau}(\hat{Y}_S + \frac{\alpha}{\omega}\hat{\chi}_S) + \frac{\tilde{\kappa}_v^{tight}}{1-\tau}(1-\alpha)\hat{q}_S + \pi_L^e - \pi^* & \text{if } \hat{Y}_t > \hat{Y}^* \\ \frac{\tilde{\kappa}}{1-\tau}(\hat{Y}_S + \frac{\alpha}{\omega}\hat{\chi}_S) + \frac{\tilde{\kappa}_v}{1-\tau}(1-\alpha)\hat{q}_S + \pi_L^e - \pi^* & \text{if } \hat{Y}_t \leq \hat{Y}^*. \end{cases} \tag{61}$$

6.1 Understanding the inflationary surge of 2020s

The AD and AS curves we derived offer a straightforward way to characterize the inflation surge during the 2020s. These curves are illustrated in Figure 11 using the parameterization we have just described.[72] As shown in the data in Figure 7, core inflation, measured at annualized quarterly rates, increased from its 2 percent target to 6.2 percent in the second quarter of 2022. The relative importance of the AD and AS shocks is calibrated to match our empirical estimates and decomposition, as depicted in Figure 7. Demand contributes 3 percentage points to the rise in core inflation above the 2 percent target, while the supply shock accounts for the remaining 1.2 percentage points, resulting in a total core inflation rate of 6.2 percent, excluding other factors.[73]

[71]Given the existence of a unique bounded solution observe that expectations in the short run for output are given by $\tau\hat{Y}_S$ and inflation $\tau\pi_S + (1-\tau)\pi_L$ so that the determination of $\hat{Y}_S$, $\hat{\pi}_S$ is summarized by the intersection of the Phillips curve (or AS equation) and the AD curve.

[72]We assume $\tau = 0.8$, representing an 80 percent probability that the short-run conditions will persist from one period to the next.

[73]Bergholt et al. (2024) also find, using a different econometric procedure, that supply shocks account for one-third of the U.S. inflationary surge.

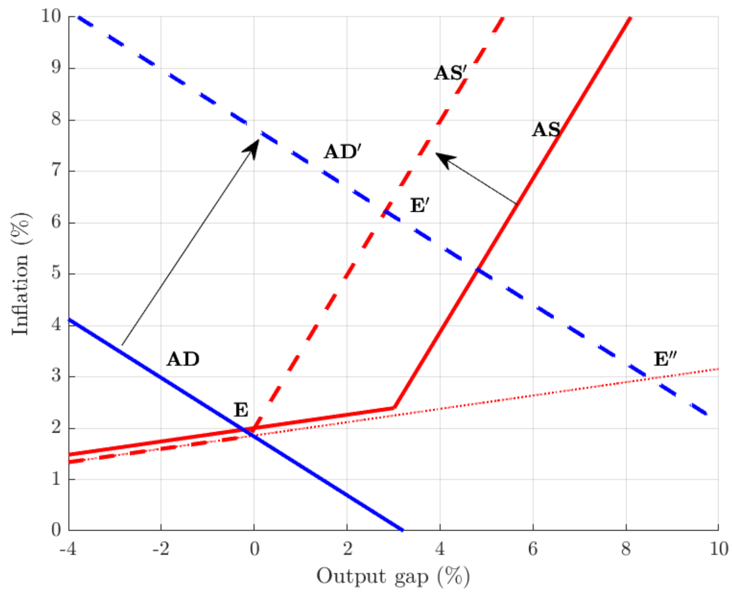


Figure 11: The 2020s Inflationary Surge: Inflation and output determination using the AS-AD model in response to an increase in demand and a supply shock. Inflation is measured at annualized quarterly rates and presented in percent, while the output gap is expressed in percentage points. The AD curve shifts from AD to AD′ following the demand shock, while the AS curve shifts from AS to AS′ in response to the supply shock. The equilibrium transitions from point E to point E′.

To see the importance of the non-linearities, consider the intersection with AD curve, assuming the Phillips curve remains flat at point E''. In this case, inflation would only have increased from 2 to 3 percent. Moreover, this increase would have been solely due to the shift in demand, as the effect of supply shocks is not statistically significant.

We have not been explicit about the drivers behind the increase in aggregate demand. As shown in equation (60), this rise can be attributed to a fiscal shock, a recovery in demand for other reasons (e.g., higher r_t^e not offset by the central bank, where $\rho < 1$), or an expansionary monetary policy shock, e_S. We summarize these forces with the term $\hat{D}_S$ in equation (59). Eggertsson and Kohn (2023) argue that all of these factors played a role.

An important corollary of the AS curve being steep is that it requires a much smaller drop in output to bring down inflation. Compare again the equilibrium points E' and E'' in the case where the Phillips curve is flat. In the former case, bringing down inflation from 6.2% to 2% reduces output by about 2.4%. In comparison, bringing down inflation from E'' implies a reduction of 8.5%.

6.1.1 Understanding the soft landing, the Beveridge curve and the informational content of θ_t

The reduction of the inflation spike from 2022 Q2 to the end of our sample in 2024 Q4 was accomplished without any increase in unemployment or reduction in output. While considering that the

AS curve is steeper when $\hat{Y}_t > \hat{Y}^*$ rationalizes why there was not a significant reduction in output compared to assuming a flat Phillips curve, there are two additional forces that explain why output did not fall and unemployment did not rise.

First, there was an increase in $\hat{\chi}_t$, which led to an endogenous reduction in labor force participation. This shift explains an even stronger backward movement of the AS curve, thus requiring a smaller increase in demand to account for the inflation surge. As we have shown, however, the characterization of the Phillips curve in terms of labor tightness means that $\hat{\theta}_t$ is a sufficient statistic that incorporates the effect of labor force participation via $\hat{\chi}_t$, which does not appear in the Phillips curve when expressed in terms of $\hat{\theta}_t$. Therefore, variations in $\hat{\chi}_t$ are only relevant to the extent that we aim to rationalize the output dynamics associated with the inflation surge. While it is plausible that $\hat{\chi}_t$ increased during the COVID-19 epidemic, its reversal to pre-COVID levels is also observed in the data. Moreover, our assumption that this represents exogenous shifts seems largely plausible, as it was driven by factors related to the spread of the COVID-19 virus, the development of vaccines, treatments, and so forth.

Second, while the decline in labor force participation can be partly explained by an increase in $\hat{\chi}_t$, this does not fully account for it for two reasons. First, not only did the labor force participation rate decline, but there was also a sharp increase in the *separation rate* between active workers and firms.[74] Additionally, the data suggest a decline in matching efficiency: the number of matches created for a given number of posted vacancies and unemployed workers decreased. Both phenomena can be captured in reduced form by time variations in z_t and m_t. However, z_t does not represent the separation rate commonly used in the literature; instead, it denotes the fraction of the labor force that is not attached to a job in a given period.[75] This fraction either becomes unemployed or represents a new hire.

Below, we present evidence of substantial movements in m_t and z_t. These shifts help explain why the informational content of θ_t was unusually high during this period compared to u_t alone. To illustrate this, combine equations (28) and (29) to derive the Beveridge curve, which describes the relationship between v_t and z_t:

$$v_t = \left(\frac{z_t - u_t}{m_t u_t^{\eta}} \right)^{\frac{1}{1-\eta}} \tag{62}$$

for $u_t \leq z_t$, which holds at any time t, unlike representing only a steady-state relationship as in many labor market models. This is due to the abstraction in our model that the labor force attached to existing jobs is independent of past employment history. If there are no changes in z_t or m_t over time then this equation shows that v_t provides no additional information.

Yet, assuming m_t and z_t being constant is inconsistent with the data, explaining the empirical relevance of θ_t. To see this, we can back out the time series for z_t directly from the data since it has a

[74] A separation between firms and workers differs from a reduction in labor force participation, as separated workers remain in the labor force but search for alternative employment.

[75] Some of these workers, who were employed last period, can be interpreted as having already been searching on the job.

natural empirical counterpart. It measures the fraction of the labor force that is not attached to existing firms in a given period. In equilibrium, the fraction of unattached workers must then equal the sum of: i) the fraction hired (h_t, i.e., hires as a fraction of labor force) in the period, and ii) the fraction that winds up unemployed (u_t, i.e., unemployment rate). Thus,

$$z_t = h_t + u_t \tag{63}$$

where both h_t and u_t can be empirically measured. The number of new hires, h_t, encompasses three flow variables: new entrants to the labor force who become employed, unemployed individuals from the previous period who find employment, and currently employed workers who transition to new jobs – thereby representing job-to-job transitions. These transitions make up approximately 3/4 of all new hires in the U.S. labor market. Using the time series for h_t, u_t and v_t, we can back out from equation (62) the time series for matching efficiency as

$$m_t = \frac{h_t}{u_t^{\eta} v_t^{1-\eta}}, \tag{64}$$

in which we set $\eta = 0.4$ as in Blanchard, Domash and Summers (2022).[76]

In the Appendix we show in Figure 30 the evolution z_t that the fraction of unattached workers, z_t, spikes at the beginning of COVID-19, indicated by a red dot, since several firms laid off workers in response to the pandemic. It then falls abruptly, corresponding to the inflation surge marked by the two dashed lines. During the inflation surge, and through to the end of the sample in 2024 Q4, it smoothly drops to pre-COVID levels and has recently stabilized. The second panel of Figure 30 shows a sharp reduction in matching efficiency at the beginning of the pandemic, which is then partially reversed and gradually improves during the inflation surge. Unlike z_t, matching efficiency still remains below pre-COVID levels at the end of our sample.

To see how the evolution of these two variables helps explaining why v/u is useful to characterize the recent inflation dynamics, beyond unemployment alone, we can rewrite (62) as:

$$\theta_t = \frac{v_t}{u_t} = \left\{ \frac{z_t - u_t}{m_t u_t} \right\}^{\frac{1}{1-\eta}} \tag{65}$$

Equation (65) clarifies that a tight labor market, e.g., $\theta > 1$, can be consistent with high unemployment, as observed during the run-up of inflation in 2021, when the vacancy rate is also high. This situation can arise either due to a large number of unattached workers z_t or because the matching efficiency m_t is low. For instance, just as the surge was about to take off in March 2021 unemployment was 6%, leading many observers to consider the labor market slack. Yet, the vacancy rate was already

[76] Benigno and Eggertsson (2024a) discuss how this compares with other proposals related to the Beveridge curve as Figura and Waller (2022), highlighting its robustness in predicting a soft landing regardless of assumptions about the elasticity η. See also Bok et al. (2022) for an alternative approach to the Beveridge curve predicting a soft landing.

rapidly rising at the time at 5.3% in march, but surpassing (6.6%) the unemployment rate (5.75%) in May 2021 implying $\theta = 1.07$. 1.07.

Figure 31 in the Appendix plots the Beveridge curve (62) since January 2020, reflecting the shifts in z_t and m_t. It is notable how the data on vacancy and unemployment rates since March 2022—when the Federal Reserve began tightening policy—closely align with a stable Beveridge curve. This suggests strong predictive performance of our model during the period of falling inflation, primarily driven by declines in vacancies. This alignment supported our 2023 prediction that reducing inflation might not entail significant economic costs.

While the spike in separations and therefore unattached workers is clearly traced to the COVID-19 crisis, we believe that the same factors affected matching efficiency. A reasonable explanation for the decline in matching efficiency – indicated by the need for employment agencies to post more vacancies to generate an employment match for a given level of unemployment – is a structural change in aggregate spending. It likely takes a worker longer to find a suitable job match when moving across sectors than when switching jobs within the same sector. Eggertsson and Kohn (2023) illustrate that the COVID-19 recovery was uneven, with spending in the goods market significantly outpacing the recovery in services. The spending mix has yet to return to its pre-COVID composition. While we have not explicitly modeled matching efficiency, an important extension of the framework is to make matching efficiency endogenous. Another area for future research is extending the model to incorporate multiple sectors along with network structure, reflecting the uneven increase in spending across sectors observed in the data, and to assume higher varying degree of search costs for finding a job in alternative sectors. Analysis of this kind can provide useful insights into the endogenous evolution of matching efficiency due to sectoral reallocation.[77]

In a followup paper, Benigno and Eggertsson (2024a), we extend our framework to achieve a more realistic analytical representation of the Beveridge curve by modeling variations in z_t in greater detail. Specifically, we show that under conditions of a loose labor market (i.e., when $\theta < 1$), z is empirically inversely related to θ, making its dynamics depend on crossing the Beveridge threshold. The bottom line is that our extension captures the dynamics of the vacancy rate and unemployment rate both under normal circumstances, and when labor is scarce.

6.2 Understanding the inflation surge in the 1970s, the disinflation and Volcker recession in the 1980s

We next consider how the model explains the increase in inflation during the 1970s, often referred to as the "Great Inflation." This narrative will be familiar to most readers, and our interpretation does not differ substantially from the conventional account (see, e.g., Erceg and Levin (2003) and Goodfriend and King (2006)). Nevertheless, it is helpful to articulate this explanation within our model so that

[77]Guerrieri et al. (2022) present a two-sector model to characterize the aggregate implications on inflation and economic activity of asymmetric shocks.

we can contrast it with our account of the inflationary surge of the 2020s. As in the previous section, it is useful to consider both the short run and the long run, defined by a stochastic reversal.

First, we assume a short-run supply shock given by $\hat{v}_S = (1-\alpha)\hat{q}_S$, meaning we consider the possibility that inflation was triggered by the large oil shocks observed during the 1970s. Second, we assume that in the short run, the public held different beliefs about the central bank's long-run inflation target, which we denote as π^*. This implies that even though long-run inflation eventually stabilized (i.e., $\pi_L = \pi^*$), as was eventually the case in the U.S., the public's expectations in the 1970s did not align with how things actually turned out. In other words, π_L^e is assumed to differ from where inflation settled ex post.

The rationale for this assumption is straightforward: several long-term inflation expectation measures in the 1970s and early 1980s, as cited in the introduction, suggested that 5-year 5-year forward inflation expectations reached 10% and only gradually declined during the 1980s associated with the so-called "Volcker recession". This indicates that the public's long-run inflation expectations in the 1970s were "incorrect" ex post.[78]

In the long run, we assume the labor market is back to "normal." In the absence of shocks, the unique bounded solution is simply given by $\pi_L = \pi^*$ and $\hat{Y}_L = 0 < \hat{Y}^*$.

Let us now consider the short run. Suppose that expected long-term inflation expectations are above π^*, as observed in the data, and constant at $\pi_L^e > \pi^*$. Then inflation expectations in the short run are

$$E_S \pi_{S+1} = \tau \pi_S + (1-\tau)\pi_L^e.$$

Since our model is characterized by long-run monetary policy neutrality, and we assume in the long run people are fully rational, then even though people expect $\pi_L^e > \pi^*$ in the short run, it remains the case that a unique bounded solution is given by $\hat{Y}_L = 0$.

Setting all shocks to zero (except the oil-price/energy shock) the AD equation is:

$$\hat{Y}_S = -\sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}(\pi_S - \pi^*) + \sigma^{-1}(\pi_L^e - \pi^*), \tag{66}$$

while the Inv-L NK Phillips Curve is:

$$\pi_S - \pi^* = \begin{cases} -\frac{\tilde{c}}{1-\tau} + \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v^{tight}}{1-\tau}\hat{v}_S + \pi_L^e - \pi^* & \text{if } \hat{Y}_t > \hat{Y}^* \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_v}{1-\tau}\hat{v}_S + \pi_L^e - \pi^* & \text{if } \hat{Y}_t \leq \hat{Y}^*. \end{cases} \tag{67}$$

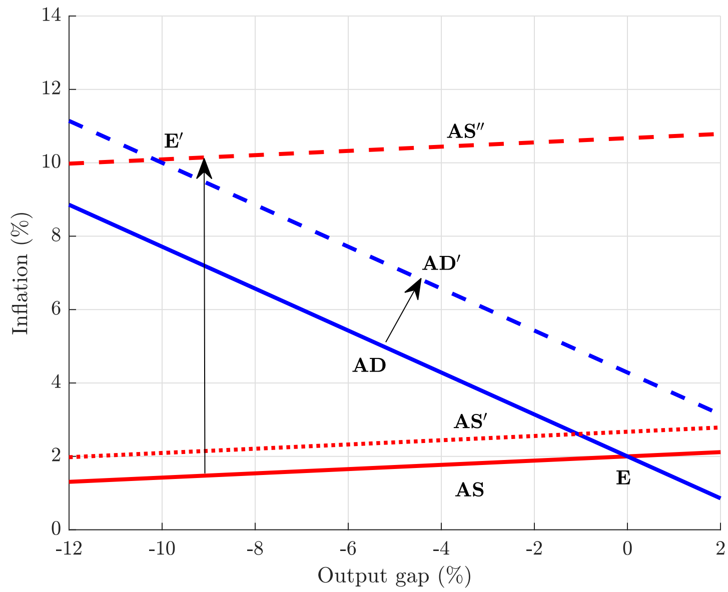


Figure 12: The 1970s Great Inflation: Inflation and output determination using the AS-AD model in response to an increase in long-run inflation expectations and an oil shock. Inflation is measured at annualized quarterly rates and presented in percent, while the output gap is expressed in percentage points. The AS curve shifts from AS to AS′ following a 30% rise in oil prices. Subsequently, with a 10% increase in long-run inflation expectations, the AS′ curve moves to AS″ and the AD curve to AD′. The equilibrium transitions from E to E′.

See Appendix E for further details of the derivations.

These two equations are plotted in Figure 12, the aggregate demand equation (66) in blue and the Inv-L NK Phillips Curve in red. Entering the 1970s, θ was less than one and held at that level during the period so that that $Y_t < Y^*$. Equilibrium is determined by the intersection of the AD and AS curves. First, consider the initial equilibrium when inflation is at the target and output is at potential, represented by point E. The numerical example highlights that the Inv-L NK Phillips curve is very flat since $\hat{Y}_t^*$. To fully understand the short-run movements, we need to consider two key factors.

First, the supply shock and the extent to which inflation expectations become "unanchored." For this, we use the principal component measure of the supply shock, shown in Figure 22, which peaked at 30 percent. We also assume that inflation expectations rose to 10 percent. Second, we consider the expected duration of the short run, which we again assume persists with a probability of 0.8 from one period to the next.

Consider first the effect of a supply shock. This directly shifts the red curve upward, leading to higher inflation and lower output, as shown by the movement from AS to AS'. The oil shocks of the 1970s clearly played a role in the inflation surge. Of greater importance, however, is the effect of a rise in the central bank's inflation target as *perceived* by the private sector and reflected in expected inflation, i.e.,

[78] Of course, there is nothing that necessarily implies these beliefs were irrational but could instead be a classic example of the so called "peso problem".

$\pi_L^e > \pi^*$. If the public believes that the central bank will set a higher inflation target in the long run, this results in a one-to-one upward shift of the red curve, from AS' to AS''. Hence, it immediately generates inflation just like a supply shock, for a given level of demand. However, this is not the whole story. As shown by equation (66), the rise in inflation expectations also increases demand in the short run by reducing the real interest rate, making borrowing cheaper (real rates were negative through much of the 1970s). This shifts the blue curve upward from AD to AD'. It is easy to show, however, that the effect on output is always negative if the model has a unique equilibrium, i.e., under the condition that $\phi_\pi > 1$.[79]

The numerical example can, therefore, capture – in rough orders of magnitudes – the rise in inflation. While we see that the supply shocks played a role, the major contributor is the unanchoring of inflation expectations, can there for account for the decomposition in Figure 8.

We can use the same framework to understand why disinflation can be very costly if the private sector believes that the long-term inflation target is high, despite the central bank's claims to the contrary. The extent of this cost fundamentally depends on how perceptions about future inflation are formed. Consider the possibility that people will only reconsider their perception of the long-run inflation target if they see a significant reduction in current inflation. In this scenario, due to the flatness of the Phillips curve, it would be very costly to bring inflation down. This is a common narrative for the Volcker recession in the early 1980s. While we will not spell out this process explicitly here, it should be evident that we have all the main ingredients to tell that story by linking inflation expectations to realized output and inflation, which could be interpreted by the public as measures of the central bank's resolve in bringing down inflation.

