# Gibson's Paradox and the Natural Rate of Interest*

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**Abstract**

Gibson's paradox—the strong positive correlation between the price level and long-term nominal interest rates observed under the Gold Standard—is not inherently linked to the Gold Standard per se. Rather, it originates from low-frequency movements in the natural rate of interest (commonly referred to as $r^*$) under monetary regimes in which inflation is strongly mean-reverting and has an approximately zero mean. While the Gold Standard is the only historical instance of such a regime, Gibson's paradox can in principle emerge under a broader class of monetary arrangements. Indeed, once the deterministic component of the price level's drift is removed, the same co-movement patterns can be recovered from data generated under contemporary inflation-targeting regimes. In line with the inefficiencies highlighted by the literature on metallic standards, this finding suggests that modern regimes are not immune to criticism, as fluctuations in $r^*$ account for significant portions of the variation of inflation and output.

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# 1 Introduction

Gibson's paradox, the strong positive correlation between the price level and long-term nominal interest rates that had prevailed under the Gold Standard, is one of the most robust stylized facts in empirical macroeconomics. It is also one of the most mysterious since, in spite of more than a century of theoretical and empirical investigations, there is no widespread consensus on what, exactly, had produced it. Figure 1*a* shows the price level and a long-term nominal rate for the United Kingdom under the Gold Standard, whereas Figure 1*b* shows the same evidence for six additional countries: a strong positive correlation between the two series is near-uniformly apparent.

Writing in the mid-1970s, Friedman and Schwartz (1976, p. 288) pointed out that '*[t]he Gibson paradox remains an empirical phenomenon without a theoretical explanation.*' Following Friedman and Schwartz's (1982) observation about the temporal coincidence of Gibson's paradox with the Gold Standard era, the only broad agreement in the literature appears to be that, since the paradox had only appeared under the Gold Standard, and it has instead been absent from post-WWII data (see Figure 1*d*), it had likely originated from the peculiar workings of monetary regimes based on commodity money.[1]

A primary reason why Gibson's paradox has long occupied some of the profession's brightest minds—including Wicksell, Fisher, Keynes, Friedman, Schwartz, and Sargent—is that it appears to contradict a key tenet of macroeconomics, i.e. that expected inflation should be 'priced in' nominal interest rates. This notion is captured by the Fisher equation, which lies at the heart of any linearized general equilibrium model:

$$i_t = r_t + E_t(p_{t+1} - p_t),$$

where $i_t$ is the nominal rate, $r_t$ the real rate, $p_t$ the logarithm of the price level, $t$ denotes the time period, and $E_t$ is the time-$t$ expectation operator.

Taken at face value, the Fisher equation implies a positive relationship between the nominal interest rate and expected inflation, rather than the current price level. Indeed, for a given expectation of the future price level, the relationship between $i_t$ and $p_t$ is *negative*. As originally suggested by Fisher (1930), the relationship could become positive if inflation were positively serially correlated, so that expected inflation would depend positively on current inflation. Sargent (1973), however, refuted Fisher's explanation as inconsistent with the stochastic properties of inflation under the Gold Standard: as it has been extensively documented, indeed, under that monetary regime inflation had been statistically indistinguishable from a zero-mean white noise process (see e.g. Barsky, 1987, and Benati, 2008). Finally, the mystery deepens when one considers that, as we document below, under the Gold Standard the correlation between interest rates and prices had been especially strong based on

[1] See e.g. the discussion in Barsky and Summers (1988).

long rates and at the very *low frequencies*.[2]

The preceding observations allow for one, and *only one* possible explanation that is compatible with the Fisher equation: the variation in nominal interest rates underlying Gibson's paradox ought to originate from fluctuations in the real interest rate $r_t$, i.e. what in the recent literature has been labelled as the natural rate of interest. Further, the fact that the paradox had been especially apparent based on long rates, and at the very low frequencies, logically suggests that it originates from highly persistent shocks to the natural rate of interest.

Our first contribution to the literature is to demonstrate that this simple intuition is indeed correct. We develop a straightforward theoretical mechanism through which persistent fluctuations in the natural rate of interest generate Gibson's paradox under the Gold Standard, and we present strong empirical support based on either structural VAR (SVAR) methods or estimated DSGE models.

This finding is most intriguing because it *logically* suggests that—contrary to the profession's dominant view following Friedman and Schwartz (1982)—the paradox has *nothing* to do with the Gold Standard per se. Rather, it may originate from low-frequency fluctuations in the natural rate of interest under *any* monetary regime that makes inflation strongly *mean-reverting*. The implication is that Gibson's paradox could in principle appear under a wide array of monetary arrangements.

Consistent with this insight, our second contribution is to show, based on either SVAR methods, or estimated DSGE models, that the inflation-targeting regimes that have become widespread since the early 1990s exhibit the very same phenomenon: highly persistent shocks to the natural rate of interest generate a strong positive long-horizon correlation between prices and nominal interest rates. A crucial point here is that, although Gibson's paradox is present under these regimes, a correlation between interest rates and prices is not visible in the raw data (as it had instead been under the Gold Standard), due to the deterministic upward drift in the price level induced by a positive inflation target. This logically suggests that, once removing such deterministic drift from the price level, Gibson's paradox should manifest itself. We show that this is in fact the case, and very starkly so.

The discussion so far naturally suggests that whether Gibson's paradox is, or is not in the data—even in a 'hidden' form as under inflation targeting—hinges in a crucial, although not exclusive manner on the nature of the monetary regime. In turn, this suggests exploring the relationship between prices and nominal interest rates under a variety of alternative monetary regimes, such as price level targeting, nominal GDP targeting, and monetary targeting.

Unfortunately, throughout recorded history, none of these regimes have been implemented in a sufficiently 'pure' and stable form over a prolonged period. For instance, although Sweden officially adopted a price level targeting regime after aban-

[2]By itself, the fact that the correlation is especially strong based on long-term interest rates—which encode expectations of future short-term rates—highlights the fact that Gibson's paradox should be regarded as a long-horizon phenomenon.

doning the Gold Standard in 1931, the Riksbank effectively began 'shadowing' the British pound sterling from 1933 onward (see Jonung, 1979).[3] Because of this, we conduct our exploration uniquely by appropriately modifying the monetary policy rule within estimated DSGE models. Our evidence suggests that Gibson's paradox would not appear under either price level or nominal GDP targeting, whereas it would indeed manifst itself under a policy of targeting the (log) level of the money stock.

Our work not only resolves a long-standing puzzle in monetary economics, but it also demonstrates its relevance for contemporary monetary policy analysis. The natural rate of interest (usually denoted $r^*$) plays a central role in the current debate about monetary policy within either academia or policymaking circles, as well as in the financial press. Central bankers are routinely questioned by journalists about their views of $r^*$, in order to gauge an idea about their future policy intentions. This emphasis on $r^*$ is deeply rooted in the 'Neo-Wicksellian' framework for monetary policy analysis that is dominant within the New Keynesian literature (see Woodford, 2003). This approach contends that under optimal policy the actual real rate should mimic the natural rate of interest, since rates above $r^*$ are contractionary, whereas rates below $r^*$ are expansionary. A *logical* implication of this position is that the appearance of Gibson's paradox in the raw data reflects the sub-optimality of monetary policies.

In fact, based on SVAR methods we show that fluctuations in the natural rate of interest had, and have driven significant fractions of business-cycle frequency fluctuations in key macroeconomic aggregates under either the Gold Standard or inflation targeting regimes. In the light of the long-standing criticism of the inflexibility of the Gold Standard (see e.g. Keynes, 1925, and Eichengreen, 1996), the evidence for this regime is hardly surprising. More intriguing is the fact that qualitatively similar evidence holds for inflation targeting regimes, because it shows that, in fact, actual policymaking under these regimes has deviated from the notion of optimality discussed in the recent New Keynesian literature.

Finally, our analysis also provides a powerful illustration of the progress made by our discipline over the last several decades from both a theoretical and an empirical point of view. Only this progress—in particular, the developement of DSGE models, and of empirical methods such as SVARs—has allowed us to resolve a puzzle that, for decades, had consistently eluded some of the profession's brightest minds, who had been compelled to rely on significantly less powerful techniques (if any).

[3]By the same token, the policy of money growth targeting introduced by the Bundesbank following the collapse of Bretton Woods was characterized until the end of the 1980s by repeated changes in the specific monetary aggregate that was being targeted, and a progressive downward drift in the numerical targets for the growth rates.

## 1.1 The mechanism underlying Gibson's paradox

Intuitively, the mechanism at the root of Gibson's paradox under the Gold Standard hinges on the interaction between (1) the Fisher equation, described above, and (2) an asset-pricing condition that determines the current value of money as the discounted expected future stream of liquidity services. The Fisher equation translates persistent fluctuations in the natural rate of interest into corresponding fluctuations in nominal interest rates at all maturities. The asset-pricing condition, on the other hand, implies that an increase in the natural rate—which is the discount factor for money's future expected liquidity services—causes a decrease in the expected present value of those services, which is obtained via an increase in the current price level. Persistent increases (decreases) in the natural rate therefore generate corresponding increases (decreases) in both the price level (via the asset-pricing condition) and nominal rates (via the Fisher equation).

We show that the same mechanism is also present under other monetary regimes, thus implying that Gibson's paradox is *not specific* to the Gold Standard, and it can also arise under alternative monetary frameworks.

An important point to stress is that, for the paradox to appear in the raw data, neither the Fisher equation, nor the asset-pricing condition, must be perturbed by features that weaken the link between prices and interest rates created by the natural rate of interest. For example a positive inflation target, by introducing an upward drift in prices, causes Gibson's paradox to become 'hidden' in the raw data, although the correlation between interest rates and prices can still be recovered by simply removing the deterministic component of the drift in the price level. By the same token, under price-level targeting—and more generally under regimes in which price-level shocks are strongly countered, or even ultimately neutralized—the paradox may be significantly weakened, or even disappear altogether from the data.

The paper is organized as follows. The next section documents the evolution of the relationship between (long-term) nominal interest rates and the price level since the early XVIII century, whereas Section 3 briefly reviews the previous literature on Gibson's paradox. Section 4 outlines a theory of Gibson's paradox based on standard general equilibrium models. Section 5 presents evidence from SVAR methods for both the Gold Standard and inflation targeting regimes. In Section 6 we explore the long-horizon relationship between prices and long-term nominal interest rates induced by alternative monetary policy rules. Section 7 shows that, once controlling for the deterministic component of the drift in the price level induced by the presence of a positive inflation target, a positive long-horizon relationship between the price level and long-term nominal interest rates can be recovered from the data generated by inflation-targeting regimes. Section 8 studies the share of the forecast error variance of macroeconomic time-series explained by shocks to the natural rate, thus highlighting the sub-optimality of both the Gold Standard and contemporary inflation-targeting regimes. Section 9 concludes.

# 2 Stylized Facts

Figures 1*a*-1*d* illustrate the evolution of the relationship between long-term nominal interest rates[4] and the price level since the early XVIII century. Gibson's paradox had been near-uniformly apparent in the data up until the outbreak of World War I, sometimes strikingly so.[5] This is the case in particular for the United Kingdom since 1850, Norway since 1822, Denmark since 1839, and the United States during the Classical Gold Standard period (January 1879-July 1914). Interestingly, for the United Kingdom a positive low-frequency co-movement between the two series is clearly apparent also in the data from the XVIII century, whereas the correlation had been weaker during the period between the re-establishment of the prewar gold parity following the end of the Napoleonic Wars, in May 1821, and the mid-XIX century.

In fact, the evidence for the Gold Standard period is so strong that in most cases statistical tests detect *cointegration* between the price level and long-term nominal interest rates. Specifically, Johansen's tests of the null of no cointegration, which is predicated on the assumption that the series feature exact unit roots,[6] uniformly detects cointegration based on monthly data,[7] with bootstrapped $p$-values[8] for the maximum eigenvalue tests ranging between 0.0000 and 0.0259. On the other hand, based on annual data the null hypothesis is never rejected.[9] By the same token, Wright's (2000) test,[10] which is valid for both exact and near unit roots, detects

[4] The data and their sources are discussed in detail in Online Appendix A. The interest rate series in Figure 1*a* are all rates on consols (i.e., perpetuities), whereas those in Figure 1*d* are yields on 10-year government bonds. As for the interest rates in Figures 1*b*-1*c*, the original sources near-uniformly label them as 'Yield on long-term government bonds', so that we do not know what exactly the maturity is. Given the very strong correlation between long-term nominal interest rates at different maturities, however, for our own purposes this is irrelevant.

[5] At first sight, a possible concern about the evidence reported in Figures 1*a*-1*c* is that, with near certainty, old price series are plagued by a non-negligible extent of measurement error (see e.g. Cogley and Sargent, 2015, and Cogley et al., 2015). Further, it can plausibly be assumed that the older the price indices, the greater the extent of measurement error they suffer from. (On the other hand, since interest rates had been quoted on financial markets, and their quotes had typically been recorded in official publications, the problem is likely virtually non-existent for long-term interest rates.) In fact, for the purpose of documenting the evolution of the long-horizon relationship between long-term interest rates and prices, since measurement error pertains to the price *level* its presence should not introduce any material distortion.

[6] For all countries and sample periods evidence from Elliot, Rothenberg and Stock (1996) tests suggests that the null hypothesis of a unit root cannot be rejected. We interpret these results as pointing towards either an exact or a near unit root.

[7] I.e., for the United Kingdom based on the series plotted in the second, third, and fourth panel of Figure 1*a*, and for the United States, Norway, and Germany based on the series plotted in Figure 1*b*.

[8] We bootstrap the $p$-values as in Cavaliere, Rahbek, and Taylor (2012).

[9] I.e., for the United Kingdom based on the series plotted in the first panel of Figure 1*a*, and for Denmark, France, and Canada based on the series plotted in Figure 1*b*.

[10] We bootstrap Wright's (2000) test via the procedure proposed by Benati, Lucas, Nicolini, and Weber (2021).

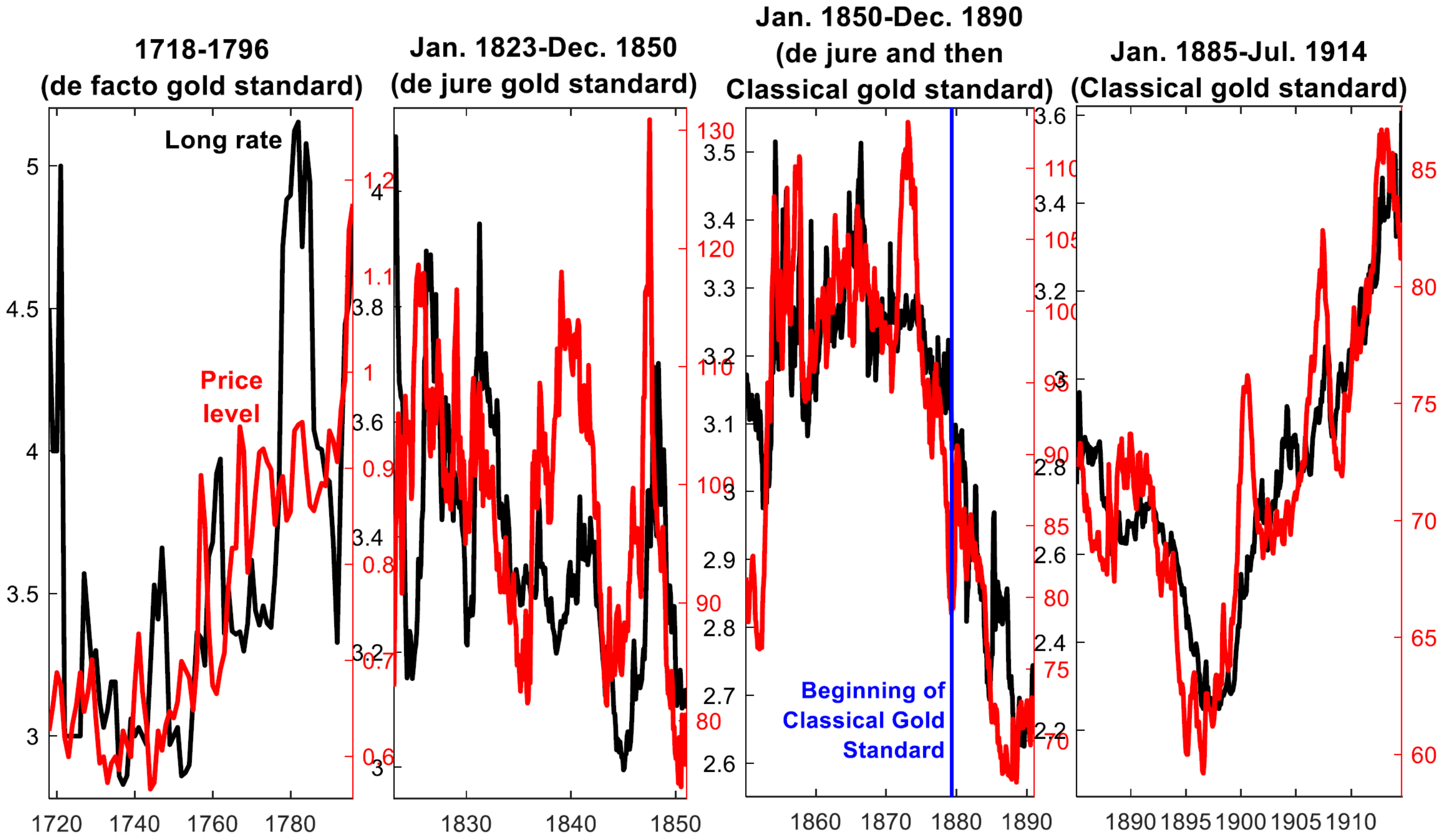


Figure 1a Gibson's paradox in the United Kingdom under the Gold Standard
(red: price level; black: long-term nominal interest rate)