One prominent hypothesis on the causes of inflation in the 1970s, proposed by Clarida, Galí, and Gertler (2000), is that the central bank did not react strongly enough to rising inflation by increasing the interest rate, implying $\phi_\pi < 1$. This assumption leads to equilibrium indeterminacy, meaning that there are infinite possible inflation paths that could satisfy the model's equilibrium conditions. However, our focus here is on highlighting the effect of inflation expectations becoming unanchored. We are interested in comparative statics concerning the central bank's long-run inflation target. If the model allows for an infinite number of equilibria, comparative statics become meaningless. Therefore, it is useful to assume $\phi_\pi > 1$. Moreover, since we incorporate a policy shock e_t, the policy rule remains flexible enough to account for the possibility that monetary policy did not respond strongly enough to the rise in inflation. This captures the essence of the argument by Clarida, Galí, and Gertler (2000), which we consider a complementary way of modeling this period, as both emphasize the Fed losing control of longer-term inflation expectations.

6.3 The missing 2008 disinflation

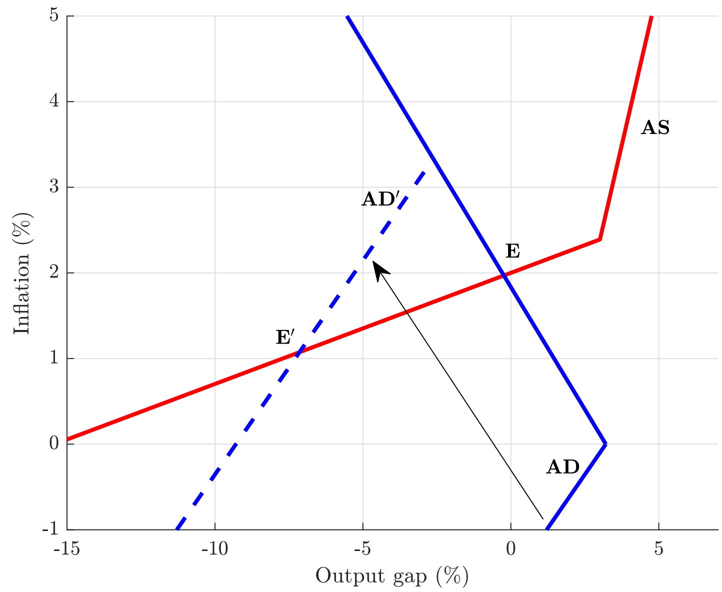


Figure 13: The 2008 missing disinflation: Inflation and output determination using the AS-AD model in response to a fall in aggregate demand. Inflation is measured at annualized quarterly rates and presented in percent, while the output gap is expressed in percentage points. The AD curve shifts from AD to AD′ following a negative demand shock. The equilibrium transitions from E to E′.

As illustrated in Figure 13, our model effectively explains why inflation did not decrease significantly following the Financial Crisis of 2008. This outcome was often cited as a key challenge to the New Keynesian Phillips curve. While various authors have proposed alternative approaches to address this issue, our model replicates the observed inflation dynamics during this period with relative ease.

In Figure 13, as shown in Appendix E, the zero lower bound causes the AD curve to slope upward at low inflation rates. We begin at the initial equilibrium, represented by point E, where inflation is at the 2% target and the output gap is stabilized at zero. At this equilibrium, the real interest rate is 1%. To define the shock characterizing the 2009 financial crisis, we follow the literature, such as Eggertsson and Krugman (2012), and hypothesize a negative shock to the natural rate of interest, reducing it to -4%. With the zero lower bound binding, the upward sloping segment of the AD curve becomes relevant, determining the new equilibrium at point E′, where it intersects the flat segment of the AS curve. As a result, inflation falls to 1%, a level observed in the U.S. economy during the financial crisis for core CPI inflation, while output contracts significantly. The flat Phillips curve under weak labor market conditions explains why this shock did not lead to a more substantial reduction in the inflation rate or cause deflation.

[79]It would be incorrect, however, to conclude that $\phi_\pi < 1$ implies that higher long-term inflation expectations increase output. If $\phi_\pi < 1$, there is an infinite number of equilibria. The model cannot predict which equilibrium will be chosen in response to a change in any of the exogenous variables if $\phi_\pi < 1$. Comparative statics are thus meaningless.

7 Conclusion

In this paper we have proposed a reformulation of what has become known as the canonical New Keynesian Phillips curve and replaced it with one that admits significant nonlinearities. Our hypothesis is that the nonlinearity is responsible for the increase in inflation in the 2020s. We conjecture that a key reason why policymakers and market participants alike failed to foresee the surge in inflation, or its persistence is that they implicitly or explicitly assumed a "flat" Phillips curve. Even after substantial inflation had already occurred, the reassurance that expectations were holding stable further induced the belief that the surge was merely transitory. One question is why the Federal Reserve did not raise interest rates more quickly. Possibly the new policy framework announced in 2020 put greater emphasis on the employment side of the Fed's dual objective. Yet, at the same time, it acknowledged that there was no agreement on any precise measure of how close the US economy was to full employment at any given point in time. This, of course, contrasts very sharply with the other side of the mandate, i.e. inflation, for which there is broad consensus on how the Fed can attain its objective.

Figure 32 sheds some light on why policymakers may have believed in 2021 that even though the traditional gauge of labor slack, i.e. unemployment, was very low, this did not capture the full picture. The unemployment rate only tells us how many active job seekers there are. As the figure reveals, however, participation collapsed with the COVID-19 epidemic, which might have suggested to many that there was still considerable room for employment to grow further. Moreover, given the flat Phillips curve – the professional consensus at the time – and stable inflation expectations, it might have been tempting for policymakers to explore the possibility that the US economy could attract greater labor force participation, e.g. similar to pre-pandemic level with relatively low risk of inflation. In terms of the dual mandate, conditional on a flat Phillips curve, this could easily have been seen at the time as a situation with possible high reward and relatively limited downside risk. The bottom line of this paper, however, is that the inflationary risk of allowing the labor market to tighten too much, to a degree we have defined as labor shortage, generates much greater upside risk for inflation than has been commonly thought. An important reason for this underestimation of inflation risk is no doubt the unprecedented labor shortage, historically unprecedented except in wartime, and the countless estimates of the slope of the Phillips curve that did not incorporate wartime. We have sought first to show this empirically and then to build a model to explain it. The good news, in any case, is that if our theory is correct the cost of taming inflation triggered by a labor shortage, but with stable inflation expectations, can be expected to be much lower than it was in the 1970s.

References

[1] Abo-Zaid, Salem and Xuguang Simon Sheng. 2024. Labor Markets, Fiscal Policy and Inflation Dynamics: A Pandemic Perspective. Unpublished Manuscript. University of Maryland-Baltimore County.

[2] Adrjan, Pawel and Reamonn Lydon. 2023. What Do Wages in Online Job Postings Tell Us About Wage Growth? Central Bank of Ireland.

[3] Ball, Laurence, Daniel Leigh, and Prachi Mishra. 2022. Understanding U.S. Inflation During the COVID Era. Brookings Papers on Economic Activity.

[4] Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk. 2014. Some Evidence on the Importance of Sticky Wages. American Economic Journal: Macroeconomics, 6(1): 70–101.

[5] Barnichon, Regis. 2010a. Productivity and Unemployment over the Business Cycle. Journal of Monetary Economics, 57(8): 1013–1025.

[6] Barnichon, Regis. 2010b. Building a Composite Help-Wanted Index. Economics Letters, 109(3): 175–178.

[7] Barnichon, Regis and Adam H. Shapiro. 2022. What's the Best Measure of Economic Slack? FRBSF Economic Letter 2022-04.

[8] Barnichon, R. and Shapiro, A.H., 2024. Phillips meets beveridge. Journal of Monetary Economics, 148, p.103660.

[9] Barnichon, Regis, Luiz E. Oliveira, and Adam H. Shapiro. 2021. Is the American Rescue Plan Taking Us Back to the '60s? FRBSF Economic Letter 2021-27.

[10] Beaudry, Paul, Chenyu Hou, and Franck Portier. 2024. The Dominant Role of Expectations and Broad-Based Supply Shocks in Driving Inflation. NBER Working Paper No. 32322.

[11] Beaudry, Paul, Chenyu Hou, and Franck Portier. 2025. On the Fragility of the Nonlinear Phillips Curve View of Recent Inflation. CEPR Discussion Paper No. 19950.

[12] Benati, Luca. 2010. Evolving Phillips Trade-Off. ECB Working Paper No. 1176.

[13] Benigno, Pierpaolo and Gauti B. Eggertsson. 2024a. Revisiting the Beveridge and Phillips Curves: Insights from the 2020s Inflation Surge. Jackson Hole Economic Policy Symposium Proceedings.

[14] Benigno, Pierpaolo and Gauti B. Eggertsson. 2024b. Slanted-L Phillips Curve. AEA Papers and Proceedings, Vol. 114, pp. 84–89.

[15] Benigno, Pierpaolo and Luca Antonio Ricci. 2011. The Inflation–Output Trade-Off with Downward Wage Rigidities. American Economic Review, 101(4): 1436–1466.

[16] Bergholt, Drago, Fabio Canova, Francesco Furlanetto, Nicolò Maffei-Faccioli, and Päl Ulvedal. 2024. What Drives the Recent Surge in Inflation? The Historical Decomposition Roller Coaster. CEPR Discussion Paper No. 19005.

[17] Bernanke, Ben S. 2007. Inflation Expectations and Inflation Forecasting. Federal Reserve Bank of Governors, `https://www.federalreserve.gov/newsevents/speech/bernanke20070710a.html`.

[18] Bernanke, Ben S. and Olivier Blanchard. 2024. What Caused the US Pandemic-Era Inflation? American Economic Journal: Macroeconomics, forthcoming.

[19] Beveridge, William Henry. 1944. Full Employment in a Free Society. George Allen & Unwin.

[20] Bewley, Truman F. 1999. Why Wages Don't Fall During a Recession. Harvard University Press.

[21] Blanchard, Olivier. 1989. Les Courbes de Beveridge et de Phillips Comme Outils d'Analyse du Chômage. L'Actualité Économique, 65(3): 396–422.

[22] Blanchard, Olivier. 2016. The Phillips Curve: Back to the 60s? American Economic Review, 106(5): 31–34.

[23] Blanchard, Olivier and Jordi Galí. 2010a. Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment. American Economic Journal: Macroeconomics, 2(2): 1–30.

[24] Blanchard, Olivier and Jordi Galí. 2010b. The Macroeconomic Effects of Oil Price Shocks: Why Are the 2000s So Different from the 1970s? In International Dimensions of Monetary Policy, edited by Jordi Galí and Mark Gertler, pp. 373–428. Chicago: University of Chicago Press.

[25] Blanchard, Olivier, Eugenio Cerutti, and Lawrence Summers. 2015. Inflation and Activity–Two Explorations and Their Monetary Policy Implications. NBER Working Paper No. 21726.

[26] Blanchard, Olivier, Alex Domash, and Lawrence Summers. 2022. Bad News for the Fed from the Beveridge Space. Peterson Institute for International Economics Policy, Policy Brief 22–7.

[27] Blinder, Alan. 2022. A Monetary and Fiscal History of the United States, 1961–2021. Princeton University Press.

[28] Bok, Brandyn, Nicolas Petrosky-Nadeau, Robert G. Valletta, and Mary Yilma. 2022. Finding a Soft Landing along the Beveridge Curve. FRBSF Economic Letter 2022-24.

[29] Carlson, John A. 1977. A Study of Price Forecasts. Annals of Economic and Social Measurement, 6(1): 27–56.

[30] Cerrato, Andrea and Giulia Gitti. 2023. Inflation Since COVID: Demand or Supply. Unpublished Manuscript. University of California at Berkeley.

[31] Chodorow-Reich, G.. 2024. Discussion of Beaudry, Hou, Portier: The Dominant Role of Expectations and Broad-Based Supply Shocks in Driving Inflation.

[32] Christ, Carl F. 1994. The Cowles Commission's Contributions to Econometrics at Chicago, 1939–1955. Journal of Economic Literature, 32(1): 30–59.

[33] Christiano, Lawrence J., Mark S. Eichenbaum, and Matthias Trabandt. 2016. Unemployment and Business Cycles. Econometrica, 84(4): 1523–1569.

[34] Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. Quarterly Journal of Economics, 115(1): 147–180.

[35] Coibion, Olivier, Yuriy Gorodnichenko, and Rupal Kamdar. 2018. The Formation of Expectations, Inflation, and the Phillips Curve. Journal of Economic Literature, 56(4): 1447–1491.

[36] Crump, Richard K., Stefano Eusepi, Marc Giannoni, and Ayşegül Şahin. 2024. The Unemployment–Inflation Trade-Off Revisited: The Phillips Curve in COVID Times. Journal of Monetary Economics, 145.

[37] Domash, Alex and Lawrence Summers. 2022. How Tight Are U.S. Labor Markets? NBER Working Paper No. 29739.

[38] Dupraz, Stéphane, Emi Nakamura, and Jón Steinsson. 2019. A Plucking Model of Business Cycles. NBER Working Paper No. 26351.

[39] Erceg, Christopher and Andrew Levin. 2003. Imperfect Credibility and Inflation Persistence. Journal of Monetary Economics, 50: 915–944.

[40] Eggertsson, Gauti B. and Donald Kohn. 2023. The Inflation Surge of the 2020s: The Role of Monetary Policy. Presentation at Hutchins Center, Brookings Institution.

[41] Eggertsson, Gauti B. and Paul Krugm an. 2012. Debt, Deleveraging, and the Liquidity Trap: A Fisher–Minsky–Koo Approach. Quarterly Journal of Economics, 127(3): 1241–1280.

[42] Eggertsson, Gauti B. and Sanjay Singh. 2019. Log-Linear Approximation versus an Exact Solution at the ZLB in the New Keynesian Model. Journal of Economic Dynamics and Controls, 105.

[43] Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob Robbins. 2019. A Model of Secular Stagnation: Theory and Quantitative Evaluation. American Economic Journal: Macroeconomics, 11(1).

[44] Fallick, Bruce, Michael Lettau, and William Wascher. 2011. Downward Nominal Wage Rigidity in the United States During the Great Recession. Mimeo, Federal Reserve Board.

[45] Figura, Andrew and Chris Waller. 2022. What Does the Beveridge Curve Tell Us About the Likelihood of a Soft Landing? FEDS Notes, July 29, 2022.

[46] Fortin, Pierre. 2015. The Macroeconomics of Downward Nominal Wage Rigidity: A Review of the Issues and New Evidence for Canada. In K. Banting, R. Chaykowski, and S. Lehrer (eds.), Thinking Outside the Box: A Conference Volume Celebrating Thomas J. Courchene. McGill-Queen's University Press, Montréal.

[47] Friedman, Milton. 1964. Monetary Studies of the National Bureau. In The National Bureau Enters Its 45th Year, New York: National Bureau of Economic Research, pp. 7–25.

[48] Friedman, Milton. 1993. The "Plucking Model" of Business Fluctuations Revisited. Economic Inquiry, 31: 171–177.

[49] Furman, Jason and Wilson Powell III. 2021. What's the Best Measure of Labor Market Tightness? Peterson Institute for International Economics, November 22, 2021.

[50] Gagliardone, Luca and Mark Gertler. 2023. Oil Prices, Monetary Policy and Inflation Surges. Unpublished Manuscript. New York University.

[51] Gagnon, Joseph E. and Christopher G. Collins. 2019. Low Inflation Bends the Phillips Curve. Peterson Institute for International Economics, Working Paper No. 19/6.

[52] Galí, Jordi. 2009. Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective. MIT Press, Boston, MA.

[53] Galí, Jordi. 2015. Monetary Policy, Inflation, and the Business Cycle. Princeton University Press, Princeton, NJ.

[54] Gertler, Mark, Christopher Huckfeldt, and Antonella Trigari. 2020. Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires. The Review of Economic Studies, 87(4): 1876–1914.

[55] Gertler, Mark, Luis Sala, and Antonella Trigari. 2008. An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining. Journal of Money, Credit and Banking, 40(8): 1713–1764.

[56] Gitti, Giulia. 2023. Nonlinearities in the Regional Phillips Curve with Labor Market Tightness. Unpublished Manuscript. Brown University.

[57] Goodfriend, Marvin and Robert G. King. 2005. The Incredible Volcker Disinflation. Journal of Monetary Economics, 52: 981–1015.

[58] Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman. 1988. Investment, Capacity Utilization, and the Real Business Cycle. The American Economic Review: 402–417.

[59] Groen, Jan and Menno Middledorp. 2013. Creating a History of U.S. Inflation Expectations. Liberty Street Economics.

[60] Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Iván Werning. 2022. Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages? American Economic Review 112 (5): 1437–74.

[61] Hall, Robert E. 2005. Employment Fluctuations with Equilibrium Wage Stickiness. American Economic Review, 95(1): 50–65.

[62] Hamilton, James D. 2018. Why You Should Never Use the Hodrick–Prescott Filter. The Review of Economics and Statistics, 100(5): 831–843.

[63] Harding, Martin, Jesper Linde, and Mathias Trabandt. 2023. Understanding Post-COVID Inflation Dynamics. Unpublished Manuscript. Bank of Canada.

[64] Hazell, Jonathon, Juan Herreño, Emi Nakamura, and Jón Steinsson. 2022. The Slope of the Phillips Curve: Evidence from U.S. States. The Quarterly Journal of Economics, 137(3): 1299–1344.

[65] Kalantzis, Yannick (@kalantzis). 2023. Why Does Everybody Now Talk About V/U and Hardly Anyone Mentioned It Before. Published online on June 28, 2023. Accessed 08/02/24. `https://x.com/kalantzis/status/1674097958298451980`.

[66] Keynes, John Maynard. 1936. The General Theory of Employment, Interest, and Money. New York.

[67] Keynes, John Maynard. 1940. How to Pay for the War. London: Macmillan.

[68] Kimball, Miles S. 1997. The Quantitative Analytics of the Basic Neomonetarist Model. Journal of Money, Credit, and Banking, 27(4): 1241–1277.

[69] Krause, Mark U. and Thomas A. Lubik. 2007. The (Ir)Relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions. Journal of Monetary Economics, 54(3): 706–727.

[70] Krause, Mark U., Diego Lopez-Salido, and Thomas A. Lubik. 2008. Inflation Dynamics with Search Frictions: A Structural Econometric Analysis. Journal of Monetary Economics, 55(5): 892–916.

[71] Krugman, Paul R. 1998. It's Baaack: Japan's Slump and the Return of the Liquidity Trap. Brookings Papers on Economic Activity, 1998(2): 1–45.

[72] Krugman, Paul R. 2021. Credible Irresponsibility Revisited. Working Paper, City University of New York.

[73] Lenoir, Marcel. 1913. Études sur la Formation et le Mouvement des Prix. Paris: Giard et Brière.

[74] Malthus, Thomas R. 1798. An Essay on the Principle of Population. J. Johnson, London.

[75] Martin, W.M. Jr. (1955) Address before the New York Group of the Investment Bankers Association of America [speech], Waldorf Astoria Hotel, New York City, 19 October. Available at: Federal Reserve Bank of St. Louis FRASER https://fraser.stlouisfed.org/title/448/item/7800 (Accessed 12 May 2025).

[76] Matheson, Troy and Emil Stavrev. 2013. The Great Recession and the Inflation Puzzle. Economics Letters, 120(3): 468–472.

[77] Mavroeidis, Sophocles, Mikkel Plagborg-Moller, and James H. Stock. 2014. Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve. Journal of Economic Literature, 52(1): 124–188.

[78] McLeay, Michael and Silvana Tenreyro. 2019. Optimal Inflation and the Identification of the Phillips Curve. NBER Macroeconomics Annual, 34: 199–255.

[79] Michaillat, Pascal. 2012. Do Matching Frictions Explain Unemployment? Not in Bad Times. American Economic Review, 102(4): 1721–1750.