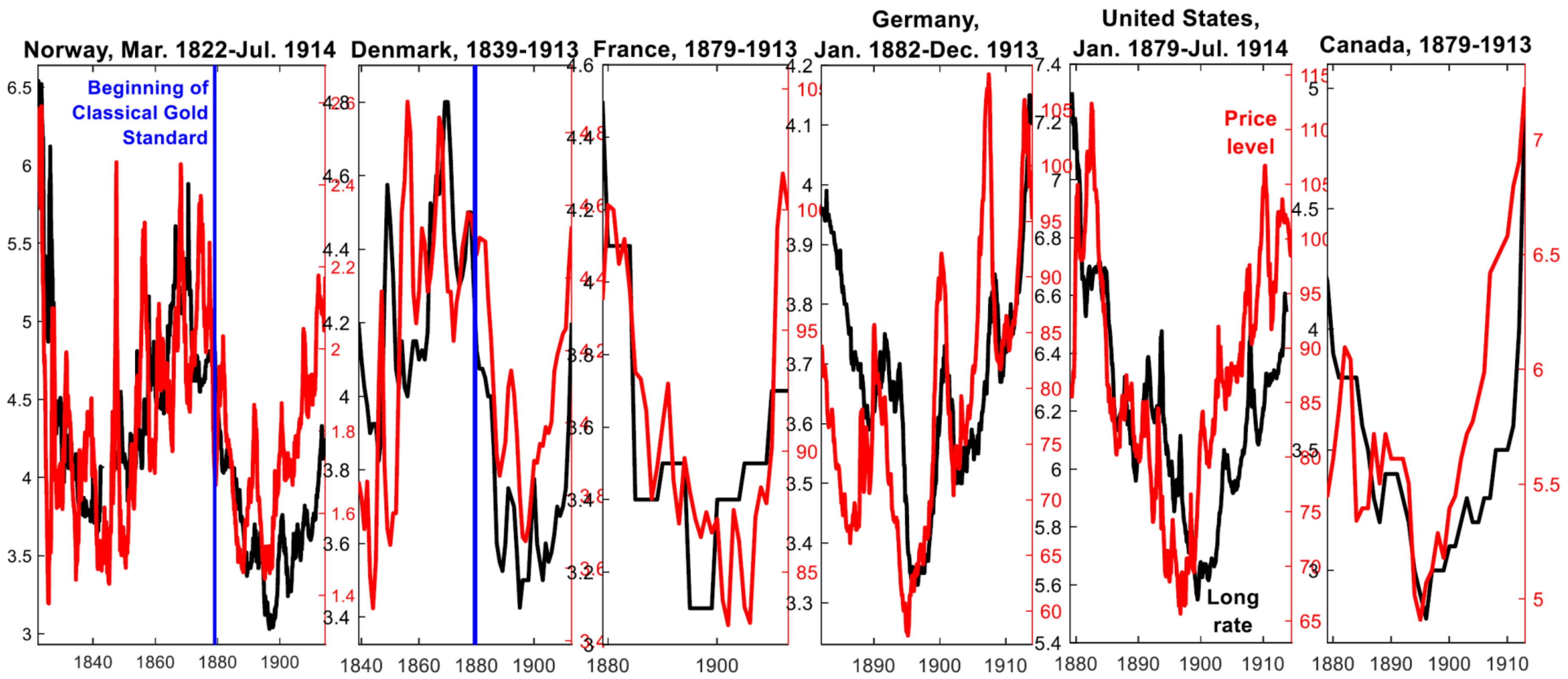


Figure 1b Gibson's paradox in other countries under the Gold Standard
(red: price level; black: long-term nominal interest rate)

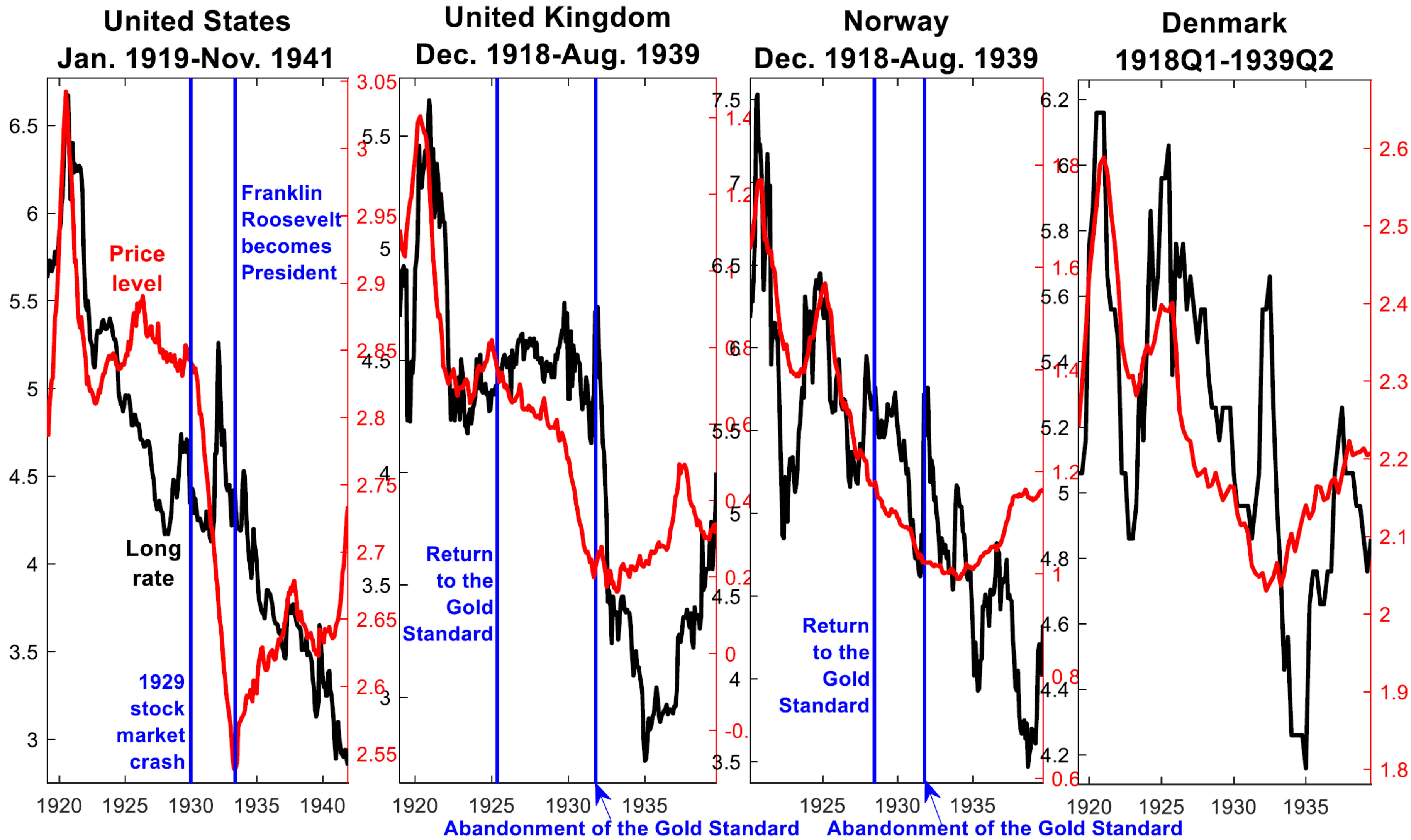


Figure 1c Gibson's paradox during the interwar period (red: price level; black: long-term nominal interest rate)

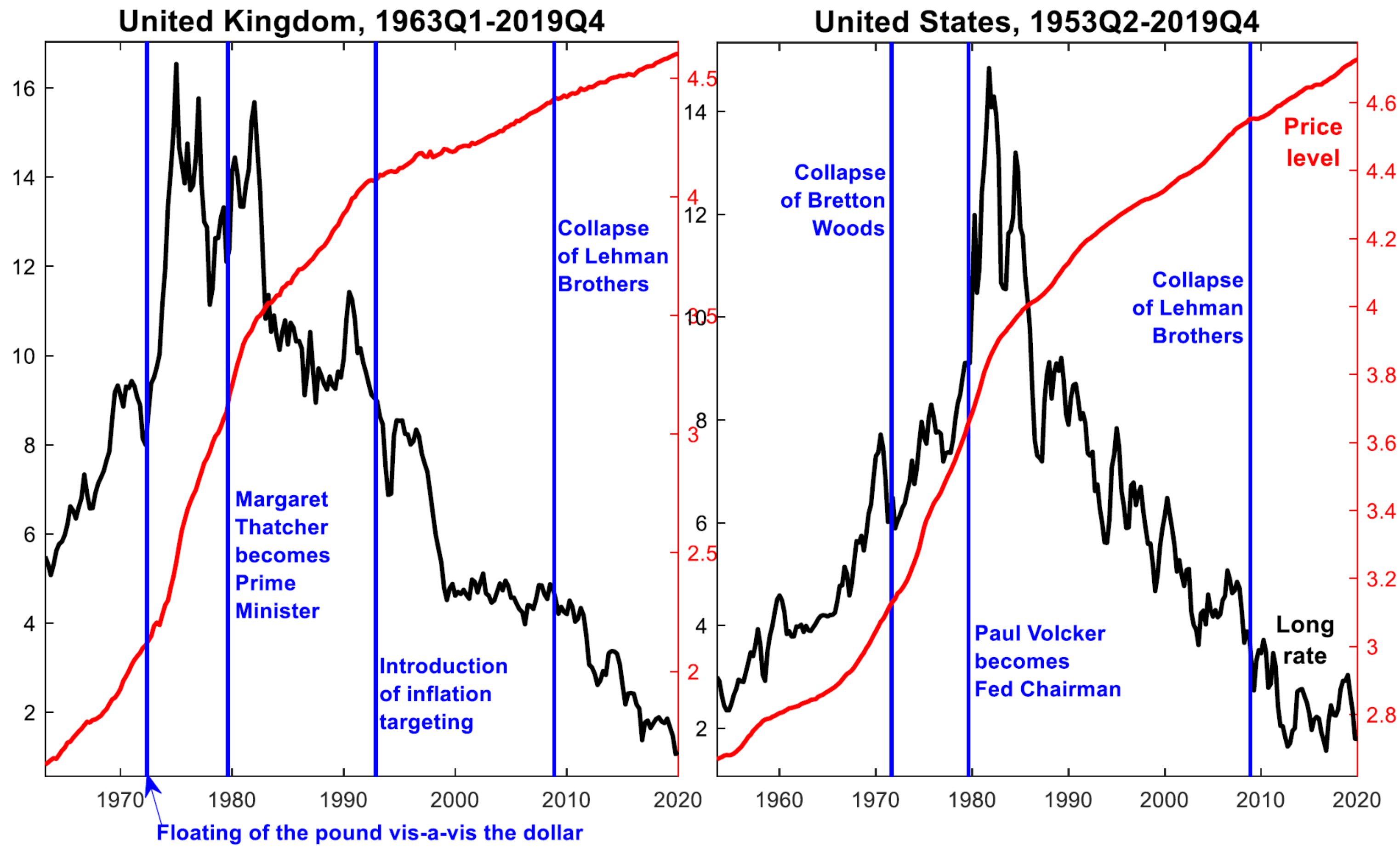


Figure 1d The price level and long-term nominal interest rates over the post-WWII period in the United Kingdom and the United States (red: price level; black: long-term nominal interest rate)

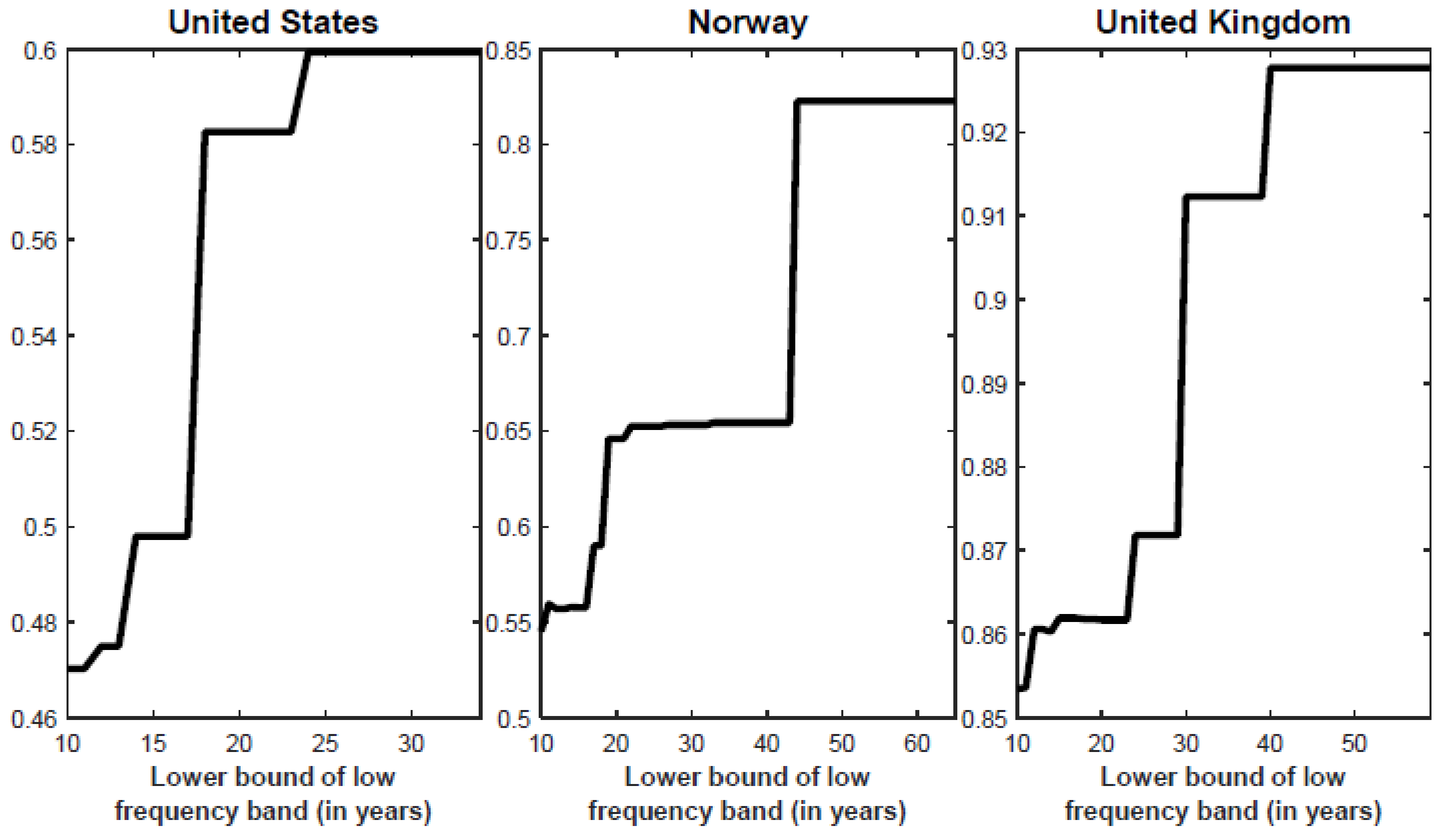


Figure 2 Evidence from Müller and Watson's low-frequency regressions: $R^2$ in the regression of log prices on the long-term nominal interest rate by frequency band

cointegration based on monthly data for all countries except Germany, whereas based on annual data it only detects it for France.[11]

Both the visual evidence in Figures 1*a*-1*b* and the previous discussion suggest that Gibson's paradox is a very low-frequency phenomenon, which however may also sometimes be apparent at higher frequencies. Figure 2 reports some simple evidence on this. The figure shows, for the United States, Norway, and the United Kingdom,[12] the $R^2$ by frequency band in the low-frequency regression of log prices on a long-term nominal interest rate based on the methodology proposed by Müller and Watson (2018, 2020). As the regression focuses on lower and lower frequencies the $R^2$ consistently increases. For example, for Norway it is equal to 'just' 0.55 for the frequency band associated with cycles slower then ten years, and it increases to 0.825 for cycles slower than 43 years or more. Findings for the other two countries are qualitatively the same.

Evidence for the interwar period is uniformly weaker, but overall it appears to still point towards a positive long-horizon co-movement between long-term rates and the price level. Any evidence of a long-horizon co-movement between the two series however vanished altogether following the end of World War II, with the price level acquiring a consistently positive drift, and long-term nominal interest rates exhibiting instead a hump-shaped pattern mostly attributable to the rise, and then fall of inflation associated with the Great Inflation episode, and in more recent years to the progressive decline of the natural rate of interest. In fact, as we will show in Section 7, the positive long-horizon correlation between the price level and long-term nominal interest rates that is the hallmark of Gibson's paradox can be recovered from the data generated by inflation-targeting regimes.

# 3 Previous Literature on Gibson's Paradox

Although Keynes (1930) labelled the paradox after Gibson (1923), in fact a long-horizon positive correlation between prices and long-term nominal interest rates had already been discussed by Tooke (1844) and Wicksell (1898, 1907). As we now discuss our explanation of Gibson's paradox, as intuitively outlined in Section 1.1, is entirely different from previous explanations that had been proposed in the literature.

Wicksell and Keynes proposed an explanation centered on the workings of the commercial banking system. In reaction to an increase in the productivity of capital, and therefore in the demand for credit, commercial banks increase lending, and therefore the money supply, but they only do so with a lag. As a result nominal

[11] Overall, our results therefore question the evidence of Corbae and Ouliaris (1989), who based on Phillips and Ouliaris's (1990) tests and asymptotic critical values do not detect cointegration for the United States and the United Kingdom.

[12] We focus on these three countries because they feature long samples of monthly data. Evidence for the other countries in Figure 1*b* is in line with that in Figure 2, with the partial exception of Denmark and Canada.

interest rates rise, but they consistently lag behind the natural rate of interest, and the resulting economic expansion leads to an increase in the price level. Cagan (1965) and Shiller and Siegel (1977) refuted Wicksell and Keynes' explanation on empirical grounds, by pointing out that an increase in the money supply due to an increase in bank lending increases the money multiplier, thus counterfactually inducing a positive correlation between the multiplier and the price level.[13]

Fisher (1930) argued that since expected inflation is positively correlated with prices, fluctuations in inflation expectations cause corresponding fluctuations in the same direction in both nominal interest rates and the price level. Several authors[14] expressed skepticism of this explanation, because Fisher's hypothesis that agents form inflation expectations based on inflation's past behavior implies that, in order to produce Gibson's paradox, they should be implausibly slow in adjusting their expectations in response to changes in actual inflation.[15] Further, from a rational expectations perspective a crucial problem with this explanation is that under the Gold Standard, during which Gibson's paradox had appeared, inflation had been near-uniformly indistinguishable from zero-mean white noise,[16] thus implying that expected inflation had been essentially constant at zero.[17]

Sargent (1973) builds an IS-LM-type model featuring inertia in the adjustment of wages and prices to their long-run equilibrium values in response to shocks. Following a permanent, one-off increase in the money supply at $t=t_0$ prices slowly increase towards their new long-run equilibrium, whereas the nominal interest rate jumps downwards at $t_0$ and then slowly reverts to its original value (see Figure 13, p. 443). During the transition between steady-states the model therefore generates Gibson's paradox. As stressed by Sargent (1973), *'[t]he key reason that the Gibson paradox may infest the data generated by the model is the failure of wages and prices to adjust sufficiently quickly to keep output always at its full-employment level.'* Under this respect our analysis does not rely on price or wage rigidities to explain the paradox, and in fact it could work in an idealised world with fully flexible prices.

Shiller and Siegel (1977) proposed an explanation based on the impact of unanticipated changes in the price level on the distribution of wealth between creditors and debtors. An unanticipated increase in the price level causes a decrease in the real value of nominal bonds, thus causing an increase in the real wealth of debtors, and

[13] Cagan (1965) further argued that, at least in the United States, changes in the money supply had originated to a dominant extent from changes in high-powered money, rather than changes in lending on the part of commercial banks. Jonung (1976) produced qualitatively the same evidence for Sweden.

[14] See Macauley (1938), Cagan (1965), and Sargent (1973).

[15] Fisher's (1930) own estimates implied that the average lag of expected inflation on actual inflation should have ranged between 7.3 and 10.7 years.

[16] See Barsky (1987) and Benati (2008).