[80] Michaillat, Pascal. 2014. A Theory of Countercyclical Government Multiplier. American Economic Journal: Macroeconomics, 6(1): 190–217.

[81] Michaillat, Pascal and Emmanuel Saez. 2022. $u^* = \sqrt{uv}$. Unpublished Manuscript. Brown University.

[82] Michaillat, Pascal and Emmanuel Saez. 2024a. Moen Meets Rotemberg: An Earthly Model of the Divine Coincidence. Unpublished Manuscript. University of California at Santa Cruz.

[83] Michaillat, Pascal and Emmanuel Saez. 2024b. Beveridgean Phillips Curve. Unpublished Manuscript. University of California at Santa Cruz.

[84] Mortensen, Dale T. and Christopher A. Pissarides. 1994. Job Creation and Job Destruction in the Theory of Unemployment. Review of Economic Studies, 61: 397–415.

[85] Nakamura, Emi and Jón Steinsson Steinsson. 2008. Five Facts About Prices: A Reevaluation of Menu Cost Models. The Quarterly Journal of Economics, 123(4): 1415–1464.

[86] Petrosky-Nadeau, Nicolas and Lu Zhang. 2021. Unemployment Crises. Journal of Monetary Economics, 117: 335–353.

[87] Phelps, Edmund S. 1967. Phillips Curves, Expectations of Inflation and Optimal Unemployment Over Time. Economica, 34: 254–281.

[88] Phillips, Alban W. 1958. The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957. Economica, 25(100): 283–299.

[89] Ravenna, Francesco and Christopher E. Walsh. 2008. Vacancies, Unemployment, and the Phillips Curve. European Economic Review, 52(8): 1494–1521.

[90] Ravenna, Francesco and Christopher E. Walsh. 2011. Welfare-Based Optimal Monetary Policy with Unemployment and Sticky Prices: A Linear–Quadratic Framework. American Economic Journal: Macroeconomics, 3(2): 130–162.

[91] Rockoff, Hugh. 1981. Price and Wage Controls in Four Wartime Periods. The Journal of Economic History, 41(2): 381–401.

[92] Rotemberg, Julio. 1982. Monopolistic Price Adjustment and Aggregate Output. The Review of Economic Studies, 49(4): 517–531.

[93] Samuelson, Paul A. and Robert M. Solow. 1960. Analytical Aspects of Anti-Inflation Policy. The American Economic Review, 50(2): 177–194.

[94] Schmitt-Grohé, Stephanie and Martin Uribe. 2016. Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment. Journal of Political Economy, 124(5): 1466–1514.

[95] Sedláček, Petr. 2016. The Aggregate Matching Function and Job Search from Employment and Out of the Labor Force. Review of Economic Dynamics, 21: 16–28.

[96] Smith, Simon C., Allan Timmermann, and Jonathan Wright. 2023. Breaks in the Phillips Curve: Evidence from Panel Data. NBER Working Paper No. 31153.

[97] Walsh, Christopher E. 2005. Labor Market Search, Sticky Prices, and Interest Rate Policies. Review of Economic Dynamics, 8(4): 829–849.

[98] Woodford, Michael. 2003. Interest Rate and Prices. Princeton University Press, Princeton, NJ.

A Appendix: Additional Tables

Table 2 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the threshold for the dummy is set at $\theta^* = 0.5026$ for the sample 1960-2024, and at $\theta^* = 0.9812$ for the sample 2008-2024, which corresponds to the thresholds in the respective sample that maximize the likelihood of the regressions across different thresholds, as shown in Figure 26.

Table 2: **Phillips Curve Estimates when θ^* is chosen to maximize the likelihood function of the model**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.3696^{***} (0.0947)	0.273 (0.2445)	0.3492^{***} (0.0905)	-0.1438 (0.196)
$\ln\theta$	0.6722^{***} (0.1758)	0.7235^{*} (0.3642)	0.3188 (0.3756)	0.4846 (0.3025)
$(\ln\theta - \ln\theta^*)\quad \theta \geq \theta^*$			0.7411 (0.493)	5.3553^{***} (0.9062)
Supply shock (ϱ)	0.0378^{**} (0.0192)	0.0187 (0.0395)	-0.0335 (0.0293)	-0.0102 (0.0227)
$\theta \geq \theta^*$			0.1112^{**} (0.0434)	0.2758^{**} (0.1202)
Inflation expectations	0.6612^{***} (0.1064)	0.7608 (0.6038)	0.6466^{***} (0.0966)	0.5101 (0.4637)
Constant	0.5522^{***} (0.1513)	0.9027^{**} (0.3892)	0.1487 (0.3357)	0.3451 (0.331)
R^2 adjusted	0.8134	0.5063	0.8288	0.662
Observations	250	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 3 presents the OLS estimates of regression (2) with the same variables as Table 1, except that we proxy the supply shock with the four-quarter average of the CPI headline shock.

Table 3: **Phillips Curve Estimates using only CPI headline shock as proxy for supply disturbances**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.3898^{***} (0.1019)	0.2365 (0.2373)	0.2992^{***} (0.1127)	-0.1158 (0.1965)
$\ln\theta$	0.6678^{***} (0.1738)	0.7664^{**} (0.3747)	0.3382^{*} (0.2044)	0.5842^{*} (0.3384)
$\theta \geq 1$			2.5943^{***} (0.9754)	3.2743^{**} (1.2401)
Supply shock (ϱ)	0.1255 (0.0835)	0.239 (0.1947)	0.0975 (0.0872)	0.0028 (0.1189)
$\theta \geq 1$			0.2131 (0.3091)	0.8203^{*} (0.4507)
Inflation expectations	0.6656^{***} (0.1093)	0.6249 (0.5743)	0.7907^{***} (0.1166)	0.5096 (0.5312)
Constant	0.5069^{***} (0.1477)	0.8924^{**} (0.3658)	0.2233 (0.1694)	0.4947 (0.3774)
R^2 adjusted	0.8084	0.528	0.8159	0.6317
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.
· Newey-West standard errors.
· (1) and (3): sample 1960 Q1 – 2024 Q4
· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 4 presents the OLS estimates of regression (2) with the same variables as Table 1, except that we proxy the supply shock with the four-quarter average import-price shock.

Table 4: **Phillips Curve Estimates using import-price relative to GDP deflator as measure of supply shock**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.3689^{***} (0.0946)	0.2734 (0.2449)	0.2531^{***} (0.0925)	-0.1503 (0.1906)
$\ln\theta$	0.674^{***} (0.1761)	0.7218^{**} (0.3636)	0.2373 (0.2004)	0.5535^{*} (0.3168)
$\theta \geq 1$			3.8979^{***} (0.8626)	5.7171^{***} (0.9048)
Supply shock (ϱ)	0.0384^{**} (0.0195)	0.0179 (0.0403)	0.046^{**} (0.0207)	-0.0093 (0.0238)
$\theta \geq 1$			0.1005 (0.0992)	0.2812^{**} (0.1191)
Inflation expectations	0.6616^{***} (0.1063)	0.7666 (0.6045)	0.8106^{***} (0.1006)	0.5241 (0.4489)
Constant	0.5707^{***} (0.153)	0.9098^{**} (0.3975)	0.2198 (0.1681)	0.4322 (0.3505)
R^2 adjusted	0.8135	0.5058	0.8263	0.6615
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 5 presents the OLS estimates of regression (2) with the same variables as Table 1, except that 2-year Cleveland-Fed inflation expectation is replaced by the 1-year CPI inflation expectations of the U.S. Survey of Professional Forecasters until 1981 Q3, which is patched backward using the interpolated 12-month Livingston survey until 1960 Q1.

Table 5: **Phillips Curve Estimates using 1-year CPI expectation of the U.S. Survey of Professional Forecasters**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.3031^{***} (0.0972)	0.102 (0.2745)	0.2151^{**} (0.0993)	-0.1846 (0.1907)
$\ln\theta$	0.6988^{***} (0.1715)	0.5046 (0.3373)	0.3452^{*} (0.1834)	0.4816 (0.324)
$\theta \geq 1$			3.0533^{***} (0.8831)	5.3397^{***} (1.1552)
Supply shock (ϱ)	0.0355^{*} (0.0195)	0.0135 (0.0388)	0.0406^{**} (0.0206)	-0.0055 (0.0242)
$\theta \geq 1$			0.0983 (0.0874)	0.2571^{**} (0.1155)
Inflation expectations	0.7706^{***} (0.1095)	1.6821 (0.6779)	0.8856^{***} (0.1109)	0.6724 (0.5092)
Constant	0.476^{***} (0.1383)	0.3511^{**} (0.3658)	0.178 (0.1447)	0.1456 (0.2984)
R^2 adjusted	0.8184	0.5314	0.8273	0.6571
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1, 5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 6 presents the OLS estimates of regression (2) with the same variables as Table 1, except that inflation expectations are proxied by the 5-year inflation expectations of the Cleveland Fed until 1982 Q2, which are patched with PFS 1-year inflation expectations for the GDP deflator until 1970 Q2 and the interpolated 12-month Livingston survey inflation expectations until 1960 Q1.

Table 6: **Phillips Curve Estimates Using Cleveland Fed's 5-year inflation expectation measure**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.3957*** (0.1002)	0.3237 (0.239)	0.279*** (0.1026)	−0.1438 (0.1927)
$\ln\theta$	0.7413*** (0.1829)	0.8577** (0.3957)	0.3282 (0.2032)	0.5756* (0.3141)
$\theta \geq 1$			3.7349*** (0.888)	5.9778*** (0.9355)
Supply shock (ϱ)	0.0482** (0.0190)	0.0305 (0.0383)	0.0574*** (0.0204)	0.0007 (0.0239)
$\theta \geq 1$			0.1026 (0.0988)	0.2611** (0.1153)
Inflation expectations	0.6443*** (0.1202)	0.3463 (0.6506)	0.8022*** (0.121)	0.129 (0.5165)
Constant	0.5865*** (0.1541)	0.8651** (0.3855)	0.2393 (0.1691)	0.2793 (0.3374)
R^2 adjusted	0.8055	0.4826	0.8178	0.6512
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1, 5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 7 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the 2-year Cleveland-Fed inflation expectations are replaced by the 5-year 5-year forward inflation expectations back-casted by Groen and Middledorp (2013) until 1971 Q4. The expectations are patched with the interpolated 12-month Livingston survey inflation expectations until 1960 Q1.

Table 7: **Phillips Curve Estimates using 5-year 5-year forward inflation expectations**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.5818***	0.1401	0.5695***	−0.187
	(0.0839)	(0.2411)	(0.0895)	(0.2214)
$\ln\theta$	0.849***	1.6901***	0.7503***	1.2537**
	(0.2008)	(0.4985)	(0.2145)	(0.5401)
$\theta \geq 1$			1.0295	4.7254***
			(0.6914)	(1.0571)
Supply shock (ϱ)	0.0686***	0.0071	0.0703***	−0.017
	(0.0212)	(0.0293)	(0.0224)	(0.0229)
$\theta \geq 1$			0.0655	0.2651**
			(0.0717)	(0.1025)
Inflation expectations	0.335***	1.4705***	0.3481***	1.0459**
	(0.0827)	(0.4452)	(0.0863)	(0.48)
Constant	0.5256***	0.8216**	0.4402***	0.4147
	(0.1243)	(0.3632)	(0.1413)	(0.3222)
R^2 adjusted	0.7814	0.5637	0.7811	0.687
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 8 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the 2-year Cleveland-Fed inflation expectations are replaced by the 12-month households inflation expectations of the Michigan survey until 1978 Q1 using the median of the survey and backward until 1960 Q1 using the mean.

Table 8: **Phillips Curve Estimates using 12-month Michigan inflation expectations**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.4656^{***} (0.0887)	-0.143 (0.1697)	0.4669^{***} (0.089)	-0.2352 (0.1426)
$\ln\theta$	0.3475^{***} (0.1162)	1.1007^{***} (0.2845)	0.4346^{***} (0.1479)	0.9305^{***} (0.3027)
$\theta \geq 1$			-0.152 (0.4876)	2.7944^{**} (1.3328)
Supply shock (ϱ)	-0.0148 (0.0195)	-0.0375 (0.0248)	-0.0213 (0.0208)	-0.0377^{*} (0.0222)
$\theta \geq 1$			0.1271^{**} (0.0624)	0.171^{**} (0.077)
Inflation expectations	0.661^{***} (0.1337)	1.4115^{***} (0.2726)	0.6672^{***} (0.1351)	1.0115^{***} (0.2822)
Constant	-0.1546 (0.1177)	-0.6398^{***} (0.2112)	-0.1022^{*} (0.1282)	-0.4619^{*} (0.2467)
R^2 adjusted	0.7997	0.6895	0.801	0.7172
Observations	253	66	253	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.
· Newey-West standard errors.
· (1) and (3): sample 1960 Q1 – 2024 Q4
· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 9 presents the OLS estimates of regression (2) with the same variables as Table 1, except that inflation expectations are proxied by the 1-year inflation expectations of the Cleveland Fed until 1982 Q2, which are patched with the interpolated 12-month Livingston survey inflation expectations until 1960 Q1.

Table 9: **Phillips Curve Estimates using Cleveland Fed's 1-year inflation expectations**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.3433^{***} (0.0964)	0.2072 (0.233)	0.2533^{***} (0.0965)	-0.1554 (0.1803)
$\ln\theta$	0.6188^{***} (0.1701)	0.6097^{**} (0.304)	0.225 (0.1902)	0.4713 (0.2942)
$\theta \geq 1$			3.1963^{***} (0.8694)	4.9982^{***} (0.9221)
Supply shock (ϱ)	0.033 (0.0203)	-0.001 (0.0376)	0.0388^{*} (0.0215)	-0.0216 (0.0231)
$\theta \geq 1$			0.0799 (0.0975)	0.2632^{**} (0.1212)
Inflation expectations	0.703^{***} (0.1074)	0.9096^{**} (0.4387)	0.8183^{***} (0.1055)	0.6188 (0.3752)
Constant	0.5169^{***} (0.1448)	0.8112^{**} (0.3534)	0.2024 (0.1569)	0.3751^{*} (0.3048)
R^2 adjusted	0.8175	0.549	0.8268	0.6792
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 10 presents the OLS estimates of regression (2) with the same variables as Table 1, except that inflation expectations are proxied by the 12-month Livingston survey inflation expectations.

Table 10: **Phillips Curve Estimates using 12-month Livingston survey inflation expectations**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.2882^{***} (0.0911)	0.0946 (0.2059)	0.2152^{***} (0.0917)	-0.1565 (0.1753)
$\ln\theta$	0.6761^{***} (0.1647)	0.4695^{**} (0.3052)	0.3069 (0.1791)	0.5148^{*} (0.3933)
$\theta \geq 1$			2.9682^{***} (0.8413)	5.5783^{***} (1.5284)
Supply shock (ϱ)	0.0344^{**} (0.0193)	0.0167 (0.0321)	0.0409^{**} (0.02)	0.0016 (0.0244)
$\theta \geq 1$			0.0599 (0.0882)	0.2329^{**} (0.1426)
Inflation expectations	0.7797^{***} (0.1012)	1.6724 (0.4144)	0.8863^{***} (0.1017)	0.3566 (0.6018)
Constant	0.4975^{***} (0.1337)	0.4528^{**} (0.292)	0.1974 (0.143)	0.2115 (0.327)
R^2 adjusted	0.8233	0.5644	0.8314	0.6534
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.
· Newey-West standard errors.
· (1) and (3): sample 1960 Q1 – 2024 Q4
· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 11 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the inflation lag and the log of the vacancy rate to unemployment rate θ are instrumented with the fitted values of their first lags.

Table 11: **Phillips Curve Estimates using an instrumental variable approach.**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag (Fitted)	0.2804^{***} (0.1044)	0.3971^{*} (0.2042)	0.1808^{*} (0.0972)	0.155 (0.1511)
$\ln\theta$ *(Fitted)*	0.7901^{***} (0.2074)	0.3839 (0.3882)	0.429 (0.2656)	0.3812 (0.3894)
$\theta \geq 1$ *(Fitted)*			3.8949^{***} (0.9143)	3.2892^{**} (1.4382)
Supply shock (ϱ)	0.051^{**} (0.0234)	0.0214 (0.0432)	0.0527^{**} (0.0244)	-0.0075 (0.0204)
$\theta \geq 1$			0.1804 (0.1357)	0.3398^{***} (0.1287)
Inflation expectations	0.7898^{***} (0.1058)	0.9538 (0.7451)	0.9006^{***} (0.0995)	0.8389 (0.651)
Constant	0.604^{***} (0.1835)	0.674^{*} (0.3512)	0.342 (0.225)	0.5692 (0.4113)
R^2 adjusted	0.794	0.4481	0.8057	0.5495
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.
· Newey-West standard errors.
· (1) and (3): sample 1960 Q1 – 2024 Q4
· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 12 presents the OLS estimates of regression (2) with the same variables as Table 1, except that PCE core inflation rate replaces CPI core inflation as the dependent variable.

Table 12: **Phillips Curve Estimates using PCE core.**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.5662^{***} (0.0691)	0.5058^{***} (0.1864)	0.4416^{***} (0.0751)	0.1501 (0.2061)
$\ln \theta$	0.2819^{***} (0.1076)	0.3489 (0.2605)	-0.0469 (0.1466)	0.2354 (0.2497)
$\theta \geq 1$			2.7574^{***} (0.6316)	3.518^{***} (0.9424)
Supply shock (ϱ)	0.0352^{***} (0.013)	0.0236 (0.0311)	0.0423^{***} (0.0142)	0.0193 (0.0183)
$\theta \geq 1$			0.0761 (0.0719)	0.2117^{**} (0.0901)
Inflation expectations	0.3708^{***} (0.0684)	0.4544 (0.5054)	0.5009^{***} (0.0745)	0.3949 (0.3767)
Constant	0.1916^{**} (0.0949)	0.3662^{*} (0.2126)	-0.092 (0.1314)	0.0125 (0.2581)
R^2 adjusted	0.8646	0.5773	0.8735	0.6669
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.
· Newey-West standard errors.
· (1) and (3): sample 1960 Q1 – 2024 Q4
· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 13 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the level of θ is used rather than its log.

Table 13: **Phillips Curve Estimates using the level of** θ

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Inflation lag	0.329^{***} (0.0928)	0.1375 (0.2419)	0.2634^{***} (0.0938)	-0.1338 (0.1922)
θ	1.1662^{***} (0.2438)	1.6631^{**} (0.7151)	0.5112 (0.3754)	1.1295 (0.6863)
$(\theta - 1) \quad \theta \geq 1$			2.333^{***} (0.8187)	3.0831^{***} (0.9185)
Supply shock (ϱ)	0.04^{**} (0.0194)	0.0249 (0.0396)	0.0441^{**} (0.0204)	-0.0078 (0.0228)
$\theta \geq 1$			0.0879 (0.102)	0.2537^{**} (0.1222)
Inflation expectations	0.7162^{***} (0.1029)	0.4156 (0.6607)	0.7992^{***} (0.1026)	0.4997 (0.4649)
Constant	-0.5859^{***} (0.15)	-0.7739 (0.472)	-0.2383 (0.2085)	-0.5958 (0.3731)
R^2 adjusted	0.8198	0.5463	0.8252	0.6556
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 14 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the lag of the inflation rate is replaced by the lag value of the detrended real wage.