[17] As pointed out by Sargent (1973), 'it is difficult *both* to accept Fisher's explanation of the Gibson paradox, *and* to maintain that the extraordinarily long lags in expectations are 'rational''. In fact, Sargent's own estimates suggested that for the United States during the period 1880-1914 the optimal prediction for inflation had been essentially zero.

a corresponding decrease in the real wealth of creditors. Assuming that both agents want to maintain a certain fraction of their wealth in either long or short positions in bonds, debtors, having become wealthier, want to increase their supply of bonds by more than the decrease in their real value. Symmetrically creditors, having become poorer, want to increase their holdings of bonds by less than the change in their real value. At the initial equilibrium interest rate, there is therefore an excess supply of bonds, and interest rates must rise in order to restore equilibrium in the market.

Friedman and Schwartz (1982, pp. 527-587) featured a detailed discussion of the literature up until the early 1980s. Following their observation about the temporal coincidence of Gibson's paradox with the Gold Standard, most subsequent authors have proposed explanations based on the peculiar workings of monetary regimes based on commodity money.

Lee and Petruzzi (1986) proposed an explanation based on the reallocation of wealth between gold and financial assets induced by fluctuations in real interest rates. In response to an increase in real, and therefore nominal interest rates, investors shift their wealth from gold to financial assets. In turn, the decrease in the demand for gold causes a fall in its real price in terms of goods, which obtains via an increase in the price level.

Building upon Barro's (1979) benchmark model of the Gold Standard, Barsky and Summers' (1988)[18] analysis is centered around the nature of gold as a very long-lived asset. An increase in the real interest rate, and therefore in nominal rates, causes a decrease in the demand for monetary gold via a standard money demand function. At the same time, by increasing the carrying cost of gold it decreases its demand for non-monetary purposes (jewelry, art, ...), which in the United Kingdom during the period between the end of the Napoleonic Wars and World War I was about two-thirds of the overall gold stock. The resulting decrease in the overall demand for gold causes, as in Lee and Petruzzi (1986), a decrease in the real price of gold, which is obtained via an increase in the price level. This mechanism constitutes one pillar of our explanation for Gibson's paradox under the Gold Standard, the other being the Fisher equation, which is ignored in their steady-state analysis. Crucially, whereas Barsky and Summers (1988) attribute the paradox uniquely to the peculiarities of the Gold Standard, our analysis demonstrates that it can arise under other monetary regimes as well, with fluctuations in the natural rate of interest being the fundamental driver.

Finally, some authors questioned the very existence of Gibson's paradox. Benjamin and Kochin (1984) for example argued that *'[i]n significant part the movements of both the interest rate and the price level have been produced by war. Once the influence of war is taken into account, there is virtually no evidence of any linkage between the price level and the long-term interest rate.'* Benjamin and Kochin's evidence was refuted by Barsky and Summers (1988). In fact, the evidence in Figures 1*a*-1*c* clearly

[18] Online Appendix D discusses the relationship between Barsky and Summers's (1988) analysis of the Gold Standard and ours.

speaks against Benjamin and Kochin's position.

Less drastically, some authors questioned the solidity of Gibson's paradox. Macaulay (1938), for example, stated that *'the exceptions to this appearance of relationship are so numerous and so glaring that they cannot be overlooked'* By the same token, Dwyer (1984) argued that for the period before World War I *'the only statistically significant correlations are the correlations of prices and short-term interest rates in the United States and France.'* Again, our evidence for the Gold Standard period, as well as (e.g.) Barsky and Summers' (1988), clearly refutes this position, and it rather supports Keynes' (1930) assertion that Gibson's paradox is *'one of the most completely established empirical facts in the whole field of quantitative economics'*.

We now turn to a theory of Gibson's paradox.

# 4 A Theory of Gibson's Paradox

## 4.1 A model of the Gold Standard

We consider a closed economy within a stochastic environment, where the representative agent is endowed with a perishable good, which is used for consumption, and a non-perishable commodity such as gold. Our framework is similar to previous models of the Gold Standard developed by Goodfriend (1988) and Jacobson, Leeper, and Preston (2019).

The representative agent maximizes her expected utility flow, which is given by

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\theta_t^c U(C_t)+\theta_t^g V(g_t^p)+\theta_t^m L\left(\frac{M_t}{P_t}\right)\right]\right\},$$

where $\beta$ is the utility discount factor, with $0<\beta<1$; $U(\cdot)$ is a concave function, with $C$ being the consumption of the perishable good; and $V(\cdot)$ is a concave function of the agent's gold holdings $g^p$. Gold provides direct utility to the representative agent. The agent also obtains liquidity services from holding real money balances via the utility function $L(\cdot)$. The function is concave and displays a satiation point at $\bar{m}$, meaning that $L_m(\cdot)=0$ for $M_t/P_t \geq \bar{m}$, where $L_m(\cdot)$ is the first derivative of the function $L(\cdot)$.[19] $M_t$ is the nominal money stock held at time $t$, and $P_t$ is the price of the consumption good. Finally, $\theta_t^c, \theta_t^g, \theta_t^m$ are preference disturbances.

The agent is subject to the flow budget constraint

$$B_t+M_t+P_{g,t}g_t^p+P_tC_t=B_{t-1}(1+i_{t-1})+M_{t-1}+P_{g,t}g_{t-1}^p+P_tY_t-T_t+P_{g,t}(G_t-G_{t-1})$$

[19]We assume that the second derivative of $L(\cdot)$ remains negative in the limit—see Woodford (2003)—in order to ensure a well-defined demand for real money balances as satiation is approached. Consequently, the function is only once differentiable, given that its second derivative at the satiation point is zero.

where $B_t$ are the holdings at time $t$ of risk-free nominal bonds denominated in units of currency, with interest rate $1+i_t$; $P_{g,t}$ is the price of gold in units of currency; $Y_t$ is the endowment of goods; $G_t$ is the stock of gold in the economy, with $G_t \geq G_{t-1}$; and $T_t$ are lump-sum taxes levied by the Treasury. The representative agent's problem is subject to an appropriate borrowing limit condition.

The first-order condition with respect to $B_t$ implies that

$$1 = \beta E_t \left\{ \frac{\theta_{t+1}^c U_c(C_{t+1})}{\theta_t^c U_c(C_t)} \frac{P_t}{P_{t+1}} \right\} (1+i_t); \tag{1}$$

the one with respect to $M_t$ is

$$\frac{\theta_t^c U_c(C_t)}{P_t} = \frac{\theta_t^m}{P_t} L_m \left( \frac{M_t}{P_t} \right) + \beta E_t \left\{ \frac{\theta_{t+1}^c U_c(C_{t+1})}{P_{t+1}} \right\}, \tag{2}$$

whereas that with respect to gold holdings is

$$\frac{P_{g,t}}{P_t} = \frac{\theta_t^g}{\theta_t^c} \frac{V_g(g_t^p)}{U_c(C_t)} + \beta E_t \left\{ \frac{\theta_{t+1}^c}{\theta_t^c} \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{P_{g,t+1}}{P_{t+1}} \right\}. \tag{3}$$

The set of first-order conditions is completed by the exhaustion of the intertemporal budget constraint.

The central bank and the Treasury are consolidated within the government. The central bank issues money $M_t$, a non-interest-bearing security, whereas the Treasury issues debt $B_t^g$ at the nominal interest rate $i_t$, and levies lump-sum taxes $T_t$. Their consolidated budget constraint is:

$$B_t^g + M_t = (1+i_{t-1})B_{t-1}^g + M_{t-1} - T_t.$$

Consider first the gold standard regime. The central bank issues money with full convertibility into gold, fixing the convertibility rate between gold and currency at a certain price $\bar{P}_g$, which without any loss of generality we can simply set to $\bar{P}_g = 1$. The central bank stands ready to buy and sell gold at the predetermined rate, thus automatically and proportionally expanding and contracting the money supply. Since $\bar{P}_g = 1$, we have that

$$M_t = g_t^c$$

where $g_t^c$ is the central bank's holdings of gold.

In equilibrium goods, gold and asset markets clear, i.e.

$$C_t = Y_t$$

$$g_t^p + g_t^c = G_t$$

and

$$B_t = B_t^g.$$

Substituting these equilibrium conditions into the first-order conditions (1)–(3), and using $P_{g,t} = 1$, we obtain, respectively, that

$$\frac{1}{1+i_t} = E_t \left\{ \beta \frac{\theta_{t+1}^c}{\theta_t^c} \frac{U_c(Y_{t+1})}{U_c(Y_t)} \frac{P_t}{P_{t+1}}, \right\} \tag{4}$$

$$\frac{1}{P_t} = \frac{1}{P_t} \frac{\theta_t^m}{\theta_t^c} \frac{L_m\left(\frac{g_t^c}{P_t}\right)}{U_c(Y_t)} + E_t \left\{ \beta \frac{\theta_{t+1}^c}{\theta_t^c} \frac{U_c(Y_{t+1})}{U_c(Y_t)} \frac{1}{P_{t+1}} \right\}, \tag{5}$$

$$\frac{1}{P_t} = \frac{\theta_t^g}{\theta_t^c} \frac{V_g(G_t - g_t^c)}{U_c(Y_t)} + E_t \left\{ \beta \frac{\theta_{t+1}^c}{\theta_t^c} \frac{U_c(Y_{t+1})}{U_c(Y_t)} \frac{1}{P_{t+1}} \right\}. \tag{6}$$

The stochastic sequences $\{i_t, P_t, g_t^c\}_{t=t_0}^{\infty}$ solve the equilibrium conditions (4)-(6) for given exogenous sequences $\{G_t, Y_t, \theta_t^g, \theta_t^m, \theta_t^c\}_{t=t_0}^{\infty}$. Finally, we want to highlight the important point that the central bank's gold holdings, $g_t^c$, are *endogenous*. This implies that, since $M_t = g_t^c$, the money stock is endogenous as well.