Table 14: **Phillips Curve Estimates using lagged real wages instead of lagged inflation.**

	(1) 1960-2024	(2) 2008-2024	(3) 1960-2024	(4) 2008-2024
Real wage lag	11.0018^{**}	-8.8262	14.1549^{***}	-7.825
	(5.0474)	(7.9603)	(3.968)	(9.5103)
$\ln\theta$	0.7992^{***}	1.1535^{***}	-0.055	0.6896^{*}
	(0.2613)	(0.4211)	(0.2535)	(0.4022)
$\theta \geq 1$			5.8288^{***}	4.1392^{***}
			(0.7558)	(1.091)
Supply shock (ϱ)	0.0494^{**}	0.016	0.0613^{**}	-0.013
	(0.0258)	(0.0462)	(0.0242)	(0.0202)
$\theta \geq 1$			0.0834	0.2812^{**}
			(0.1083)	(0.1149)
Inflation expectations	1.0264^{***}	0.774	1.0552^{***}	0.3616
	(0.0509)	(0.6267)	(0.0485)	(0.5175)
Constant	0.7835^{***}	1.2124^{***}	0.0928	0.4959
	(0.2138)	(0.3475)	(0.2017)	(0.3713)
R^2 adjusted	0.7848	0.4718	0.8249	0.6597
Observations	260	66	260	66

· ***,**,* denote statistical significance at the 1,5, and 10 percent level, respectively.
· Newey-West standard errors.
· (1) and (3): sample 1960 Q1 – 2024 Q4
· (2) and (4): sample 2008 Q3 – 2024 Q4

Table 15: Correlation between CPI core inflation and different measures of inflation expectations.

Inflation expectations	2008Q3 - 2024Q4	2020Q4 - 2024Q4
2-year Cleveland (baseline)	0.5998	0.1874
1 year SPF	0.7289	0.5588
5-year Cleveland	0.4332	0.0439
5y-5y Forward	-0.0595	0.2941
12-month Michigan Survey	0.7006	0.8226
1-year Cleveland	0.6805	0.4026
12-month Livingston Survey	0.7480	0.6516

Table 16: Granger causality test: Core CPI inflation causing inflation expectations.

Variable	h	p-Value
2008Q3 - 2024Q4		
2-year Cleveland (baseline)	0	0.4815
1 year SPF	0	0.9859
5-year Cleveland	0	0.3582
5y-5y Forward	0	0.4175
12-month Michigan Survey	1	0.0202
1 year Cleveland	0	0.6241
12-month Livingston Survey	0	0.6241

Table 17: Reverse Granger causality test: Inflation expectations causing CPI core inflation.

Variable	h	p-Value
2008Q3 - 2024Q4		
2-year Cleveland (baseline)	1	0.0267
1 year SPF	1	0.0000
5-year Cleveland	0	0.1077
5y-5y Forward	0	0.4446
12-month Michigan Survey	1	0.0059
1-year Cleveland	1	0.0033
12-month Livingston Survey	1	0.0014

B Appendix: empirical results using Kalman-filter estimation

As an alternative benchmark to capture nonlinearities, we consider a specification that allows for time-varying coefficients, focusing on the period from 2008 Q3 to 2024 Q4. Generally, the results support our previous findings. We follow closely the existing literature, see Blanchard, Cerutti and Summers (2015). We consider the regression reported in Table 1, but the parameters are now allowed to vary over time by a random walk. The model is estimated by a Kalman filter using as initial conditions the OLS estimates generated by a regression up to 2008 Q2. Figure 14 shows how the estimated coefficients vary over time from 2008 Q3 to 2024 Q4 with red lines. The blue lines correspond to one-standard-deviation confidence bands.

The main conclusion is that the estimated coefficients shift sharply towards the end of the sample, once $\theta > 1$. The slope of the curve steepens significantly in the post-COVID period, ending with a value close to 2. This is consistent with the results in Table 1, although smaller in magnitude. The coefficient on the supply shock also increases from zero to over 0.15. This is of the same order, even if slightly smaller, than the estimated value of the supply shock in Table 1 when $\theta > 1$.

The inflation-persistence coefficient declines over time and hovers near zero at end of the sample. This, too, is one of our model's main predictions when $\theta > 1$, and is consistent with the benchmark regression.

Figure 15 highlights how poorly a forecaster would have done using either our benchmark regression or our regression with time varying coefficients if the forecaster fails to incorporate the non-linearities. The results reported in that figure give a natural explanation for why both policy makers and professional forecasters consistently failed to forecast the scope of the surge in inflation, as well as its persistence, as shown in Figures 17 and 18 already mentioned in the introduction.

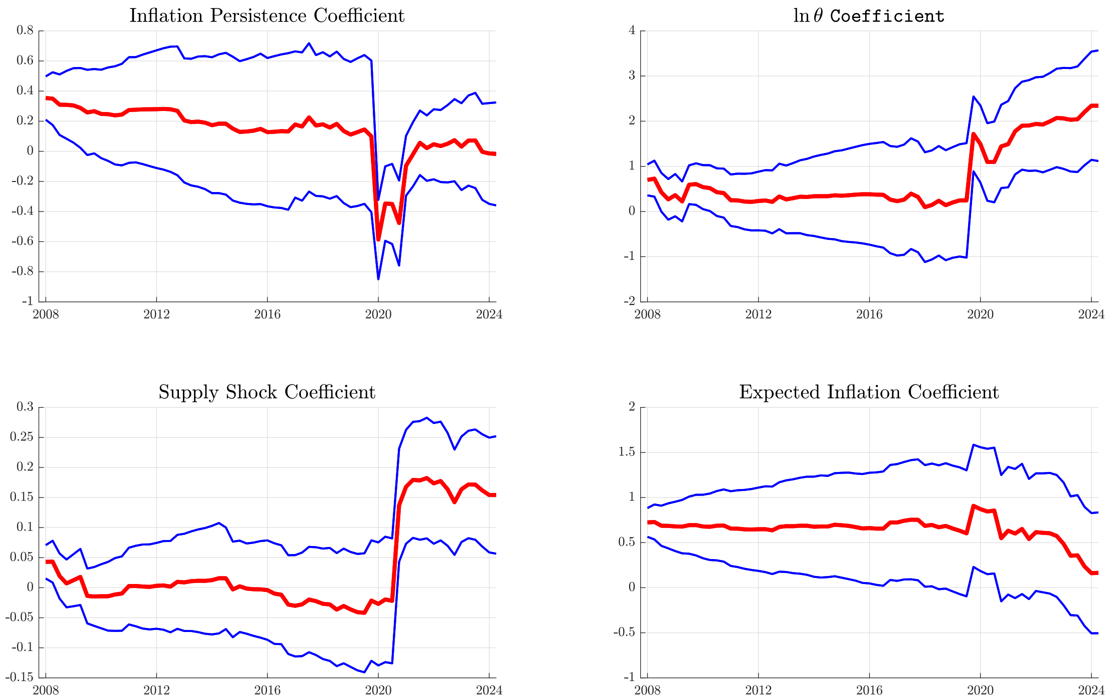


Figure 14: Estimates of the Kalman Filter with time-varying parameters on sample 2008 Q3 – 2024 Q4 with one-standard-deviation confidence bands.

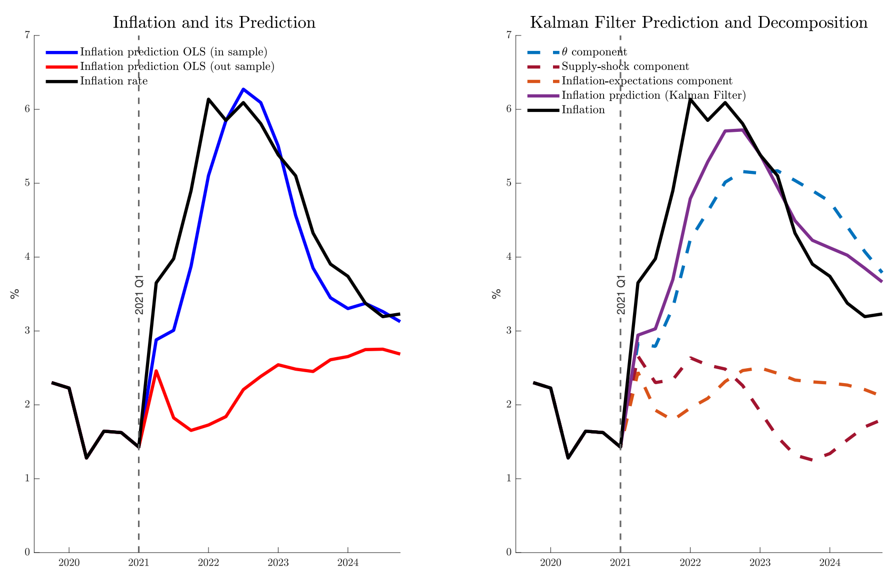


Figure 15: Left panel: CPI inflation rate at annual rates (black line); out-of-sample inflation prediction (red line) using OLS regression (2) without the dummy variable on the sample 2008 Q3 – 2021 Q1; in-sample inflation prediction (blue line) using OLS regression (2) on the sample 2008 Q3 – 2024 Q4. Right panel: CPI inflation rate at annual rates (black line); in-sample inflation prediction (purple line) using Kalman-Filter estimation with time-varying coefficients on the sample 2008 Q3 – 2024 Q4. The three dashed lines represent the inflation prediction using the Kalman-Filter estimates by restricting only to the variable θ, or the supply shock or the inflation expectations, respectively.

C Appendix: Additional Figures

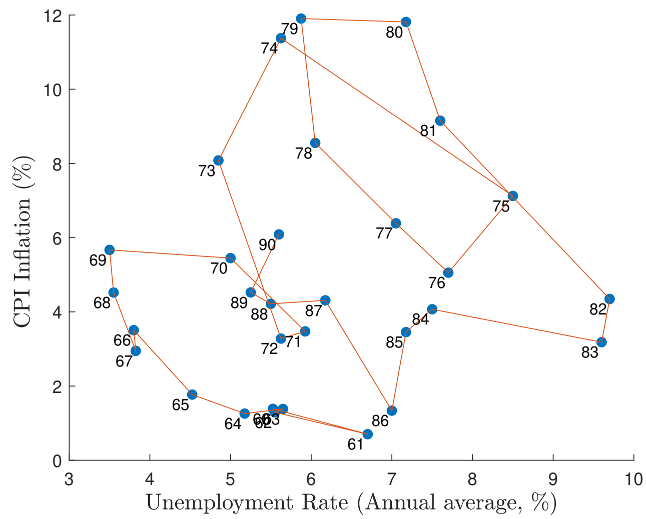


Figure 16: Empirical breakdown of the Phillips Curve in the 1970s as discussed in the Introduction, sample 1960-1990.

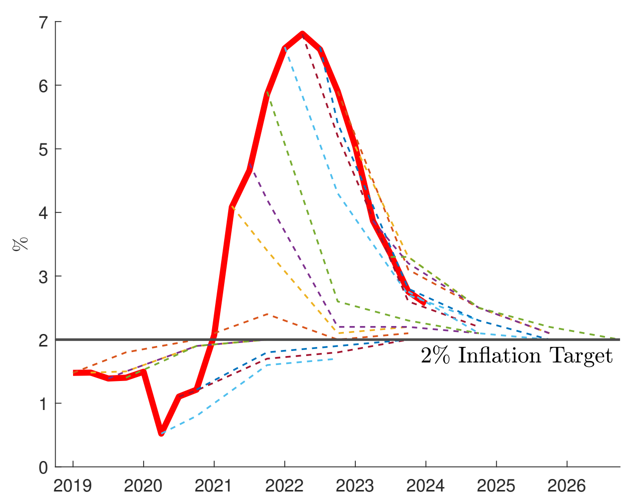


Figure 17: PCE-index inflation at annual rate (red line) and the inflation forecast of the Summary of Economic Projections (SEP) (dashed lines) of the Federal Reserve up to and during the inflation surge.

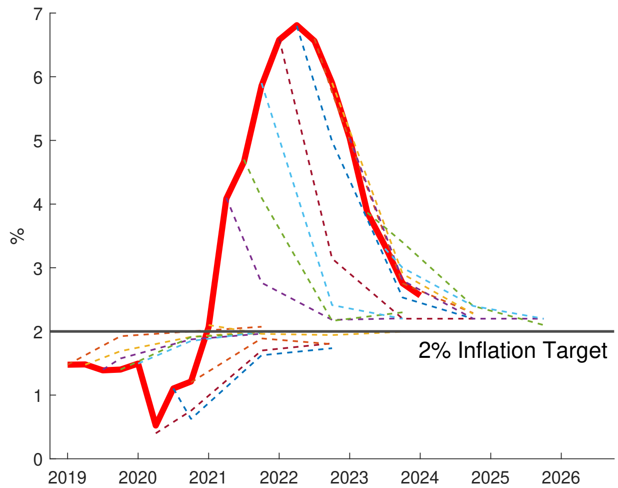


Figure 18: PCE-index inflation at annual rate (red line) and the inflation forecast of the Survey of Professional Forecasters (SPF) (dashed lines) up to, and during, the inflation surge.

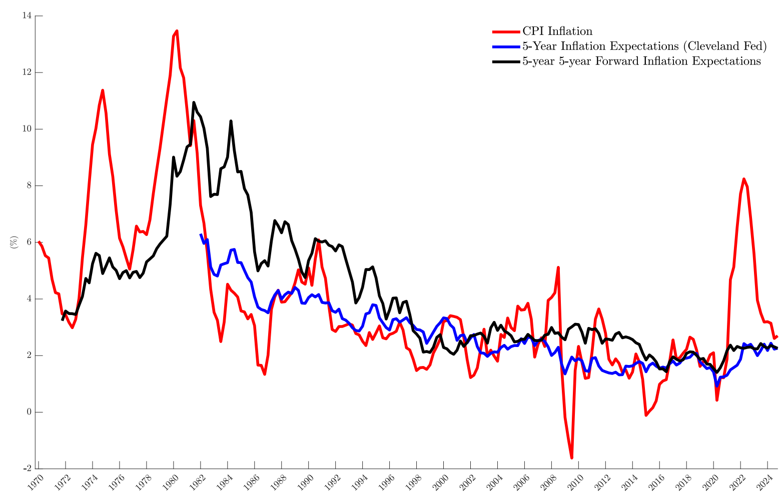
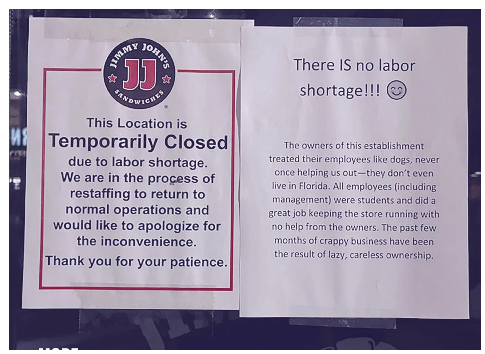


Figure 19: This figure contrasts CPI inflation at annual rates with the five-year expected inflation rate compiled by the Cleveland Fed and five-year five-year forward inflation expectations, which are market-based from 1997 and back-casted by Groen and Middledorp (2013) to 1970.

(a) Ohio labor shortage, June 2021

(b) Pennsylvania labor shortage, May 2021

(c) Virginia labor shortage, July 2022



(d) Florida labor shortage, January 2022

Figure 20: Anectodal evidence of labor shortage across different states

(a) March 2021: blockage of the Suez Canal by a vessel.

(b) September 2021: Over 100 idle cargo ships waiting to offload outside of Los Angeles

Figure 21: Anecdotal evidence of supply chain disruptions

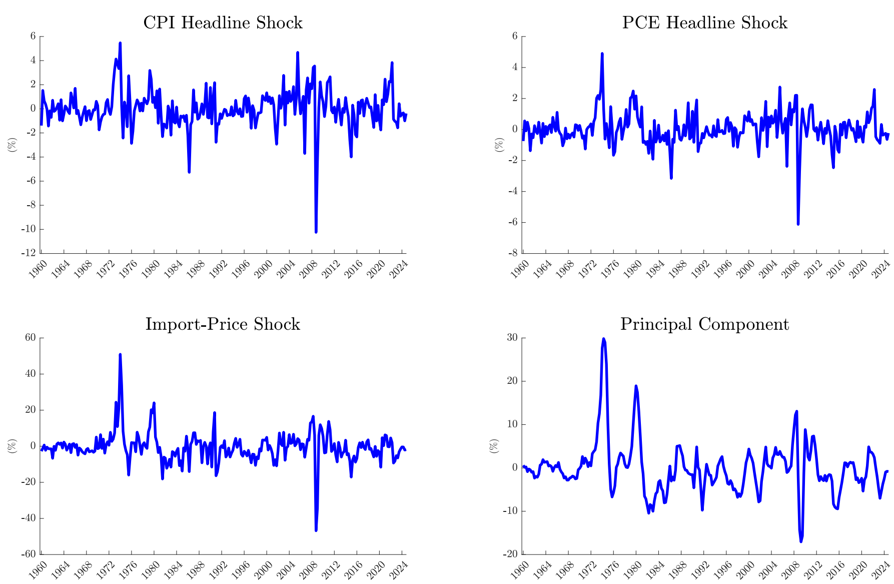


Figure 22: Measures of supply shock and their principal component (four-quarter average)

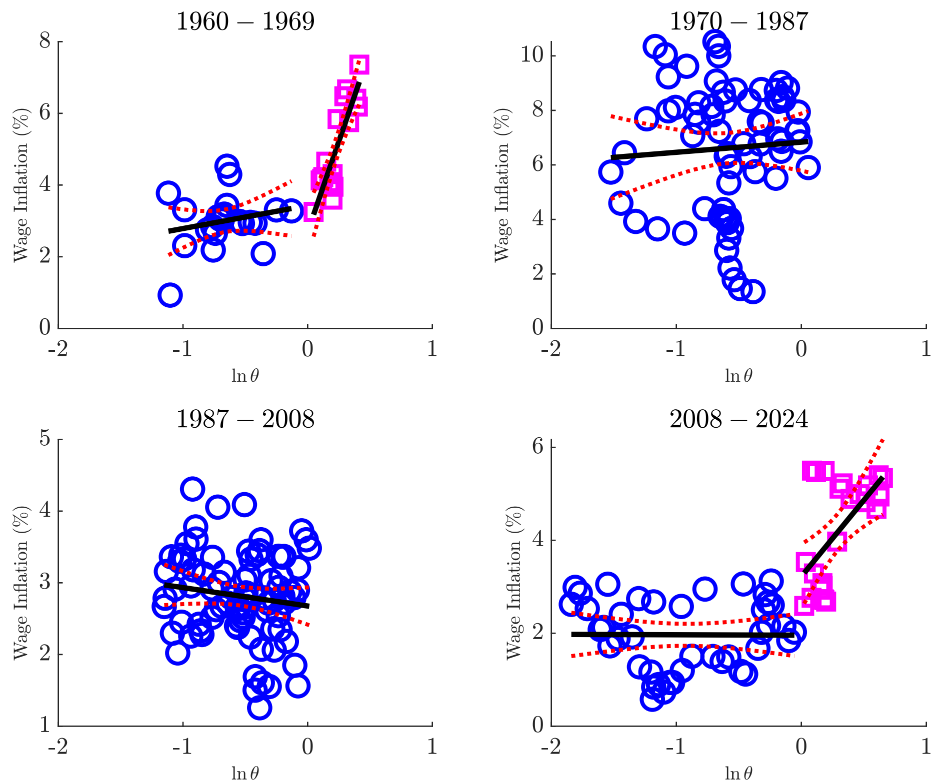


Figure 23: Wage inflation: growth rate of average hourly earnings of production and nonsupervisory employees. $\ln\theta$: log of the vacancy-to-unemployed ratio.

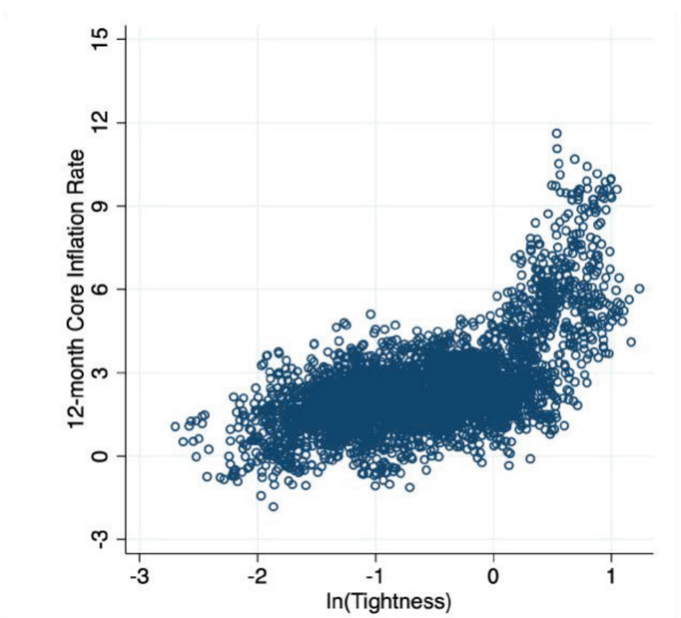


Figure 24: Inflation: CPI inflation rate at annual rates. $\ln\theta$: log of the vacancy-to-unemployed ratio for 21 Metropolitan Statistical Areas in the U.S. from 2000-2023. *Source: Gitti (2023).*

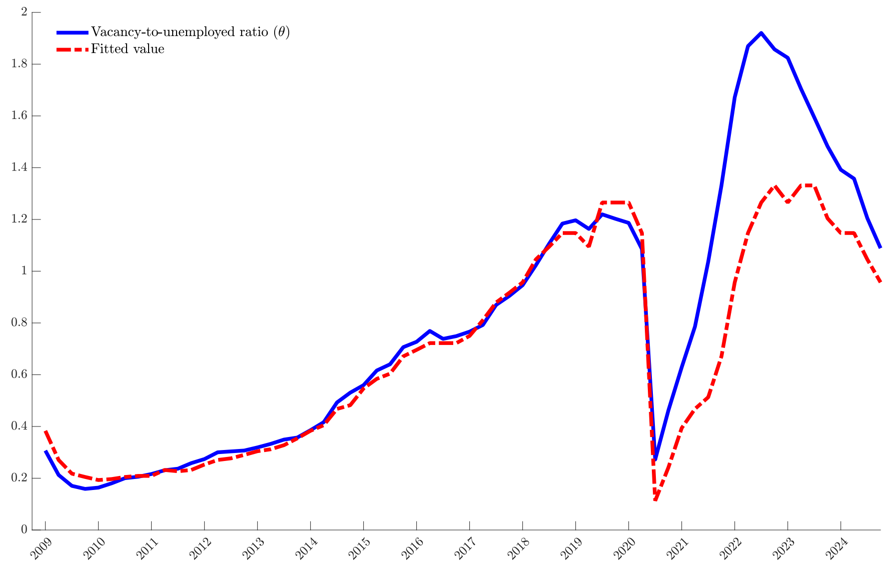


Figure 25: Vacancy-to-unemployed ratio and its fitted value using the regression $\ln\theta_t = a + b\ln(u_t/(1-u_t)) + \varepsilon_t$ on the sample 2001 Q1 – 2024 Q4, as in Kalantzis (2023). The Figure shows the time-series for the sample 2009 Q1 – 2024 Q4.

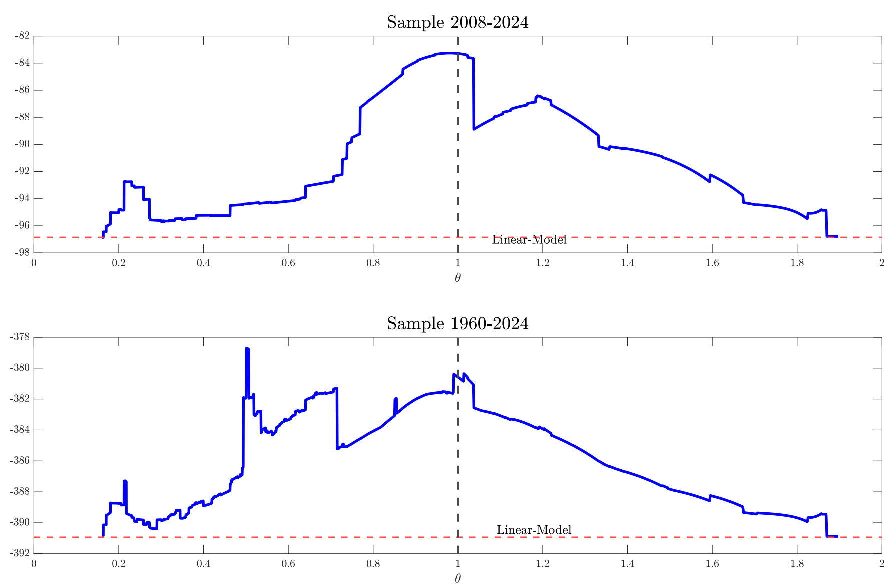


Figure 26: Maximum likelihood of OLS regression of equation (2) by varying the threshold for the dummy at different values of θ. Sample 1960-2024 and Sample 2008-2024. The red dashed line reports the maximum likelihood of the OLS regression of equation (2) without the dummy.

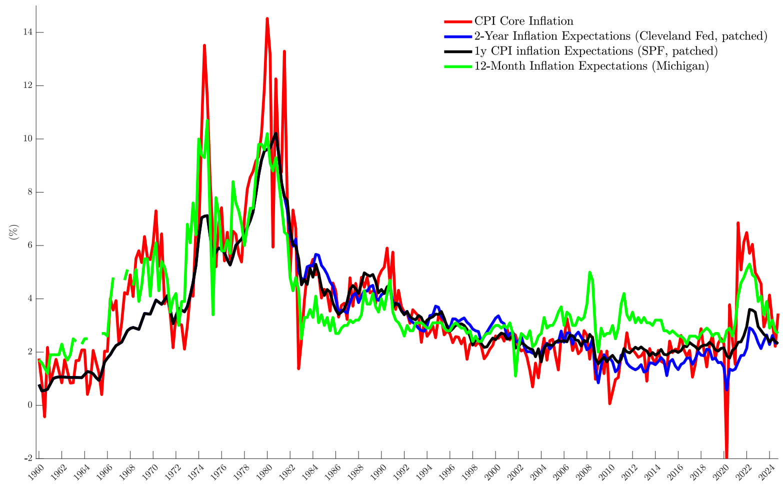


Figure 27: CPI core inflation (annualized quarterly rates). 2-year inflation expectations of the Cleveland Fed patched, before 1982 Q1, with 12-month Livingston survey inflation expectations. 1-year CPI inflation expectations of SPF patched, before 1981 Q3, with 12-month Livingston survey inflation expectations. 12-month households inflation expectations of the Michigan survey.

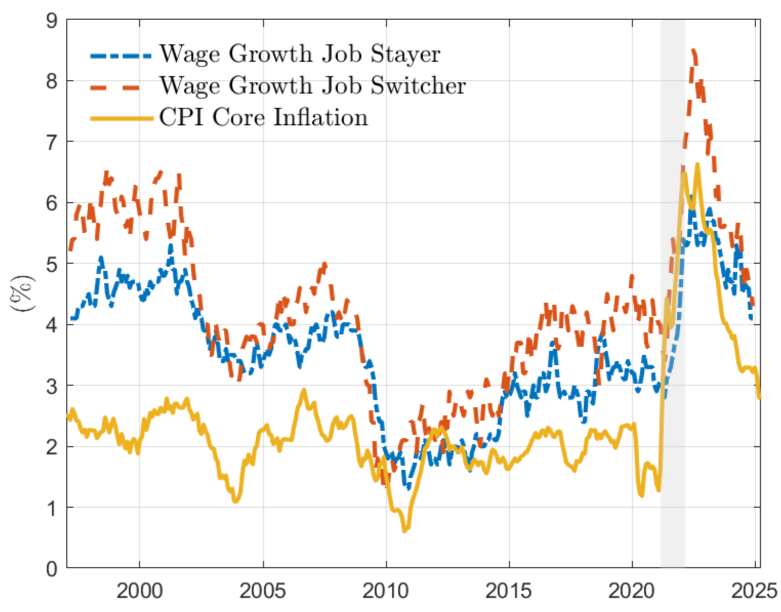


Figure 28: Wage growth (%), overall, and decomposition between job switchers and job stayers, from Wage Growth Tracker of the Federal Reserve Bank of Atlanta.

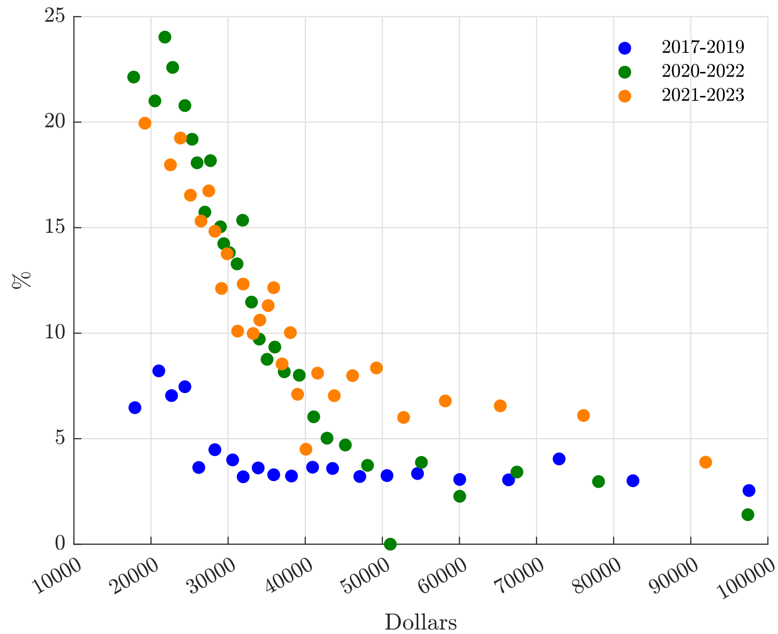


Figure 29: Nonparametric estimates of the conditional median function of two-year posted wage growth given initial wage level, based on data from Burning Glass Technologies. Source Crump et al. (2024).

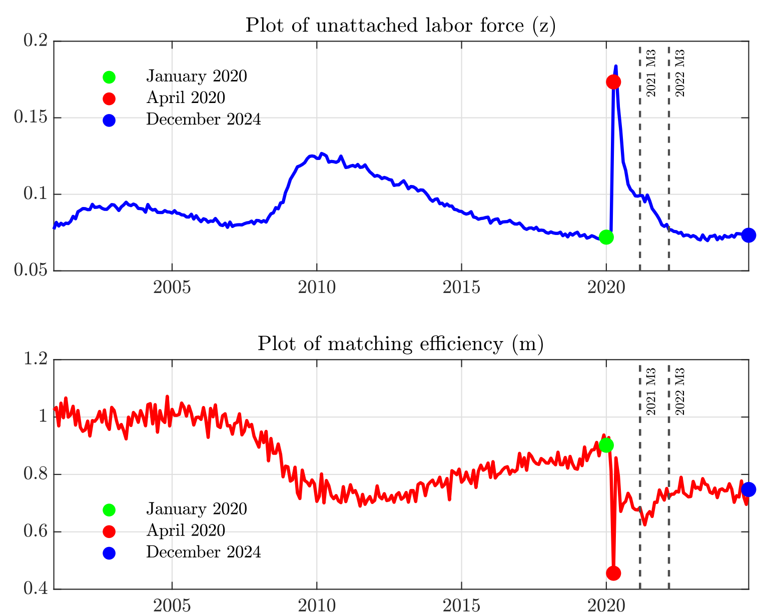


Figure 30: Plot of unattached labor force (z) and matching efficiency (m) over time.

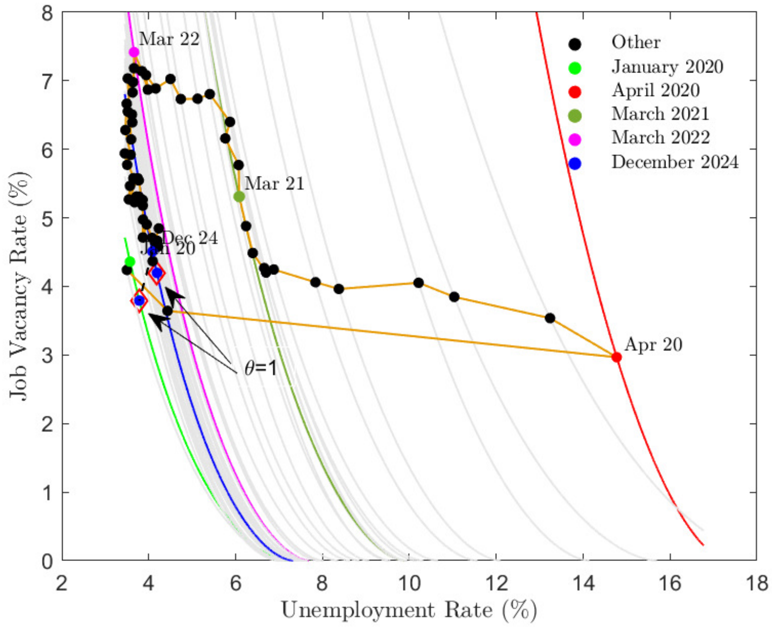


Figure 31: Scatter plot of job vacancy rate (%) and unemployment rate (%), sample 2020 M1– 2024 M6. Beveridge curve is plotted at each point in time. The points with the diamond represents the points on the January 2020 and December 2024 Beveridge curves where vacancy rate is equal to the unemployment rate.

Figure 32: Labor force participation in the U.S. during the last decade.

D Appendix: Data Description

Table 1

Table 1 presents the estimates of equation

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) \ln \theta_t + (\beta_\varrho + \beta_{\varrho_d} D_t) \cdot \varrho_t + \beta_{\pi^e} \cdot \pi_t^e + \varepsilon_t, \tag{D.1}$$

in which π_t is the annualized quarterly inflation rate, computed as log changes, in deviation from a 2% inflation target. The rate is computed using the CPI core component (net of energy and food); π_{t-1} is its lagged value. Data on CPI are from FRED, collected quarterly, using the average of monthly observations for each quarter.

$\ln \theta_t$ is the log of the ratio of vacancies to unemployed workers provided by Barnichon (2011) and updated by the author. Data are monthly. Accordingly, the quarterly series used in the regression is log of the average of the relevant monthly observations. D_t is a dummy variable taking the value one when $\theta_t \geq 1$.

ϱ_t is the four-quarter average of the principal component of the following three series: headline shocks, both to CPI and PCE, and import shock. The CPI or PCE headline shock is the difference between the annualized quarterly inflation rate computed using the CPI or PCE price index and that computed using the CPI or PCE price index excluding energy and food. The import shock is the difference between the annualized quarterly inflation rate computed using the import-price deflator and that computed using the GDP deflator. Data are from FRED and collected quarterly, using the average of the relevant monthly observations. We proxy the supply shock with the four-quarter average of the principal component of the three series. Let pr_t be the principal component of the three series described above; then ϱ_t is given by:

$$\varrho_t = (pr_t + pr_{t-1} + pr_{t-2} + pr_{t-3})/4.$$

We proxy inflation expectations (π^e) with the 2-year inflation expectations of the Federal Reserve Bank of Cleveland, which are collected from FRED quarterly and available since 1982 Q2. The series is patched backward to 1960 Q1 with the 12-month inflation expectations from the Livingston survey. Since the latter is twice yearly, missing observations are interpolated though a spline curve-preserving function. In all regressions, π_t and π_t^e are deviations with respect to a 2% annual inflation target.

Table 2

Table 2 presents the results of the regression

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + \beta_\theta \ln \theta_t + \beta_{\theta_d} D_t (\ln \theta_t - \ln \theta^*) + (\beta_\varrho + \beta_{\varrho_d} D_t) \cdot \varrho_t + \beta_{\pi^e} \cdot \pi_t^e + \varepsilon_t, \tag{D.2}$$

with the same data as in Table 1 where the threshold for the dummy is set at $\theta^* = 0.5026$ for the sample 1960 Q1-2024 Q4, and at $\theta^* = 0.9812$ for the sample 2008 Q3-2024 Q4, which correspond to the thresholds, in the respective sample, that maximize the likelihood of the regressions across different thresholds, as shown in Figure 26.

Table 3

Table 3 uses as a measure of supply shock the four-quarter average of the CPI headline shock, described under Table 1.

Table 4

Table 4 uses as a measure of supply shock the four-quarter average of the import-price shock, described under Table 1.

Table 5

Table 5 uses as a proxy of inflation expectations the 1-year CPI inflation expectations of the U.S. Professional Forecasters Surveys, retrieved from Thompson Reuters Datastream, which starts in 1981 Q3. We patch this series backward to 1960 Q1 again using interpolated 12-month Livingston survey inflation expectations.

Table 6

Table 6 uses as a proxy of inflation expectations the 5-year inflation expectations of the Federal Reserve of Cleveland, which starts in 1982 Q1, collected from the FRED database. The series is patched with the 1-year GDP-deflator inflation expectations of the U.S. Professional Forecasters Surveys, retrieved from Thompson Reuters Datastream, which starts in 1970 Q2, and finally patched backward to 1960 Q1 again using interpolated 12-month Livingston survey inflation expectations.

Table 7

Table 7 uses as a proxy of inflation expectations the 5-year 5-year forward inflation expectations backcasted by Groen and Middleddorp (2013) until 1971 Q4. The series is patched backward to 1960 Q1, again using interpolated 12-month Livingston survey inflation expectations.

Table 8

Table 8 uses as a proxy of inflation expectations the 12-month households inflation expectations of the Michigan survey until 1978 Q1 using the median of the survey and until 1960 Q1 using the mean. Missing values are present during the 1960s.

Table 9

Table 9 uses as a proxy of inflation expectations the 1-year inflation expectations of the Federal Reserve of Cleveland, which starts in 1982 Q1, collected from the FRED database. The series is patched with the 1-year GDP-deflator inflation expectations of the U.S. Professional Forecasters Surveys, retrieved from Thompson Reuters Datastream, which starts in 1970 Q2, and finally patched backward to 1960 Q1 again using interpolated 12-month Livingston survey inflation expectations.

Table 10

Table 10 uses as a proxy of inflation expectations the interpolated 12-month Livingston survey inflation expectations.

Table 11

Table 11 presents the OLS estimates of regression (2) with the same variables as Table 1, except that the inflation lag and the log of the vacancy rate to unemployment rate (θ) are instrumented with the fitted values of OLS regression on their first lags. Namely, the regressors π_{t-1} and $\ln \theta_t$ are replaced with the fitted values of the respective OLS estimates:

$$\pi_t = \beta_0 + \beta_1 \pi_{t-1} + \varepsilon_t,$$
$$\ln \theta_t = \gamma_0 + \gamma_1 \ln \theta_{t-1} + \varepsilon_t.$$

Table 12

Table 12 uses inflation measures from the core PCE at annualized quarterly rate. The core PCE price index is collected from the FRED database quarterly, as the average of the relevant monthly observations.

Table 13

Table 13 uses the level of θ rather than $\ln\theta$.

Table 14

Table 14 repeats the estimation of Table 1, but in which the lag of inflation is replaced with the detrended real wage. The real wage is built deflating the series "Nonfarm Business Sector: Unit Labor Costs for All Workers (ULCNFB)" retrieved from the BLS with the GDP deflator from FRED database. The series is not stationary and it is deetrended using the procedure of Hamilton (2018) to obtain the cyclical component.

Tables 15, 16 and 17

Table 15, 16 and 17 uses CPI core inflation rate at a annualized quarterly frequency, 1-year, 2-year and 5-year inflation expectations of the Federal Reserve of Cleveland, 1-year CPI inflation expectations of the U.S. Professional Forecasters Surveys, 5-year-5-year forward inflation expectations, 12-month households inflation expectations of the Michigan Survey, 12-month inflation expectations from the Livingston Survey. Data are collected from FRED database, Thompson Reuters Datastream, at a quarterly frequency.

Figure 1

Figure 1 presents the scatter plots of inflation and labor market tightness in the United States for the samples 1960 Q1-1969 Q4, 1970 Q1-1987 Q2, 1987 Q3-2008 Q2, 2008 Q3-2024 Q4. Inflation is annual inflation rate computed using the quarterly CPI. CPI quarterly observations are the average of the relevant monthly observations. $\ln\theta$ is the log of the ratio of vacancies to unemployed workers provided by Barnichon (2010b) and updated by the author. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations.