## 4.2 What had generated Gibson's paradox?

In order to understand which shocks are most likely to have generated the paradox, consider rewriting (4) as

$$\frac{1}{1+i_t} = E_t \left\{ \frac{1}{R_{t,t+1}} \frac{1}{\Pi_{t+1}} \right\}, \tag{7}$$

for an appropriately defined random variable $R_{t,t+1} \equiv [\theta_t^c U_c(Y_t)]/[\beta \theta_{t+1}^c U_c(Y_{t+1})]$ whereas $\Pi_{t+1}$ is the gross inflation rate between $t$ and $t+1$. Equation (7) represents a non-linear stochastic version of the Fisher equation. In order to turn it into a form that is more familiar to the reader we take a first-order approximation, thus obtaining

$$\hat{\imath}_t = r_t^n + E_t \pi_{t+1}, \tag{8}$$

where $\hat{\imath}_t \equiv \ln[(1+i_t)/(1+i)]$, $\pi_{t+1} \equiv \ln(P_{t+1}/P_t)$, and $r_t^n \equiv E_t \hat{R}_{t,t+1}$, with $\hat{R}_{t,t+1}$ being the log-approximation with respect to the steady state of the variable $R_{t,t+1}$, and $i$ being the value taken by $i_t$ in the steady-state.

The variable $r_t^n$ is the real interest rate, and it corresponds to what the New Keynesian literature defines as the *natural rate* of interest, namely the rate that would prevail under fully flexible prices (as it is in this model).[20] Notice that since we have modelled an endowment economy, within the present context the only sources of variation in the natural rate of interest are the preference shock $\theta^c$ and the endowment $Y$. In the New Keynesian model we estimate in Section 5.2.3, on the other hand, output is endogenous and the natural rate of interest is therefore also affected by the drivers of natural output.

[20] See, for instance, Justiniano and Primiceri (2010) and Laubach and Williams (2003).

According to equation (8), variation in the nominal interest rate is uniquely driven by fluctuations in the natural rate of interest and expected inflation. This implies that long-horizon variation in nominal rates can only be driven by persistent fluctuations in the natural rate and/or the inflation rate.[21] (This will also hold in the model with price rigidities we estimate in Section 5.2.3, since in the long-run the economy converges to the flexible-price equilibrium.) Under this respect, the fact that under the Gold Standard inflation had been statistically indistinguishable from a zero-mean white noise process[22] *logically* implies that, per the Fisher equation (8), under that regime the natural rate of interest had been the *only* driver of (long-horizon) fluctuations in nominal interest rates. But then, why had nominal interest rates—in particular, long rates—closely co-moved with the price level? This is the mystery. In order to unravel it, we need to obtain the solution for the price level.

By combining equations (5) and (6) we obtain that

$$\frac{1}{P_t}\theta_t^m L_m\left(\frac{g_t^c}{P_t}\right) = \theta_t^g V_g(G_t - g_t^c). \tag{9}$$

Under a Gold Standard regime the consumer equates the marginal benefits of the liquidity services of money (on the left-hand side of the equation) to the marginal benefits of gold (on the right-hand side). In a log-linear approximation we can write the above expression as

$$p_t = -\vartheta_g \hat{g}_t^c + \vartheta_G \hat{G}_t + \hat{\theta}_t^m - \hat{\theta}_t^g, \tag{10}$$

for positive parameters $\vartheta_g$ and $\vartheta_G$, which are defined in Online Appendix C. In the previous expression $p_t$ is the logarithm of the price level, and a hat over a variable denotes a logarithmic deviation from the steady-state. Since $\hat{g}_t^c$ is endogenous, in order to solve for the price level we need an additional equation. To this end, we write (6) as

$$\frac{1}{P_t} = \frac{\theta_t^g}{\theta_t^c}\frac{V_g(G_t - g_t^c)}{U_c(Y_t)} + E_t\left\{\frac{1}{R_{t,t+1}}\frac{1}{P_{t+1}}\right\} \tag{11}$$

Under a Gold Standard regime the current value of money depends on

(1) the utility services provided by gold relative to those provided by consumption, which are the first addendum on the right-hand side of this expression; and

(2) the expected discounted value of money, which is the second addendum, where the discount factor is $R_{t,t+1}$.

Performing a log-linear approximation of (11), we can write

$$p_t = (1-\beta)[(\hat{\theta}_t^c - \hat{\theta}_t^g) + \epsilon_g^{-1}(\hat{G}_t - s_g \hat{g}_t^c) - \sigma^{-1}\hat{Y}_t] + \beta(r_t^n + E_t p_{t+1}), \tag{12}$$

[21] As it is well known, indeed, highly persistent processes are also highly *forecastable* (see e.g. Granger and Newbold, 1986, or Barsky, 1987). Persistent inflation fluctuations therefore automatically map into persistent fluctuations in *expected* inflation.

[22] See Barsky (1987) and Benati (2008).

where $\sigma$, $\epsilon_g$ and $s_g$ are parameters defined in Online Appendix C. By combining (10) and (12) we obtain

$$p_t = \frac{(1-\beta)}{\phi}\left[(\hat{\theta}_t^c - \hat{\theta}_t^g) - \vartheta(\hat{\theta}_t^m - \hat{\theta}_t^g) + z\hat{G}_t - \sigma^{-1}\hat{Y}_t\right] + \frac{\beta}{\phi}(r_t^n + E_t p_{t+1}) \tag{13}$$

where $z$ , $\phi$ and $\vartheta$ are positive parameters, with $\phi > \beta$.This equation can be solved forward in order to determine the price level as a function of the exogenous disturbances in the model

$$p_t = \frac{1}{\phi}E_t\left\{\sum_{T=t}^{\infty}\left(\frac{\beta}{\phi}\right)^{T-t}\left[(1-\beta)\left((\hat{\theta}_T^c - \hat{\theta}_T^g) - \vartheta(\hat{\theta}_T^m - \hat{\theta}_T^g) + z\hat{G}_T - \sigma^{-1}\hat{Y}_T\right) + \beta r_T^n\right]\right\} \tag{14}$$

Equations (8) and (14), together with the definition of inflation,

$$\pi_t \equiv p_t - p_{t-1},$$

constitute the data-generating process for nominal interest rates and prices under the Gold Standard. They characterize the dynamics of the two variables at each point in time, given the processes for the exogenous disturbances.

#### 4.2.1 The natural rate of interest and the co-movement of prices and nominal interest rates

For the sake of simplicity, and without any loss of generality, we assume that the natural rate follows the AR(1) process

$$r_t^n = \rho_r r_{t-1}^n + \varepsilon_{r,t} \tag{15}$$

with $0 \leq \rho_r \leq 1$ and $\varepsilon_{r,t} \sim N(0, \sigma_r^2)$. We further assume that, beyond the natural rate, one (and only one) of the additional stochastic processes is non-zero. Again without any loss of generality we focus on $\hat{\theta}_t^c$, and we postulate that it likewise follows the AR(1) process

$$\hat{\theta}_t^c = \rho_c \hat{\theta}_{t-1}^c + \varepsilon_{c,t} \tag{16}$$

with $0 \leq \rho_c \leq 1$ and $\varepsilon_{c,t} \sim N(0, \sigma_c^2)$.

Combining expressions (14) to (16) we obtain the following expression for the price level:

$$p_t = \frac{\beta}{\phi - \beta\rho_r}r_t^n + \frac{1-\beta}{\phi - \beta\rho_c}\hat{\theta}_t^c. \tag{17}$$

From this we can compute expected inflation as

$$E_t\pi_{t+1} = -\frac{\beta(1-\rho_r)}{\phi - \beta\rho_r}r_t^n - \frac{(1-\beta)(1-\rho_c)}{\phi - \beta\rho_c}\hat{\theta}_t^c.$$

Substituting this expression into (8) we can solve for the nominal interest rate as

$$\hat{\imath}_t = \frac{\phi - \beta}{\phi - \beta\rho_r} r_t^n - \frac{(1-\beta)(1-\rho_c)}{\phi - \beta\rho_c} \hat{\theta}_t^c. \tag{18}$$

We start by analyzing the extreme, purely theoretical case in which the natural rate is the *only* stochastic process driving the economy, so that all other disturbances in (14) are set to zero at all times. As always, extreme cases are particularly insightful because they allow to see a mechanism, or concept in an especially stark way.

**An extreme case: the natural rate as the only random driver** With all processes other than $r_t^n$ set to zero at all $t$, expressions (17) and (18) imply that

$$p_t = \underbrace{\frac{\beta}{\phi - \beta}}_{+} \hat{\imath}_t \tag{19}$$

This expression shows that if the natural rate had been the *only* stochastic process driving the economy, under the Gold Standard Gibson's paradox would have appeared at *all times* and *all frequencies*.

As previously noted, however, under that regime the paradox had been especially apparent at the very *low frequencies* (see in particular the evidence in Section 2). This naturally suggests that

(1) at high-to-medium frequencies additional, comparatively short-lived stochastic processes had blurred the correlation between nominal rates and prices induced by variation in the natural rate, whereas

(2) at the very low frequencies, highly persistent fluctuations in the natural rate had 'swamped out' these additional processes, thus ultimately allowing Gibson's paradox to clearly emerge in the raw data.

We explore this intuition in the next sub-section.

**The influence of additional stochastic processes** With $\hat{\theta}_t^c$ now evolving according to (16), expression (19) now becomes

$$p_t = \underbrace{\frac{\beta}{\phi - \beta}}_{+} \hat{\imath}_t + \underbrace{\frac{1-\beta}{\phi - \beta}}_{+} \hat{\theta}_t^c \tag{20}$$

This expression shows that $\hat{\theta}_t^c$ introduces a 'wedge' within the relationship between prices and the nominal rate induced by the natural rate of interest, thus *blurring* it. Together with (17) and (18), equation (20) therefore highlights how the extent to which Gibson's paradox appears in the raw data under the Gold Standard crucially hinges on the *comparative stochastic properties* of $r_t^n$ and $\hat{\theta}_t^c$.