Figure 3

Figure 3 plots the annual inflation rate computed using the quarterly CPI (top panel). CPI quarterly observations are the average of the relevant monthly observations. θ is the ratio of vacancies to unemployed workers (bottom panel) derived by Petrosky-Nadeau and Zhang (2021) back to 1919. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations.

Figure 4

Figure 4 plots the annual inflation rate computed using the quarterly CPI (top panel). CPI quarterly observations are the average of the relevant monthly observations. θ is the ratio of vacancies to unemployed workers (bottom panel) provided by Barnichon (2011) and updated by the author. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations.

Figure 5

Figure 5 plots the annual inflation rate computed using the quarterly CPI. CPI quarterly observations are the average of the relevant monthly observations. Inflation expectations are from the Livingston survey of the Federal Reserve Bank of Philadelphia for the 12-month horizon on CPI. The frequency of the graph is twice yearly, consistently with the Livingston Survey data.

Figure 6

The left panel of Figure 6 uses the same data for inflation and θ as in Table 1, described above. The variable 'inflation deviations', π_t^d, on the right panel is built as

$$\pi_t^d = \pi_t - \beta_\pi \pi_{t-1} - (\beta_\varrho + \beta_{\varrho_d} D_t) \cdot \varrho_t - \beta_{\pi^e} \pi_t^e$$

using the estimates of Table 1, column (3).

Figure 7

Figure 7 builds a decomposition among the different regressors of Table 1 column 4. Consider a situation in which inflation is on target, expectation on target, and $\ln \theta_t = \ln \bar{\theta}$ where $\ln \bar{\theta}$ correspond to $\ln \theta_t$ being neither inflationary or deflationary. In this case

$$0 = \beta_c + \beta_\theta \ln \bar{\theta}$$

Hence:

$$\ln \bar{\theta} = -\frac{\beta_c}{\beta_\theta}.$$

This implies that the fitted value for inflation can be written as follows:

$$\pi_t = \underbrace{\beta_\pi \pi_{t-1}}_{\text{Lagged Inflation}} + \underbrace{\beta_\theta((\min(\ln(\theta_t), 0) - \ln \bar{\theta}))}_{\text{Labor Market Tightness}} + \underbrace{D_t \left[\beta_{\theta_d} \ln \theta_t \right]}_{\text{Labor Market Tightness when } \theta \geq 1}$$
$$+ \underbrace{\beta_\varrho \varrho_t + \beta_{\varrho_d} D_t \varrho_t}_{\text{Cost Push Shock}} + \underbrace{\beta_\pi \pi_t^e}_{\text{Inflation Expectations}}$$

Note that the contribution of the component $\ln \theta_t$ when $\theta_t \geq 1$ is given by the component $\beta_\theta((\min(\ln(\theta_t), 0) - \ln \bar{\theta}))$ up to the unitary value and the component $\beta_{\theta_d} \ln \theta_t$. Therefore the overall contribution is $\beta_c + \beta_{\theta d} \ln \theta_t$. Moreover, when $\theta_t \geq 1$ the contribution of the cost push shock is given by the sum of $\beta_\varrho \varrho_t + \beta_{\varrho_d} D_t \varrho_t$. Figure 7 shows this decomposition together with core inflation, both at annualized rates.

Figure 8

Figure 8 builds a decomposition among the different regressors of Table 1 column 3 for the sample 1960 Q1 – 2024 Q4. The procedure follows the description given under Figure 7.

Figure 9

Figure 9 plots wage growth, in its decomposition between the category job switchers and job stayers, and core CPI inflation. Data on wages are monthly and collected from the ADP Employment Report.

Figure 10

The Figure uses the estimates of Table 1, column 4, to draw inflation as a piecewise linear function of $\ln \theta$.

Figure 11

The Figure uses the estimates of Table 1, column 4, for $\kappa = 0.5185$, $\kappa_v = -0.0096$, $\kappa^{tight} = 0.5185 + 5.4627$, $\kappa_v^{tight} = -0.0096 + 0.2745$. The estimates κ and κ^{tight} are divided by 400 since in the OLS regression inflation is measured in percent and at annual rates while the model is interpreted at a quarterly frequency. The following parametrization is used: $\omega = 1$, $\alpha = 0.9$, $\lambda = 0.5$, $z = 0.0733$, $\bar{u} = 0.04$, $\eta = 0.4$, $\tau = 0.8$, $\phi_\pi = 1.5$, $\sigma = 0.5$, $\pi^* = 0.05$. The following parameters, derived in Appendix E, are obtained:

$$
\begin{aligned}
d_\theta &\equiv (1-\lambda)\eta + \frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(1+\omega)(1-\eta), \\
\tilde{d}_\theta &\equiv \left(\frac{1-z}{1-\bar{u}}(1-\lambda) + \frac{z-\bar{u}}{1-\bar{u}}\right)\eta + \frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(1+\omega)(1-\eta),
\end{aligned}
$$

$$
\tilde{\kappa} \equiv \frac{\frac{\omega\kappa}{\alpha d_\theta}}{1 - \frac{\kappa\lambda}{d_\theta}}, \tag{D.3}
$$

$$
\tilde{\kappa}_v \equiv \frac{\kappa_v}{1 - \frac{\kappa\lambda}{d_\theta}},
$$

$$\tilde{\kappa}^{tight} \equiv \frac{\frac{\omega}{\alpha}\frac{\kappa^{tight}}{\tilde{d}_\theta}}{1-\frac{\kappa^{tight}\lambda}{\tilde{d}_\theta}\frac{1-z}{1-\bar{u}}},$$

$$\tilde{\kappa}_\nu^{tight} \equiv \frac{\kappa_\nu^{tight}}{1-\frac{\kappa^{tight}\lambda}{\tilde{d}_\theta}\frac{1-z}{1-\bar{u}}}.$$

The Figure plots the AD equation given by:

$$\hat{Y}_S = \hat{G}_S - \sigma^{-1}\frac{\phi\pi-\tau}{1-\tau}(\pi_S-\pi^*)$$

and the Inv-L NK Phillips curve:

$$\pi_S = \begin{cases} \frac{c}{1-\tau} + \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_\nu^{tight}}{1-\tau}(1-\alpha)\hat{q}_S + \pi^* & \hat{Y}_t \geq \hat{Y}^* \\ \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_\nu}{1-\tau}(1-\alpha)\hat{q}_S + \pi^* & \hat{Y}_t < \hat{Y}^* \end{cases},$$

where $\hat{Y}^*$ is set such that when $\hat{Y}_S = \hat{Y}^*$, $\pi_S = (2.3906/400)$ consistent with the estimates of Table 1, column 4. The demand shock is set at $\hat{G}_S = 0.015$ and the supply shock at $\hat{q}_S = 0.075$. Demand and supply shocks are picked in a way that inflation rate reaches 6.2%, consistently with the value reached in Q2 2022 as shown in Figure 7, and that demand shock contributes to 2/3 to the rise, while the remaining fraction is explained by the supply shock. The parameter c is such that

$$c = \left(\tilde{\kappa} - \tilde{\kappa}^{tight}\right)\hat{Y}^*.$$

In Figure **11**, inflation, π_S, is at annual rates and in percent, output gap $\hat{Y}_S$ is in percentage points.

Figure 12

The Figure uses the estimates of Table 1, column 3, for $\kappa = 0.2315$, $\kappa_\nu = 0.0447$, $\kappa^{tight} = 0.2315 + 3.7753$, $\kappa_\nu^{tight} = 0.0447 + 0.1038$. The estimates κ and κ^{tight} are divided by 400 since in the OLS regression inflation is measured in percent and at annual rates while the model is interpreted at a quarterly frequency. The following parametrization is used: $\omega = 1$, $\alpha = 0.9$, $\lambda = 0.5$, $z = 0.0733$, $\bar{u} = 0.04$, $\eta = 0.4$, $\tau = 0.8$, $\phi_\pi = 1.5$, $\sigma = 0.5$, $\pi^* = 0.05$. The following parameters, derived in Appendix E, are obtained:

$$\begin{aligned} d_\theta &\equiv (1-\lambda)\eta + \frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(1+\omega)(1-\eta), \\ \tilde{d}_\theta &\equiv \left(\frac{1-z}{1-\bar{u}}(1-\lambda) + \frac{z-\bar{u}}{1-\bar{u}}\right)\eta + \frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(1+\omega)(1-\eta), \end{aligned}$$

$$\tilde{\kappa} \equiv \frac{\frac{\omega\kappa}{\alpha d_\theta}}{1-\frac{\kappa\lambda}{d_\theta}}, \tag{D.4}$$

$$\tilde{\kappa}_{\nu} \equiv \frac{\kappa_{\nu}}{1 - \frac{\kappa\lambda}{d_{\theta}}},$$

$$\tilde{\kappa}^{tight} \equiv \frac{\frac{\omega}{\alpha}\frac{\kappa^{tight}}{\tilde{d}_{\theta}}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{d}_{\theta}}\frac{1-z}{1-\bar{u}}},$$

$$\tilde{\kappa}_{\nu}^{tight} \equiv \frac{\kappa_{\nu}^{tight}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{d}_{\theta}}\frac{1-z}{1-\bar{u}}}.$$

The Figure plots the AD equation given by:

$$\hat{Y}_S = -\sigma^{-1}\frac{\phi\pi - \tau}{1-\tau}(\pi_S - \pi^*) + \sigma^{-1}(\pi_L^e - \pi^*).$$

and the inv-L NK Phillips curve:

$$\pi_S - \pi^* = \begin{cases} \frac{c}{1-\tau} + \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_{\nu}^{tight}}{1-\tau}(1-\alpha)\hat{q}_S + (\pi_L^e - \pi^*) & \hat{Y}_t \geq \hat{Y}^* \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_{\nu}}{1-\tau}(1-\alpha)\hat{q}_S + (\pi_L^e - \pi^*) & \hat{Y}_t < \hat{Y}^* \end{cases}.$$

where $\hat{Y}^*$ is set such that when $\hat{Y}_S = \hat{Y}^*$, $\pi_S = (2.1922/400)$ consistently with the estimates of Table 1, column 3. The supply shock is set at $\hat{q}_S = 0.0725$, consistently with the maximum 30% increase, during the 1970s inflationary surge, in the proxy of the supply shock used in the regression as shown in Figure 22; the inflation expectations shock is set at $\pi_L^e = 0.025$, consistently with the 10% increase in inflation expectations observed during the 1970s, according to Figure 5. The parameter c is such that

$$c = \left(\tilde{\kappa} - \tilde{\kappa}^{tight}\right)\hat{Y}^*.$$

In Figure **12**, inflation, π_S, is at annual rates and in percent, output gap $\hat{Y}_S$ is in percentage points.

Figure 13

The Figure uses the same estimates and calibration described under Figure 11.

The Figure plots the AD equation given by:

$$\hat{Y}_S = -\sigma^{-1}\frac{\phi\pi - \tau}{1-\tau}(\pi_S - \pi^*)$$

which applies whenever the economy is above the zero lower bound, i.e. under the condition that

$$\hat{r}_S^e > \phi_{\pi}(\pi_S - \pi^*) - \ln(1+i),$$

in which i is the steady state interest rate, which is set at $i = 0.0075$. When the zero lower bound binds, the AD equation is given by

$$\hat{Y}_S = \sigma^{-1}\frac{\tau}{1-\tau}(\pi_S - \pi^*) + \sigma^{-1}\frac{\ln(1+i)+\hat{r}^e_S}{1-\tau}.$$

The Inv-L NK Phillips curve:

$$\pi_S = \begin{cases} \frac{c}{1-\tau} + \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \pi^* & \hat{Y}_t \geq \hat{Y}^* \\ \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \pi^* & \hat{Y}_t < \hat{Y}^* \end{cases},$$

where $\hat{Y}^*$ is set such that when $\hat{Y}_S = \hat{Y}^*$, $\pi_S = (2.3906/400)$ consistent with the estimates of Table 1, column 4. The parameter c is such that

$$c = \left(\tilde{\kappa} - \tilde{\kappa}^{tight}\right)\hat{Y}^*.$$

The shocks to the natural rate of interest is set at $\hat{r}^e_S = -0.0125$.

Figure 14

Figure 14 presents the estimates through Kalman Filter of the measurement equation

$$\pi_t = \beta_{c,t} + \beta_{\pi,t}\pi_{t-1} + \beta_{\theta,t}\ln\theta_t + \beta_{\varrho,t}\varrho_t + \beta_{\pi^e,t}\pi^e_t + \varepsilon_t,$$

in which ε_t is distributed as $N(0,\sigma^2_\varepsilon)$ with the state equations given by

$$\begin{aligned} \beta_{c,t} &= \beta_{c,t-1} + \epsilon_{c,t} \\ \beta_{\pi,t} &= \beta_{\pi,t-1} + \epsilon_{\pi,t} \\ \beta_{\theta,t} &= \beta_{\theta,t-1} + \epsilon_{\theta,t} \\ \beta_{\varrho,t} &= \beta_{\varrho,t-1} + \epsilon_{\varrho,t} \\ \beta_{\pi^e,t} &= \beta_{\pi^e,t-1} + \epsilon_{\pi^e,t} \end{aligned}$$

in which $\epsilon_{c,t} \sim N(0,\sigma^2_\epsilon)$, $\epsilon_{\pi,t} \sim N(0,\sigma^2_{\epsilon_\pi})$, $\epsilon_{\theta,t} \sim N(0,\sigma^2_{\epsilon_\theta})$, $\epsilon_{\varrho,t} \sim N(0,\sigma^2_{\epsilon_\varrho})$, $\epsilon_{\pi^e,t} \sim N(0,\sigma^2_{\epsilon_{\pi^e}})$. The Kalman Filter is initialized by running an OLS regression of the measurement equation with constant coefficients on the sample period 1960 Q1 – 2008 Q2. Then, the Kalman Filter estimation runs from 2008 Q3 to 2024 Q4. σ^2_ε is initialized as the variance of the residuals of the OLS regression on the pre-sample; β_c, β_π, β_θ, β_ϱ and β_{π^e} are initialized with OLS estimates of the respective coefficients on the pre-sample; σ^2_ϵ, $\sigma^2_{\epsilon_\pi}$, $\sigma^2_{\epsilon_\theta}$, $\sigma^2_{\epsilon_\varrho}$, $\sigma^2_{\epsilon_{\pi^e}}$ are initialized with the variance of the respective coefficients of the OLS regression on the pre-sample. Figure 14 plots the estimated time-varying coefficients $\beta_{\pi,t}$, $\beta_{\theta,t}$, $\beta_{\varrho,t}$ and $\beta_{\pi^e,t}$ using the Kalman Filter and their one-standard-deviation confidence bands.

Figure 15

The black line of the left panel of Figure 15 is the annual CPI inflation rate excluding food and energy sectors. The red line represents the out-of-sample prediction of equation

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + \beta_\theta \ln \theta_t + \beta_\varrho \varrho_t + \beta_{\pi^e} \pi_t^e + \varepsilon_t,$$

estimated for the sample 2008 Q3 – 2021 Q1 for the period 2021 Q2 – 2024 Q4. The model produces forecasts for the quarterly inflation rate in deviations of a 2% target. Accordingly, we build the corresponding predicted inflation at annual rates.

The blue line represents the in-sample prediction of equation

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) \cdot \ln \theta_t + (\beta_\varrho + \beta_{\varrho_d} D_t) \varrho_t + \beta_{\pi^e} \pi_t^e + \varepsilon_t,$$

estimated for the sample 2008 Q3 – 2024 Q4 for the period 2021 Q2 – 2024 Q4. The model produces predictions for the quarterly inflation rate in deviation of a 2% target. Accordingly, we build the corresponding predicted inflation at annual rates.

The black line in the right panel of Figure 15 is the annual CPI inflation rate excluding food and energy. The purple line is the in-sample prediction using the non-linear Kalman Filter estimates, while 'θ component', 'Supply Shock component', 'Inflation Expectations component' correspond, respectively, to the in-sample prediction derived from the following three equations using the Kalman Filter estimates.

$$\begin{aligned}
\pi_t^\theta &= \beta_{\pi,t} \pi_{t-1}^\theta + \beta_{\theta,t} (\ln \theta_t - \ln \bar{\theta}_t), \\
\pi_t^\varrho &= \beta_{\pi,t} \pi_{t-1}^\varrho + \beta_{\varrho,t} \varrho_t, \\
\pi_t^* &= \beta_{\pi,t} \pi_{t-1}^* + \beta_{\pi^e,t} \pi_t^e.
\end{aligned}$$

in which

$$\ln \bar{\theta}_t = -\beta_{c,t}^{-1} \beta_{\theta,t},$$

and initial conditions are given by the inflation rate in 2021 Q1. The model produces inflation predictions at quarterly frequency in deviations of a 2% target, so we build the corresponding annual inflation predictions plotted in the Figure.

Figure 16

Data are taken from the FRED Database. Inflation is computed using the CPI annual inflation rate (Q4 on Q4) for the reference year. The unemployment rate is the annual average.

Figure 17

Data for the PCE-index inflation and its forecasts of the Summary of Economic Projections are from the FRED database.

Figure 18

Data on PCE-index inflation and its forecasts of the Survey of Professional Forecasters are from the FRED database.

Figure 19

Figure 19 plots the annual inflation rate computed using the quarterly CPI. CPI quarterly observations are the average of the relevant monthly observations. Inflation expectations are the 5-year inflation expectations of the Federal Reserve of Cleveland. Data are from the FRED database. The Figure also plots the 5-year 5-year forward inflation expectations of Groen and Middledorp (2013) update from FRED database using the series T5YIFR with end-of-month data.

Figure 22

Figure 22 presents the three different measures of the supply shock that we use to build the proxy for ϱ_t, namely the four-quarter averages of the principal component of the two headline shocks (using CPI and PCE price index) and the import-price shock, as described under Table 1.

Figure 23

Figure 23 shows scatter plots of wage inflation and $\ln \theta$ at quarterly frequency and for different samples. The wage inflation is the annual growth rate using Average Hourly Earnings of Production and Nonsupervisory Employees, Manufacturing, Dollars per Hour, retrieved from the FRED database. $\ln \theta$ is the logarithm of the ratio of vacancies to unemployed workers as in Figure 1.

Figure 24

Figure 24 is taken from Gitti (2023).

Figure 25

Figure 25 plots the vacancy-to-unemployment ratio (θ) and its fitted valued using the regression

$$\ln \theta_t = a + b \ln(u_t/(1-u_t)) + \varepsilon_t$$

estimated with OLS on the sample 2001 Q1 – 2024 Q4. This regression is suggested by Kalantzis(2023). Data are from the Job Openings and Labor Turnover Survey of the BLS. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations.

Figure 26

Figure 26 reports the likelihood value of the regressions

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + \beta_\theta \ln \theta_t + \beta_{\theta_d} D_t (\ln \theta_t - \ln \theta^*) + (\beta_\varrho + \beta_{\varrho_d} D_t) \cdot \varrho_t + \beta_{\pi^e} \cdot \pi_t^e + \varepsilon_t, \tag{D.5}$$

in which $D_t = 1$ whenever $\theta_t \geq \theta^*$ otherwise $D_t = 0$. The threshold θ^* can take values between $\theta_{\min} = \min \theta_t$ and $\theta_{\max} = \max \theta_t$. The Figure plots the likelihood value as a function of the threshold θ^* estimating the regressions in the samples 1960 Q1- 2024 Q4 (bottom panel) and 2008 Q3- 2024 Q4 (top panel).

Figure 27

Figure 27 plots the inflation rate and inflation expectations used in Table 1, Table 5 and Table 8. Inflation rate is the annualized quarterly inflation rate computed using core CPI. Inflation expectations, used in Table 1, are the 2-year inflation expectations of the Federal Reserve Bank of Cleveland, collected from FRED quarterly and available since 1982 Q2. The series is patched backward to 1960 Q1 with the 12-month inflation expectations from the Livingston survey. Since the latter is twice yearly, missing observations are interpolated though a spline curve-preserving function. Inflation expectations in Table 5 are the 1-year CPI inflation expectations of the Survey of Professional Forecasters, retrieved from Thompson Reuters Datastream, which starts in 1981 Q3. We patch this series backward to 1960 Q1 again using interpolated 12-month Livingston inflation expectations. Inflation expectations in Table 8 represent the 12-month consumer inflation expectations from the University of Michigan survey, retrieved from the FRED database at a quarterly frequency. This data starts in 1979 Q1 and corresponds to the median of the survey. The series is extended backward to 1960 Q1 using the mean of the same survey.

Figure 28

Figure 28 plots wage growth, in its decomposition between the category job switchers and job stayers, and core CPI inflation. Data on wages are monthly and collected from the website of the Atlanta Fred at Wage Growth Tracker - Federal Reserve Bank of Atlanta (atlantafed.org).

Figure 29

Figure 29 reproduces Figure 8, top panel, of Crump et al. (2024). It shows the nonparametric estimates of the conditional median function of two-year posted wage growth given initial wage level, based on data from Burning Glass Technologies.

Figure 30

Considering a time-varying z_t, equation (28) implies that

$$N_t = [1 - z_t + m_t u_t^{\eta} v_t^{1-\eta}] F_t. \tag{D.6}$$

First using data on the hiring rate (h_t), vacancy rate (v_t) and unemployment rate (u_t) from JOLTS of the U.S. Bureau of Labor Statistics, m_t is obtained as

$$m_t = \frac{h_t}{u_t^{\eta} v_t^{1-\eta}},$$

having set $\eta = 0.4$ as in Blanchard, Domash and Summers (2022). Moreover note that (D.6) implies that

$$z_t = u_t + h_t,$$

from which we build z_t.

Figure 30 plots z_t and m_t for the sample 2000 M12 – 2024 M12.

Figure 31

Figure 31 presents a scatter plot of job vacancy rate, v_t, and unemployment rate, u_t, for the sample 2020 M1 – 2024 M12. Data are from JOLTS of the Bureau of Labor Statistics. At each date, and therefore for each couple (m_t, z_t), the Figure plots the Beveridge curve implied by (D.6) and given by

$$v = \left(\frac{z_t - u}{m_t u^{\eta}} \right)^{\frac{1}{1-\eta}},$$

for $u < z_t$; z_t and m_t are derived following the procedure described under Figure 30 and $\eta = 0.4$. Note that when $v = u$ and therefore $\theta = 1$, then $u = z/(1+m)$.

Figure 32

Data on labor force participation are taken from the FRED Database at quarterly frequency.

E Appendix: The Model

E.1 Derivation of the AS equation (26)

The firms' discounted value of current and expected future profits are:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \left\{ p_T(i) y_T(i) - W_T^{ex} N_T^{ex}(i) - (1+\gamma_T^b) W_T^{new} N_T^{new}(i) - P_T q_T O_T(i) - \frac{\varsigma}{2} \left(\frac{p_T(i)}{p_{T-1}(i)} \frac{1}{\Pi} - 1 \right)^2 P_T Y_T \right\}$$

where $Q_{t,T} \equiv \beta^{T-t} (X_T^{-\sigma}/P_T)/(X_t^{-\sigma}/P_t)$ is the stochastic discount factor the household uses at time t. Note that the maximization problem is subject to the following constraints:

$$y_t(i) = A_t (N_t^{ex}(i) + N_t^{new}(i))^{\alpha} O_t(i)^{1-\alpha}, \tag{E.7}$$

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\epsilon_t} Y_t, \tag{E.8}$$

$$0 \leq N_t^{ex}(i) \leq (1-z) F_t, \tag{E.9}$$

$$N_t^{new}(i) \geq 0. \tag{E.10}$$

We can use (E.7) to solve for $N_t^{new}(i)$ to obtain

$$N_t^{new}(i) = \left(\frac{y_t(i)}{A_t} \right)^{\frac{1}{\alpha}} O_t(i)^{\frac{\alpha-1}{\alpha}} - N_t^{ex}(i),$$

which substituted into the objective function yields to:

$$\begin{aligned} &E_t \sum_{T=t}^{\infty} Q_{t,T} \left\{ p_T(i) y_T(i) - (W_T^{ex} - (1+\gamma_T^b) W_T^{new}) N_T^{ex}(i) - (1+\gamma_T^b) W_T^{new} \left(\frac{y_T(i)}{A_T} \right)^{\frac{1}{\alpha}} O_T(i)^{\frac{\alpha-1}{\alpha}} + \right. \\ &\left. - P_T q_T O_T(i) - \frac{\varsigma}{2} \left(\frac{p_T(i)}{p_{T-1}(i)} \frac{1}{\Pi} - 1 \right)^2 P_T Y_T \right\}. \end{aligned}$$

Note that whenever $W_t^{ex} < (1+\gamma_t^b) W_t^{new}$, it follows, using (E.9), that the optimal choice for $N_t^{ex}(i)$ is $N_t^{ex}(i) = (1-z) F_t$. Given this result, first-order conditions with respect to $p_t(i)$ and $O_t(i)$ imply

$$\begin{aligned} 0 \quad = \quad & (1-\epsilon_t) y_t(i) + \epsilon_t (1+\gamma_t^b) W_t^{new} \frac{1}{\alpha} \left(\frac{y_t(i)}{A_t} \right)^{\frac{1}{\alpha}-1} \frac{y_t(i)}{A_t p_t(i)} O_t(i)^{\frac{\alpha-1}{\alpha}} + \\ & - \varsigma \left(\frac{p_t(i)}{p_{t-1}(i) \Pi} - 1 \right) \frac{1}{p_{t-1}(i) \Pi} P_t Y_t + \varsigma E_t \left\{ \beta \left(\frac{X_{t+1}}{X_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i) \Pi} - 1 \right) \frac{p_{t+1}(i)}{(p_t(i))^2 \Pi} P_{t+1} Y_{t+1} \right\} \end{aligned}$$

and

$$\frac{1-\alpha}{\alpha} (1+\gamma_t^b) W_t^{new} \left(\frac{y_t(i)}{A_t} \right)^{\frac{1}{\alpha}} O_t(i)^{-\frac{1}{\alpha}} = P_t q_t.$$

We can combine the second first-order condition into the first to substitute for $O_t(i)$ and obtain

$$
\begin{aligned}
0 \quad = \quad & (1-\epsilon_t) y_t(i) + \epsilon_t \left(\frac{(1+\gamma_t^b) W_t^{new}}{\alpha} \right)^{\alpha} \left(\frac{P_t q_t}{1-\alpha} \right)^{1-\alpha} \frac{y_t(i)}{A_t p_t(i)} + \\
& - \varsigma \left(\frac{p_t(i)}{p_{t-1}(i)\Pi} - 1 \right) \frac{1}{p_{t-1}(i)\Pi} P_t Y_t + \varsigma E_t \left\{ \beta \left(\frac{X_{t+1}}{X_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i)\Pi} - 1 \right) \frac{p_{t+1}(i)}{(p_t(i))^2 \Pi} P_{t+1} Y_{t+1} \right\}.
\end{aligned}
$$

All firms are going to set the same price therefore $p_t(i) = P_t$ and $y_t(i) = Y$. We can then obtain

$$
\begin{aligned}
0 \quad = \quad & (1-\epsilon_t) + \frac{\epsilon_t}{A_t} \left(\frac{1+\gamma_t^b}{\alpha} \frac{W_t^{new}}{P_t} \right)^{\alpha} \left(\frac{q_t}{1-\alpha} \right)^{1-\alpha} - \varsigma \left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} + \\
& + \varsigma E_t \left\{ \beta \left(\frac{X_{t+1}}{X_t} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \right\},
\end{aligned}
$$

from which we can obtain equation (26), here restated as

$$
\begin{aligned}
\left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} \quad = \quad & \frac{\epsilon_t - 1}{\varsigma} \left(\frac{\mu_t}{A_t} \left(\frac{1+\gamma_t^b}{\alpha} \frac{W_t^{new}}{P_t} \right)^{\alpha} \left(\frac{q_t}{1-\alpha} \right)^{1-\alpha} - 1 \right) + \\
& + \beta E_t \left\{ \left(\frac{X_{t+1}}{X_t} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \right\}, \qquad \text{(E.11)}
\end{aligned}
$$

in which we have defined $\mu_t \equiv \epsilon_t / (\epsilon_t - 1)$.

Note that whenever $W_t^{ex} = (1+\gamma_t^b) W_t^{new}$, the above derivation applies too implying the same AS equation.

E.2 Inv-L Phillips curve characterization

In this Section we derive the Inv-L Phillips curve characterization through a log-linear approximation of equation (E.11) considering that

$$
w_t^{new} = \max(w_t^{ex}, w_t^{flex}), \qquad \text{(E.12)}
$$

where

$$
w_t^{flex} = \frac{\gamma_t^c}{\gamma_t^b} \frac{1}{m_t} \theta_t^{\eta}, \qquad \text{(E.13)}
$$

and

$$
w_t^{ex} = \left(w_{t-1}^{ex} \frac{(\Pi_{t+1}^e)^{\delta}}{\Pi_t} \right)^{\lambda} (w_t^{flex})^{1-\lambda} \phi_t. \qquad \text{(E.14)}
$$

The approximation is going to deliver a piece-wise log-linear function as it is shown in equation (49) in the main text. First, we characterize the steady state of the general equilibrium model, then we take a first-order log-linear approximation of (E.11), (E.12), (E.13) and (E.14).

E.2.1 Steady state

Let us first consider a steady state in which $\xi_t = \xi$, $A_t = A$, $\epsilon_t = \epsilon$, $q_t = q$, $\bar{O}_t = \bar{O}$, $G_t = G$, $m_t = m$, $z_t = z$, $\chi_t = \chi$, $\Psi_t = \Psi$, $\gamma_t^c = \gamma^c$, $\gamma_t^b = \gamma^b$, $\phi_t = \phi$, $\Pi_t = \Pi$.[80] The steady-state versions of equations (14), (26), (28), (29) and (35) imply:

$$\chi F^{\omega} = (1-z)w^{ex} + um\theta^{1-\eta}w^{new}, \tag{E.15}$$

$$w^{new} = \frac{\alpha}{1+\gamma^b}\left(\frac{\epsilon-1}{\epsilon}A\right)^{\frac{1}{\alpha}}\left(\frac{q}{1-\alpha}\right)^{-\frac{1-\alpha}{\alpha}}, \tag{E.16}$$

$$N = F(1-z+um\theta^{1-\eta}), \tag{E.17}$$

$$1 = \frac{N}{F} + u \tag{E.18}$$

$$w^{flex} = \frac{1}{m}\frac{\gamma^c}{\gamma^b}\theta^{\eta}. \tag{E.19}$$

Moreover

$$w^{new} = \max(w^{ex}, w^{flex}) \tag{E.20}$$

with

$$w^{ex} = (w^{ex}\Pi^{-1}(\Pi)^{\delta})^{\lambda}(w^{flex})^{1-\lambda}\phi. \tag{E.21}$$

We consider a steady state in which $w^{flex} \leq w^{ex}$ and therefore $w^{new} = w^{ex} = \bar{w}$, with

$$\bar{w} = \frac{\alpha}{1+\gamma^b}\left(\frac{\epsilon-1}{\epsilon}A\right)^{\frac{1}{\alpha}}\left(\frac{q}{1-\alpha}\right)^{-\frac{1-\alpha}{\alpha}}.$$

In this steady state $\bar{\theta} \leq \bar{\theta}^*$, which requires ϕ to satisfy the inequality $\phi \geq \Pi^{\lambda(1-\delta)}$. Note that $\bar{\theta}^*$ is defined when $w^{flex} = w^{new} = w^{ex} = \bar{w}$, therefore, in this case, $\phi = \Pi^{\lambda(1-\delta)}$ whereas $w^{flex} = w^{new}$ implies

$$\bar{\theta}^* = \left[\frac{\alpha m\gamma^b}{\gamma^c(1+\gamma^b)}\left(\frac{\epsilon-1}{\epsilon}A\right)^{\frac{1}{\alpha}}\left(\frac{q}{1-\alpha}\right)^{-\frac{1-\alpha}{\alpha}}\right]^{\frac{1}{\eta}}.$$

In our approximation, we consider a steady state in which $\bar{\theta} < \bar{\theta}^*$, requiring, therefore, $\phi > \Pi^{\lambda(1-\delta)}$. For a given ϕ, satisfying the inequality, we can then use (E.19) into (E.21) noting that $w^{ex} = \bar{w}$, to determine $\bar{\theta}$. We can then combine (E.15) – (E.18) to obtain

$$\chi\bar{F}^{\omega} = (1-\bar{u})\bar{w}$$

$$\bar{u} = \frac{z}{1+m\bar{\theta}^{1-\eta}},$$

[80]We are generalizing the analysis by having z stochastic.

which determine $\bar{F}$ and $\bar{u}$, given $\bar{w}$ and $\bar{\theta}$. $\bar{N}$ is then determined by (E.18).

E.2.2 Derivation of equation (49)

In a log-linear approximation of equation (E.11), the AS equation is:

$$\pi_t - \pi = \frac{(\epsilon - 1)}{\varsigma}(\hat{\mu}_t + \alpha(\hat{w}_t^{new} + d_\gamma \hat{\gamma}_t^b) - \hat{A}_t + (1-\alpha)\hat{q}_t) + \beta E_t(\pi_{t+1} - \pi), \tag{E.22}$$

in which $d_\gamma \equiv \gamma^b/(1+\gamma^b)$.

Consider first the case in which $\theta_t \geq \theta_t^*$ and $w_t^{new} = w^{ex}$, then it follows that $\hat{w}_t^{new} = -c_w + \hat{w}_t^{flex}$ where $c_w = \ln(\bar{w}/\bar{w}^{flex})$, with $c_w \geq 0$ and, in particular, $c_w = 0$ whenever the steady-state approximation is taken at the kink point, $\bar{\theta} = \bar{\theta}^*$. A log-linear approximation of equation (E.13) implies that:

$$\hat{w}_t^{flex} = \eta\hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t. \tag{E.23}$$

Using (E.23) into the AS equation, we obtain:

$$\pi_t - \pi = -c + \kappa^{tight}\hat{\theta}_t + \kappa_\nu^{tight}(\hat{\nu}_t + \hat{\vartheta}_t^{tight}) + \beta E_t(\pi_{t+1} - \pi), \tag{E.24}$$

given the following parameters

$$\kappa_c^{tight} = \frac{(\epsilon-1)\alpha}{\varsigma}$$
$$\kappa^{tight} = \frac{(\epsilon-1)\alpha\eta}{\varsigma},$$
$$\kappa_\nu^{tight} = \frac{(\epsilon-1)}{\varsigma},$$
$$c = \kappa_c^{tight} c_w,$$

having defined

$$\hat{\nu}_t \equiv \hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t,$$
$$\hat{\vartheta}^{tight} = \alpha(\hat{\gamma}_t^c - (1-d_\gamma)\hat{\gamma}_t^b - \hat{m}_t).$$

Now consider the state in which $\theta_t < \theta_t^*$ and $w_t^{new} = w_t^{ex}$, then it follows that $\hat{w}_t^{new} = \hat{w}_t^{ex}$, and therefore using (E.14) that:

$$\begin{aligned}
\hat{w}_t^{new} &= \lambda\hat{w}_{t-1} - \lambda(\pi_t - \pi) + \lambda\delta E_t(\pi_{t+1} - \pi) + (1-\lambda)\hat{w}_t^{flex} + \hat{\phi}_t \\
&= \lambda\hat{w}_{t-1} - \lambda(\pi_t - \pi) + \lambda\delta E_t(\pi_{t+1} - \pi) + (1-\lambda)(\eta\hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t) + \hat{\phi}_t,
\end{aligned}$$

in which we have used $\hat{w}_{t-1}$ in place of $\hat{w}_{t-1}^{ex}$ and equation (E.13).

We can then substitute the wage norm into (E.22) to write it as

$$\pi_t - \pi = \frac{(\epsilon-1)}{\zeta}\Big\{\hat{\mu}_t + \alpha[\lambda\hat{w}_{t-1} - \lambda(\pi_t - \pi) + \lambda\delta E_t(\pi_{t+1} - \pi) + d_\gamma \hat{\gamma}_t^b +] + $$
$$+(1-\lambda)(\eta\hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t) + \hat{\phi}_t - \hat{A}_t + (1-\alpha)\hat{q}_t\Big\} + \beta E_t(\pi_{t+1} - \pi),$$

which can be written more compactly as

$$\pi_t - \pi = \kappa_w \hat{w}_{t-1} + \kappa\hat{\theta}_t + \kappa_\nu(\hat{\nu}_t + \hat{\vartheta}_t) + \kappa_\beta E_t(\pi_{t+1} - \pi), \tag{E.25}$$

given the following parameters

$$\kappa_w = 1 - \psi,$$
$$\kappa = (1-\lambda)\psi\kappa^{tight},$$
$$\kappa_\nu = \psi\kappa_\nu^{tight},$$
$$\kappa_\beta = (1-\psi)\delta + \psi\beta,$$

with ψ being a positive parameter with $0 < \psi \leq 1$ defined as

$$\psi \equiv \frac{1}{1 + \frac{(\epsilon-1)}{\varsigma}\alpha\lambda}.$$

in which

$$\hat{\vartheta}_t = \alpha(1-\lambda)(\hat{\gamma}_t^c - \hat{m}_t) + \alpha\hat{\phi}_t + \alpha(d_\gamma - (1-\lambda))\hat{\gamma}_t^b.$$

Note that $\kappa < \kappa^{tight}$ and $\kappa_\nu < \kappa_\nu^{tight}$, since $0 < \psi \leq 1$ and $0 < \lambda \leq 1$.

Note that

$$\pi_t - \pi = -c + \kappa^{tight}\hat{\theta}_t + \kappa_\nu^{tight}(\hat{\nu}_t + \hat{\vartheta}_t^{tight}) + \beta E_t(\pi_{t+1} - \pi),$$

applies when $\hat{\theta}_t \geq \hat{\theta}_t^*$ while

$$\pi_t - \pi = \kappa_w \hat{w}_{t-1} + \kappa\hat{\theta}_t + \kappa_\nu(\hat{\nu}_t + \hat{\vartheta}_t) + \kappa_\beta E_t(\pi_{t+1} - \pi),$$

whenever $\hat{\theta}_t < \hat{\theta}_t^*$. A requirement for this to be a log-linear approximation is that c must be of the same order as the norm of the shocks.

F Derivations of Section 6

In Section 6, we make the assumption that the wage norm is

$$w_t^{new} = \begin{cases} w_t^{flex} & \text{for } \theta_t > \theta_t^* \\ \bar{w}^\lambda(\Pi_t^{-1})^\lambda(\Pi_{t+1}^e)^{\delta\lambda}(w_t^{flex})^{1-\lambda} & \text{for } \theta_t \leq \theta_t^*, \end{cases}$$

We take a first-order approximation of equations (14), (28), (29) to obtain

$$\omega\hat{F}_t + \hat{\chi}_t = \frac{1-z}{1-\bar{u}}\hat{w}_t^{ex} + \frac{z-\bar{u}}{1-\bar{u}}\hat{w}_t^{new} - \frac{\bar{u}}{1-\bar{u}}\hat{u}_t, \tag{F.26}$$

$$\hat{N}_t = \hat{F}_t - \frac{\bar{u}}{1-\bar{u}}\hat{u}_t \tag{F.27}$$

$$\hat{u}_t = \hat{z}_t - \frac{z-\bar{u}}{z}(\hat{m}_t + (1-\eta)\hat{\theta}_t). \tag{F.28}$$

Note that we are also allowing for time variations in z. We can combine (F.27) and (F.26) to obtain

$$\hat{N}_t = \frac{1}{\omega}\left(\frac{1-z}{1-\bar{u}}\hat{w}_t^{ex} + \frac{z-\bar{u}}{1-\bar{u}}\hat{w}_t^{new}\right) - \frac{1}{\omega}\hat{\chi}_t - \frac{1+\omega}{\omega}\frac{\bar{u}}{1-\bar{u}}\hat{u}_t, \tag{F.29}$$

which holds independently of the wage mechanism.