At one extreme, if the variance of $\hat{\theta}_t^c$ is sufficiently larger than the variance of $r_t^n$ Gibson's paradox may be blurred beyond recognition. In the limit, if $\sigma_c^2 \rightarrow \infty$ and/or $\rho_c \rightarrow 1$, whereas $\rho_r < 1$, the paradox disappears altogether from the data. If, on the other hand, $\rho_c < 1$, whereas $\rho_r \rightarrow 1$, prices and the nominal interest rate tend to cointegrated processes driven by the common random-walk trend $r_t^n$, thus highlighting the paradox in the starkest way possible.

In order to analyse intermediate cases, and to explore how the comparative stochastic properties of $r_t^n$ and $\hat{\theta}_t^c$ shape the extent to which the paradox manifests itself in the data, we can either $(i)$ working in the time domain, analyse the covariance between the *forecast errors* of prices and nominal rates at horizon $k$ induced by random variation in $r_t^n$ and $\hat{\theta}_t^c$, or $(ii)$ working in the frequency domain, study the *cross-spectrum* between prices and nominal rates at frequency $\omega$. In either case, we are interested in how the comparative stochastic properties of $r_t^n$ and $\hat{\theta}_t^c$ impact upon the statistic of interest.

Straightforward calculations show that the covariance between $(p_{t+k} - E_t p_{t+k})$ and $(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})$ is equal to[23]

$$E[(p_{t+k} - E_t p_{t+k})(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})] =$$

$$= \sigma_r^2 \frac{1-\rho_r^{2k}}{1-\rho_r^2} \frac{\beta(\phi-\beta)}{(\phi-\beta\rho_r)^2} - \sigma_c^2 \frac{(1-\rho_c)(1-\rho_c^{2k})}{1-\rho_c^2} \frac{(1-\beta)^2}{(\phi-\beta\rho_c)^2}. \tag{21}$$

We now explore how the stochastic properties of $r_t^n$ and $\hat{\theta}_t^c$ impact upon this statistic.

We start by assuming that $\sigma_r^2 = \sigma_c^2 = 1$, so that we can focus on the persistence of $r_t^n$ and $\hat{\theta}_t^c$. Let us first assume that $\rho_c = 0$ and $\rho_r \rightarrow 1$. Under these circumstances[24]

$$E[(p_{t+k} - E_t p_{t+k})(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})] \rightarrow \frac{\beta}{\phi-\beta} k - \frac{\overbrace{(1-\beta)^2}^{\simeq 0}}{\phi^2} \simeq \underbrace{\frac{\beta}{\phi-\beta}}_{+} k \tag{22}$$

so that

$$\lim_{k \rightarrow +\infty} E[(p_{t+k} - E_t p_{t+k})(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})] = +\infty$$

This shows that if the natural rate tends to a random walk, whereas $\hat{\theta}_t^c$ is white noise, the covariance between the forecast errors of the nominal rate and the price level

[23] By the same token, it can be easily shown that for $|\rho_r, \rho_c| < 1$ the cross-spectrum between $\hat{\imath}_t$ and $p_t$ at frequency $\omega$ is

$$\hat{\imath}_t(e^{i\omega}) p_t(e^{-i\omega}) = \frac{\sigma_r^2}{1+\rho_r^2 - 2\rho_r \cos\omega} \frac{\beta(\phi-\beta)}{(\phi-\beta\rho_r)^2} - \frac{\sigma_c^2}{1+\rho_c^2 - 2\rho_c \cos\omega} \frac{(1-\beta)^2(1-\rho_c)}{(\phi-\beta\rho_c)^2}.$$

[24] Notice that since $\beta$ is slightly smaller than 1, $(1-\beta)^2$ in (22) is negligible.

explodes to infinity with the forecast horizon $k$. The reason for this is straightforward: under these circumstances $p_t$ and $\hat{\imath}_t$ tend to two cointegrated processes.

In the opposite polar case in which $\rho_r = 0$ and $\rho_c \to 1$, we have

$$E[(p_{t+k} - E_t p_{t+k})(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})] \to \frac{\beta(\phi - \beta)}{\phi^2} > 0 \tag{23}$$

so that the covariance is positive but finite, and independent of the horizon $k$.

Finally, assuming that $\sigma_r^2 \neq \sigma_c^2$ and $|\rho_r = \rho_c = \rho|<1$, we have

$$E[(p_{t+k} - E_t p_{t+k})(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})] =$$

$$= \underbrace{\left[\frac{1-\rho^{2k}}{1-\rho^2} \frac{1}{(\phi - \beta\rho)^2}\right]}_{+} \left[\sigma_r^2 \underbrace{\beta(\phi - \beta)}_{+} - \sigma_c^2 \underbrace{(1-\rho)(1-\beta)^2}_{+}\right] \tag{24}$$

Under these circumstances, the covariance between the forecast errors is larger, and more positive, the greater $\sigma_r^2$ is compared to $\sigma_c^2$—i.e., the greater the variance of innovations to the natural rate is compared to the variance of innovations to the other stochastic processes.

**Taking stock** The previous discussion makes clear how, under the Gold Standard,

- fluctuations in the natural rate of interest, by themselves (i.e. abstracting from additional stochastic processes), had generated a positive correlation between prices and interest rates at all times and at all horizons.

- When also considering additional stochastic processes, the long-horizon covariance between prices and nominal interest rates becomes especially large when the natural rate is very highly persistent compared to these processes. If the natural rate tends to a random walk, the price level and the nominal interest rate tend to two cointegrated processes, thus 'tracing out' Gibson's paradox in the starkest way possible.

- For a given persistence of the stochastic processes driving the economy, an increase in the variance of the innovations to the natural rate causes an increase in the correlation between prices and interest rates.

The fact that an increase in the natural rate causes a corresponding increase in the nominal interest rate is a direct implication of the Fisher equation. On the other hand, within the asset pricing equation (14) an increase in the natural rate causes a decrease in the current expected value of money's future liquidity services, which is obtained via an increase in the price level.

As noted, a necessary condition for the relationship between prices and the nominal rate induced by fluctuations in the natural rate to clearly appear in the raw data is that additional sources of variation must not vary appreciably compared to the natural rate. In particular, the inflation rate ought to leave a light imprint in interest rates, which obtains if it is strongly mean-reverting, and/or the variance of its innovations is sufficiently small. In fact, under the Gold Standard this had indeed been the case since, as mentioned, inflation had been statistically indistinguishable from a zero-mean white noise process (see Barsky, 1987, and Benati, 2008). Based on the previous discussion this explains why under that regime Gibson's paradox had appeared in such a stark way: intuitively, the natural rate had been the dominant source of long-horizon variation for both prices and nominal interest rates.

**An alternative channel at odds with the stochastic properties of inflation under the Gold Standard** The previous discussion suggests that an alternative channel through which Gibson's paradox could have appeared under the Gold Standard involves highly persistent fluctuations in expected inflation. The obvious problem with this explanation is that it is at odds with the stochastic properties of inflation under that regime. With inflation indistinguishable from zero-mean white noise, its expectation had been essentially zero at all horizons.

It is however worth briefly exploring this possibility, focusing for the sake of the argument on variations in the stock of gold, $G_t$. Assume that $r_t^n = 0$ at all $t$, whereas $G_t$ evolves as

$$\Delta\hat{G}_t \equiv \hat{G}_t - \hat{G}_{t-1} = \mu_{g,t} + \varepsilon_{g,t},$$

with

$$\mu_{g,t} = \rho_\mu \; \mu_{g,t-1} + \varepsilon_{\mu,t},$$

where $0 \leq \rho_\mu \leq 1$, $\varepsilon_{g,t} \sim N(0, \sigma_g^2)$ and $\varepsilon_{\mu,t} \sim N(0, \sigma_\mu^2)$. Conditional on gold, the price level evolves as

$$p_t = \frac{z}{\phi - \beta}\,\hat{G}_t + \frac{\beta}{\phi}\,\frac{z}{\phi - \beta}\,\mu_{g,t}.$$

From this expression it follows that expected inflation and the nominal interest rate, $\hat{\imath}_t = E_t[\pi_{t+1}]$, are given by

$$\hat{\imath}_t = \left[\rho_\mu - (1 - \rho_\mu)\frac{\beta}{\phi}\right] \frac{z}{\phi - \beta}\,\mu_{g,t}. \tag{25}$$

Under these circumstances Gibson's paradox can arise from fluctuations in $\mu_{g,t}$. In particular, if $\rho_\mu \to 1$, $\hat{\imath}_t = [(\phi - \beta)/z]\;\mu_{g,t}$, so that the interest rate becomes a random walk, and

$$p_t = \left(1 + \frac{\beta}{\phi}\right)\hat{\imath}_t + \sum_{k=1}^{\infty} \hat{\imath}_{t-k} + \frac{\phi - \beta}{z} \sum_{k=0}^{\infty} \varepsilon_{g,t-k}$$

Up to the sequence of white noise shocks $\varepsilon_{g,t-k}$, the price level is driven by the random walk in $\hat{\imath}_t$, i.e. $\mu_{g,t}$. Moreover, expected inflation would also be a random walk, as shown in (25, which is at odds with the stochatic properties of inflation under the Gold Standard.

We now turn to alternative monetary regimes.

## 4.3 Alternative monetary regimes

The fact that—as we have argued theoretically, and as we will show empirically—fluctuations in the natural rate of interest had generated Gibson's paradox under the Gold Standard raises the natural possibility that, if these fluctuations were sufficiently large and persistent compared to other sources of variation, the paradox might also emerge under alternative monetary regimes.