Consider first the case when $\theta_t \leq \theta_t^*$ and therefore

$$\hat{w}_t^{new} = \hat{w}_t^{ex} = (1-\lambda)(\eta\hat{\theta}_t - \hat{m}_t) - \lambda((\pi_t - \pi) - \delta E_t(\pi_{t+1} - \pi)), \tag{F.30}$$

having set $\hat{\gamma}_t^c = \hat{\gamma}_t^b = \hat{\phi}_t = 0$.

Note that

$$\hat{N}_t = \frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t), \tag{F.31}$$

having set $\hat{O}_t = 0$. We can then plug (F.28), (F.30) and (F.31) into (F.29) to obtain

$$\begin{aligned}\frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t) \quad &= \quad \frac{1}{\omega}\left((1-\lambda)(\eta\hat{\theta}_t - \hat{m}_t) - \lambda((\pi_t - \pi) - \delta E_t(\pi_{t+1} - \pi))\right) - \frac{1}{\omega}\hat{\chi}_t \\ &\quad -\frac{1+\omega}{\omega}\frac{\bar{u}}{1-\bar{u}}\hat{z}_t + \frac{1+\omega}{\omega}\frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(\hat{m}_t + (1-\eta)\hat{\theta}_t)\end{aligned}$$

which can be simplified to obtain

$$\frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t) = \frac{d_\theta}{\omega}\hat{\theta}_t - \frac{\lambda}{\omega}((\pi_t - \pi) - \delta E_t(\pi_{t+1} - \pi)) - \frac{1}{\omega}\hat{\chi}_t - \frac{d_s}{\omega}\hat{z}_t + \frac{d_m}{\omega}\hat{m}_t$$

in which we have defined

$$d_\theta \equiv (1-\lambda)\eta + \frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(1+\omega)(1-\eta),$$

$$d_m \equiv -(1-\lambda) + \frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(1+\omega),$$

$$d_s \equiv (1+\omega)\frac{\bar{u}}{1-\bar{u}}.$$

Therefore, we can write

$$\hat{\theta}_t = \frac{\lambda}{d_\theta}((\pi_t - \pi) - \delta E_t(\pi_{t+1} - \pi) + \frac{1}{d_\theta}\hat{\chi}_t + \frac{d_s}{d_\theta}\hat{z}_t - \frac{d_m}{d_\theta}\hat{m}_t + \frac{\omega}{\alpha d_\theta}(\hat{Y}_t - \hat{A}_t).$$

Using it into (E.25) we can obtain

$$
\begin{aligned}
\pi_t - \pi &= \tilde{\kappa}\left(\hat{Y}_t + \frac{\alpha}{\omega}\hat{\chi}_t + \frac{\alpha d_s}{\omega}\hat{z}_t - \hat{A}_t - \frac{\alpha d_m}{\omega}\hat{m}_t\right) + \tilde{\kappa}_\nu\left(\hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t - \alpha(1-\lambda)\hat{m}_t\right) \\
&+ \tilde{\kappa}_\beta E_t(\pi_{t+1} - \pi),
\end{aligned}
$$

having defined:

$$
\tilde{\kappa} \equiv \frac{\frac{\omega\kappa}{\alpha d_\theta}}{1 - \frac{\kappa\lambda}{d_\theta}},
$$

$$
\tilde{\kappa}_\nu \equiv \frac{\kappa_\nu}{1 - \frac{\kappa\lambda}{d_\theta}},
$$

$$
\tilde{\kappa}_\beta \equiv \frac{\kappa_\beta - \frac{\delta\lambda\kappa}{d_\theta}}{1 - \frac{\kappa\lambda}{d_\theta}}.
$$

We now characterize the case in which $\theta_t > \theta_t^*$ where:

$$
\hat{w}_t^{new} = -c_w + \eta\hat{\theta}_t - \hat{m}_t \tag{F.32}
$$

$$
\hat{w}_t^{ex} = (1-\lambda)(\eta\hat{\theta}_t - \hat{m}_t) - \lambda(\pi_t - \pi - \delta E_t(\pi_{t+1} - \pi)) \tag{F.33}
$$

We can then plug (F.28), (F.31), (F.32), (F.33) into (F.29) to obtain

$$
\begin{aligned}
\frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t) &= \frac{1}{\omega}\frac{1-z}{1-\bar{u}}\left((1-\lambda)(\eta\hat{\theta}_t - \hat{m}_t) - \lambda(\pi_t - \pi - \delta E_t(\pi_{t+1} - \pi))\right) + \\
&+ \frac{1}{\omega}\frac{z-\bar{u}}{1-\bar{u}}(-c_w + \eta\hat{\theta}_t - \hat{m}_t) - \frac{1}{\omega}\hat{\chi}_t - \frac{d_s}{\omega}\hat{z}_t + \frac{1+\omega}{\omega}\frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(\hat{m}_t + (1-\eta)\hat{\theta}_t)
\end{aligned}
$$

which can be simplified to obtain

$$
\frac{1}{\alpha}(\hat{Y}_t - \hat{A}_t) = \frac{\tilde{d}_\theta}{\omega}\hat{\theta}_t - \frac{\lambda}{\omega}\frac{1-z}{1-\bar{u}}(\pi_t - \pi - \delta E_t(\pi_{t+1} - \pi)) - \frac{1}{\omega}\hat{\chi}_t - \frac{d_s}{\omega}\hat{z}_t + \frac{\tilde{d}_m}{\omega}\hat{m}_t + \frac{1}{\omega}\frac{z-\bar{u}}{1-\bar{u}}c_w
$$

in which we have defined

$$
\tilde{d}_\theta \equiv \left(\frac{1-z}{1-\bar{u}}(1-\lambda) + \frac{z-\bar{u}}{1-\bar{u}}\right)\eta + \frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(1+\omega)(1-\eta),
$$

$$
\tilde{d}_m \equiv -(1-\lambda)\frac{1-z}{1-\bar{u}} - \frac{z-\bar{u}}{1-\bar{u}} + \frac{z-\bar{u}}{1-\bar{u}}\frac{\bar{u}}{z}(1+\omega).
$$

Therefore, we can write

$$
\begin{aligned}
\hat{\theta}_t &= \frac{\lambda}{\tilde{d}_\theta}\frac{1-z}{1-\bar{u}}(\pi_t - \pi - \delta E_t(\pi_{t+1} - \pi)) + \frac{1}{\tilde{d}_\theta}\hat{\chi}_t + \frac{d_s}{\tilde{d}_\theta}\hat{z}_t - \frac{\tilde{d}_m}{\tilde{d}_\theta}\hat{m}_t + \frac{\omega}{\alpha\tilde{d}_\theta}(\hat{Y}_t - \hat{A}_t) \\
&- \frac{1}{\tilde{d}_\theta}\frac{z-\bar{u}}{1-\bar{u}}c_w.
\end{aligned}
$$

Using it into (E.24) we can obtain

$$\pi_t - \pi = -\tilde{c} + \tilde{\kappa}^{tight}\left(\hat{Y}_t + \frac{\alpha}{\omega}\hat{\chi}_t + \frac{\alpha d_s}{\omega}\hat{z}_t - \hat{A}_t - \frac{\alpha\tilde{d}_m}{\omega}\hat{m}_t\right) + \tilde{\kappa}_\nu^{tight}\left(\hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t - \alpha\hat{m}_t\right)$$
$$+\tilde{\kappa}_\beta^{tight} E_t(\pi_{t+1} - \pi),$$

having defined:

$$\tilde{\kappa}^{tight} \equiv \frac{\frac{\omega}{\alpha}\frac{\kappa^{tight}}{\tilde{d}_\theta}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{d}_\theta}\frac{1-z}{1-\bar{u}}},$$

$$\tilde{\kappa}_\nu^{tight} \equiv \frac{\kappa_\nu^{tight}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{d}_\theta}\frac{1-z}{1-\bar{u}}},$$

$$\tilde{\kappa}_\beta^{tight} \equiv \frac{\beta - \frac{\lambda}{\tilde{d}_\theta}\frac{1-z}{1-\bar{u}}\delta\kappa^{tight}}{1 - \frac{\kappa^{tight}\lambda}{\tilde{d}_\theta}\frac{1-z}{1-\bar{u}}},$$

$$\tilde{c} \equiv -c - \frac{\kappa^{tight}}{\tilde{d}_\theta}\frac{z-\bar{u}}{1-\bar{u}}c_w.$$

Note that the steeper curve applies whenever $\theta_t > \theta_t^*$ ($\hat{\theta}_t > \hat{\theta}_t^*$) and, therefore, when $Y_t > Y_t^*$ ($\hat{Y}_t > \hat{Y}_t^*$) for an appropriately defined Y_t^* and $\hat{Y}_t^*$.

F.1 The 2020s

To characterize the 2020s, we consider a short run in which the relevant shocks are $\hat{G}_S > 0$, $\hat{\chi}_S > 0$, $\hat{q}_S > 0$, $\hat{z}_S > 0$, $\hat{m}_S < 0$ and allow for variations in the policy shock e_S. Shocks revert to zero in the long run. This is an absorbing state that occurs with probability $1-\tau$. In the long run $\hat{Y}_L = 0$ and $\pi_L = \pi^*$.

The short-run Euler equation, substituting for the policy rule, can accordingly be written as

$$\hat{Y}_S = \hat{G}_S + \tau(\hat{Y}_S - \hat{G}_S) - \sigma^{-1}(\pi^* + \phi_\pi(\pi_S - \pi^*) + e_S + (\rho-1)\hat{r}_t^e - \tau\pi_S - (1-\tau)\pi^*)$$

which implies:

$$\hat{Y}_S = \hat{G}_S - \sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}(\pi_S - \pi^*) - \frac{\sigma^{-1}}{1-\tau}e_S + \frac{\sigma^{-1}}{1-\tau}(1-\rho)\hat{r}_t^e,$$

and, therefore,

$$\hat{Y}_S = \hat{D}_S - \sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}(\pi_S - \pi^*),$$

having defined

$$\hat{D}_S \equiv \hat{G}_S - \frac{\sigma^{-1}}{1-\tau}e_S + \frac{\sigma^{-1}}{1-\tau}(1-\rho)\hat{r}_t^e,$$

while the Inv-L NK Phillips curve is:

$$\pi_S = \begin{cases} -\tilde{c} + \tilde{\kappa}^{tight}\left(\hat{Y}_S + \frac{\alpha_n}{\omega}\hat{\chi}_S + \frac{\alpha d_s}{\omega}\hat{z}_S - \frac{\alpha \tilde{d}_m}{\omega}\hat{m}_S\right) + \tilde{\kappa}_\nu^{tight}\left((1-\alpha)\hat{q}_S - \alpha\hat{m}_S\right) + \tau\pi_S + (1-\tau)\pi^* & \hat{Y}_t \geq \hat{Y}_t^* \\ \tilde{\kappa}\left(\hat{Y}_S + \frac{\alpha_n}{\omega}\hat{\chi}_S + \frac{\alpha d_s}{\omega}\hat{z}_S - \frac{\alpha \tilde{d}_m}{\omega}\hat{m}_S\right) + \tilde{\kappa}_\nu\left((1-\alpha)\hat{q}_S - \alpha\hat{m}_S\right) + \tau\pi_S + (1-\tau)\pi^* & \hat{Y}_t < \hat{Y}_t^* \end{cases}$$

where $\hat{Y}_t^*$ is the threshold for output at which point the curve changes slope. We can also write the above equation as:

$$\pi_S = \begin{cases} -\frac{\tilde{c}}{1-\tau} + \frac{\tilde{\kappa}^{tight}}{1-\tau}\left(\hat{Y}_S + \frac{\alpha_n}{\omega}\hat{\chi}_S + \frac{\alpha d_s}{\omega}\hat{z}_S - \frac{\alpha \tilde{d}_m}{\omega}\hat{m}_S\right) + \frac{\tilde{\kappa}_\nu^{tight}}{1-\tau}\left((1-\alpha)\hat{q}_S - \alpha\hat{m}_S\right) + \pi^* & \hat{Y}_t \geq \hat{Y}_t^* \\ \frac{\tilde{\kappa}}{1-\tau}\left(\hat{Y}_S + \frac{\alpha_n}{\omega}\hat{\chi}_S + \frac{\alpha d_s}{\omega}\hat{z}_S - \frac{\alpha \tilde{d}_m}{\omega}\hat{m}_S\right) + \frac{\tilde{\kappa}_\nu}{1-\tau}\left((1-\alpha)\hat{q}_S - \alpha\hat{m}_S\right) + \pi^* & \hat{Y}_t < \hat{Y}_t^* \end{cases}.$$

Combining aggregate demand and aggregate supply we obtain the equilibrium inflation rate as

$$\pi_S - \pi^* = \begin{cases} \left(\frac{\hat{D}_S + \frac{\alpha}{\omega}\hat{\chi}_S + \frac{\alpha d_S}{\omega}\hat{d}_S - \frac{\alpha\tilde{d}_m}{\omega}\hat{m}_S + \frac{\tilde{\kappa}_\nu^{tight}}{\tilde{\kappa}^{tight}}((1-\alpha)\hat{q}_S - \alpha\hat{m}_S) - \frac{1}{\tilde{\kappa}^{tight}}\tilde{c}}{\frac{1-\tau}{\tilde{\kappa}^{tight}} + \sigma^{-1}\frac{\phi\pi - \tau}{1-\tau}}\right) & \hat{Y}_t \geq \hat{Y}_t^* \\ \left(\frac{\hat{D}_S + \frac{\alpha}{\omega}\hat{\chi}_S + \frac{\alpha d_S}{\omega}\hat{z}_S - \frac{\alpha\tilde{d}_m}{\omega}\hat{m}_S + \frac{\tilde{\kappa}_\nu}{\tilde{\kappa}}((1-\alpha)\hat{q}_S - \alpha\hat{m}_S)}{\frac{1-\tau}{\tilde{\kappa}} + \sigma^{-1}\frac{\phi\pi - \tau}{1-\tau}}\right) & \hat{Y}_t < \hat{Y}_t^* \end{cases}$$

showing that inflation will be higher when the curve is steeper, following demand shocks and oil (energy) shock.

F.2 The 1970s

To characterize the 1970s, we consider a short run in which there is a positive oil price shock, $\hat{q}_S > 0$, and we also allow for a shock to the policy rate e_S. Shocks revert to normal in the long run, which is an absorbing state that occurs with probability $1-\tau$. In the long run $\hat{Y}_L = 0$ and $\pi_L = \pi^*$ in which π^* is the central bank inflation target. In the short run, however, we also assume that private agents fear that the central bank may have changed its long-run inflation target, so their belief is $\pi_L^e > \pi^*$.

In this case, the short-run Euler equation, substituting for the policy rule, is given by

$$\hat{Y}_S = \tau\hat{Y}_S - \sigma^{-1}(\pi^* + \phi_\pi(\pi_S - \pi^*) + e_S - \tau\pi_S - (1-\tau)\pi_L^e)$$

which implies that

$$\hat{Y}_S = -\sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}(\pi_S - \pi^*) - \frac{\sigma^{-1}}{1-\tau}e_S + \sigma^{-1}(\pi_L^e - \pi^*).$$

while the Inv-L NK Phillips curve is:

$$\pi_S = \begin{cases} -\tilde{c} + \tilde{\kappa}^{tight}\hat{Y}_S + \tilde{\kappa}_\nu^{tight}(1-\alpha)\hat{q}_S + \tau\pi_S + (1-\tau)\pi_L^e & \hat{Y}_t \geq \hat{Y}_t^* \\ \tilde{\kappa}\hat{Y}_S + \tilde{\kappa}_\nu(1-\alpha)\hat{q}_S + \tau\pi_S + (1-\tau)\pi_L^e & \hat{Y}_t < \hat{Y}_t^* \end{cases}$$

implying

$$\pi_S - \pi^* = \begin{cases} -\frac{\tilde{c}}{1-\tau} + \frac{\tilde{\kappa}^{tight}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_\nu^{tight}}{1-\tau}(1-\alpha)\hat{q}_S + (\pi_L^e - \pi^*) & \hat{Y}_t \geq \hat{Y}_t^* \\ \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \frac{\tilde{\kappa}_\nu}{1-\tau}(1-\alpha)\hat{q}_S + (\pi_L^e - \pi^*) & \hat{Y}_t < \hat{Y}_t^* \end{cases}.$$

During the 1970s, θ was below the unitary value, so that $\theta_t < \theta^*$ and $\hat{Y}_t < \hat{Y}_t^*$. Therefore, the inflation rate, looking at the flat segment of the Inv-L NK curve, is given by

$$\pi_S - \pi^* = \frac{\frac{\tilde{\kappa}_\nu}{\tilde{k}}(1-\alpha)\hat{q}_S - \frac{\sigma^{-1}}{1-\tau}e_S + \sigma^{-1}(\pi_L^e - \pi^*) + \frac{1-\tau}{\tilde{\kappa}}(\pi_L^e - \pi^*)}{\frac{1-\tau}{\tilde{\kappa}} + \sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}}.$$

Short-run inflation is pushed above the target by the supply shock, the disanchoring of inflation expectations, and an accommodative monetary policy.

F.3 The 2008 missing disinflation

To characterize the 2008 missing disinflation, we consider a shock to the natural rate of interest that brings the policy rate at the zero-lower bound. The shock reverts to zero in the long run. This is an absorbing state that occurs with probability $1-\tau$. In the long run $\hat{Y}_L = 0$ and $\pi_L = \pi^*$.

Note that the interest rate policy is bounded by the zero lower bound and then given by

$$\hat{\imath}_S = \max(\rho\hat{r}_S^e + \phi_\pi(\pi_S - \pi^*), -\ln(1+i)),$$

in which i is the steady-state interest rate. Under the condition

$$\hat{r}_S^e > \frac{\phi_\pi(\pi^* - \pi_S) - \ln(1+i)}{\rho}$$

the interest rate is positive and the zero lower bound is not binding. In this case, the short-run aggregate demand is given by

$$\hat{Y}_S = \tau\hat{Y}_S - \sigma^{-1}(\pi^* + (\rho-1)\hat{r}_S^e + \phi_\pi(\pi_S - \pi^*) - \tau\pi_S - (1-\tau)\pi^*),$$

which implies:

$$\hat{Y}_S = \sigma^{-1}\frac{(1-\rho)\hat{r}_S^e}{1-\tau} - \sigma^{-1}\frac{\phi_\pi - \tau}{1-\tau}(\pi_S - \pi^*). \tag{F.34}$$

On the contrary, when the zero lower bound is binding, the AD equation is

$$\hat{Y}_S = \tau\hat{Y}_S - \sigma^{-1}(\pi^* - \ln(1+i) - \tau\pi_S - (1-\tau)\pi^* - \hat{r}^e_S)$$

and therefore

$$\hat{Y}_S = \sigma^{-1}\frac{\ln(1+i) + \hat{r}^e_S}{1-\tau} + \sigma^{-1}\frac{\tau}{1-\tau}(\pi_S - \pi^*) \tag{F.35}$$

showing a positive relationship between output and inflation.

The AS equation characterizing the missing disinflation is that on the flat part of the inv-L NK Phillips curve and is given by:

$$\pi_S = \tilde{\kappa}\hat{Y}_S + \tau\pi_S + (1-\tau)\pi^*$$

and therefore

$$\pi_S = \frac{\tilde{\kappa}}{1-\tau}\hat{Y}_S + \pi^*.$$

The economy starts from a condition in which $\hat{r}^e_S = 0$ and the equilibrium is on the downward part of the AD equation (F.34) at the intersection with the AS equation resulting in $\hat{Y}_S = 0$ and $\pi_S = \pi^*$. With a negative shock on $\hat{r}^e_S$ the equilibrium moves at the intersection of the upward sloping part of the AD equation at intersection with the AS equation. The equilibrium inflation rate is given by:

$$\pi_S = \pi^* + \sigma^{-1}\tilde{\kappa}\frac{\ln(1+i) + \hat{r}^e_S}{(1-\tau)^2 - \sigma^{-1}\tau\tilde{\kappa}},$$

which is pushed downward below the target by the negative shock to $\hat{r}^e_S$.