Under this respect, the experience of inflation-targeting regimes is especially intriguing. As it has been documented (see Benati, 2008), under these regimes inflation has consistently been strongly mean-reverting, and in many cases statistically indistinguishable from a white noise process with a positive mean. At the same time, since the early 1990s (when these regimes were first introduced) the natural rate of interest has near-uniformly trended downward by substantial amounts.[25] This suggests that inflation-targeting regimes could in principle have given rise to Gibson's paradox. However, the presence of (1) a positive drift in the price level due to a positive inflation target, and (2) a negative drift in long-term rates due to the decline in the natural rate, has likely prevented the paradox from appearing in the raw data as starkly as it did under the Gold Standard. As we will see, our empirical evidence in Sections 5.2.2 and 8 suggest that this has indeed been the case.

By the same token, the evidence in Figure 1*c* of a low-frequency relationship between the price level and long-term interest rates during the interwar period has a natural interpretation. As argued, for example, by Eggertsson (2008), the Great Depression was characterized by a dramatic fall in the natural rate of interest. This suggests that variation in the natural rate may well have been *dominant* relative to other sources of variation, thereby causing Gibson's paradox to appear in the data.

Formally, a 'building block' of our explanation of Gibson's paradox that is common to all monetary regimes is the Fisher equation (7). What differs across regimes, on the other hand, is the particular form taken by the asset-pricing condition that determines the current price level as the expected discounted future flow of the liquidity services provided by money.

### 4.3.1 An inflation-targeting regime

Under a monetary framework in which the central bank implements its policy by manipulating an interest rate—such as an inflation targeting regime—there would not seem to be any corresponding asset-pricing equation. This is however not quite right. Consider for example an interest rate rule such as

$$1 + i_t = (1 + i_t^*) \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\pi}$$

[25]For instance, Holston et al. (2017) estimate that between the early 1990s and the mid-2010s, the natural rate declined from 2.5% to just above 1% in Canada. The corresponding figures for the Euro Area are approximately 2.5% and -0.3%, and for the United Kingdom about 2.1% and 1.5%.

for some process $i_t^*$ controlled by the monetary authority, in which $\phi_\pi > 1$ and $\Pi$ is the constant inflation objective that is targeted by the central bank. This expression can still be combined with (7) in order to obtain the 'asset-pricing condition'

$$q_t \left(\frac{1}{P_t}\right)^{1+\phi_\pi} = E_t \left\{ \frac{1}{R_{t,t+1}} \frac{1}{P_{t+1}} \right\}, \tag{26}$$

with $q_t$ given by

$$q_t = \frac{(P_{t-1}\Pi)^{\phi_\pi}}{(1+i_t^*)}$$

The expression (26) relates the current value of money to the expected future value of money, discounted once again by $R_{t,t+1}$. To the extent that variation in $q_t$ in (26) and in the inflation rate in (7) are sufficiently small compared to variation in the natural rate of interest, a positive relationship between the natural rate and the price level should clearly appear in the data.

Taking a log-linear approximation of equation (26) we obtain

$$\hat{\imath}_t^* + \phi_\pi(\pi_t - \pi) = r_t^n + E_t(\pi_{t+1} - \pi),$$

which under the standard Taylor principle, $\phi_\pi > 1$, has a solution of the form

$$p_t = p_{t-1} + \pi + \frac{1}{\phi_\pi} \sum_{T=t}^{\infty} \left(\frac{1}{\phi_\pi}\right)^{T-t} (r_T^n - \hat{\imath}_T^*). \tag{27}$$

This expression produces once again a positive relationship between prices and the natural rate of interest at all horizons, provided that the process $\{\hat{\imath}_t^*\}$ does not closely track (i.e., offset) movements in the natural rate. Further, note that movements in expected inflation driven by the natural rate of interest, which are implied by (27), reinforce the direct positive effect of fluctuations in the natural rate on the nominal interest rate in the Fisher equation (8), thus generating a positive correlation between the nominal interest rate and prices. To see this, assume that $r_t^n$ and $\hat{\imath}_t^*$ follow AR(1) processes with autoregressive coefficient $\rho_r$ and $\rho_i$, and variances of the white-noise innovations $\sigma_r^2$ and $\sigma_i^2$, respectively. Equation (27) then implies

$$p_t = p_{t-1} + \pi + \frac{1}{\phi_\pi - \rho_r} r_t^n - \frac{1}{\phi_\pi - \rho_i} \hat{\imath}_t^*. \tag{28}$$

This equation can be used to compute expected inflation, which can then be substituted into the Fisher equation (8) in order to obtain the expression for the nominal interest rate:

$$\hat{\imath}_t = \frac{\phi_\pi}{\phi_\pi - \rho_r} r_t^n - \frac{\rho_i}{\phi_\pi - \rho_i} \hat{\imath}_t^*. \tag{29}$$

Expressions (28) and (29) show that both $r_t^n$ and $\hat{\imath}_t^*$ can generate a positive co-movement between prices and the nominal interest rate. The covariance between $(p_{t+k} - E_t p_{t+k})$ and $(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})$ is indeed equal to

$$E[(p_{t+k} - E_t p_{t+k})(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})] =$$

$$= \underbrace{\frac{\phi_\pi}{(\phi_\pi - \rho_r)^2} \frac{(1-\rho_r^k)(1-\rho_r^{k+1})}{(1+\rho_r)(1-\rho_r)^2}\sigma_r^2}_{+} + \underbrace{\frac{\rho_i}{(\phi_\pi - \rho_i)^2} \frac{(1-\rho_i^k)(1-\rho_i^{k+1})}{(1+\rho_i)(1-\rho_i)^2}\sigma_i^2}_{+}$$

The extent to which Gibson's paradox is driven by natural rate fluctuations, as opposed to variation in $\hat{\imath}_t^*$, hinges on their comparative stochastic properties. A formal illustration of this point can be trivially performed along the lines of what precedes, and it is therefore not pursued here.

An important point to stress is rather that the Taylor rule coefficient $\phi_\pi$ also plays a crucial role. An increase in $\phi_\pi$ causes a decrease in the covariance between the forecast errors of prices and the nominal interest rate at all horizons, and as $\phi_\pi \to \infty$—such as under strict inflation targeting—the covariance is driven to zero. For $\phi_\pi \ll +\infty$, on the other hand, highly persistent fluctuations in the natural rate produce a large covariance at all horizons, since as shown by equation (28) the process for the price level would be highly persistent as well.

**How a positive inflation target hides Gibson's paradox in the raw data** The previous discussion makes clear that if fluctuations in $r_t^n$ and/or $\hat{\imath}_t^*$ are sufficiently large compared to those in other stochastic processes inducing a negative correlation between nominal interest rates and prices, the positive covariance between the two series's forecast errors that is one manifestation of Gibson's paradox will appear in the data. Different from the Gold Standard, however, a positive correlation between prices and the nominal rate would *not appear* in the raw data. The reason for this is straightforward: whereas, per the Fisher equation, fluctuations in the nominal rate originate from variation in the natural rate and expected inflation, the presence of a positive inflation target $\pi > 0$ introduces an *upward drift* in the price level—as shown in (27). This automatically *hides* any correlation between the *raw* prices and nominal rate data.[26] At the same time, however, it is important to stress that the correlation only becomes hidden in the raw data, whereas it is by no means destroyed. This can be trivially understood by noting that if $\pi = 0$ Gibson's paradox would actually appear in the raw data. Although, so far, all inflation-targeting regimes have set $\pi > 0$, the experiment of setting a zero inflation target can be performed via either VARs or estimated DSGE models. In fact, in Section 7 we will show that once removing from the raw data the deterministic components of the series' trends, Gibson's paradox starkly emerges from the raw data generated under inflation-targeting regimes.

### 4.3.2 A price level-targeting regime

Under price level-targeting, on the other hand, the evolution of inflation following *any* stationary shock ultimately brings the price level back to its target (see Woodford,

[26] A further reason for this is that since (at least) the early 1990s, when inflation-targeting regimes started being introduced, the natural rate of interest has been broadly trending downwards.

2003). This feature has crucial implications. Consider the policy rule

$$1 + i_t \;=\; (1 + i_t^*) \left( \frac{P_t}{P^*} \right)^{\phi_p},$$

where $i_t^*$ is the instrument controlled by the central bank, $\phi_p > 0$ is the response coefficient, and $P^*$ is the price-level target. Taking logarithms and substituting into the Fisher equation yields

$$\hat{\imath}_t^* + \phi_p \left(p_t - p^*\right) = r_t^n + E_t\big(p_{t+1} - p_t\big).$$

Solving this expression forward produces

$$p_t = p^* + \frac{1}{1 + \phi_p - \rho_r} r_t^n - \frac{1}{1 + \phi_p - \rho_i} \hat{\imath}_t^*.$$

This expression can be used to compute expected inflation which, substituted in the Fisher equation yields the following expression for the nominal rate:

$$\hat{\imath}_t = \frac{\phi_p}{1 + \phi_p - \rho_r} r_t^n + \frac{(1 + \phi_p)(1 - \rho_i)}{1 + \phi_p - \rho_i} \hat{\imath}_t^*.$$

As in the previous section, we assume that $r_t^n$ and $\hat{\imath}_t^*$ follow AR(1) processes with autoregressive coefficient $\rho_r$ and $\rho_i$, and variances of the white-noise innovations $\sigma_r^2$ and $\sigma_i^2$, respectively. The covariance between $(p_{t+k} - E_t p_{t+k})$ and $(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})$ is then equal to

$$E[(p_{t+k} - E_t p_{t+k})(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})] =$$

$$= \underbrace{\frac{\phi_p}{\left(1 + \phi_p - \rho_r\right)^2} \frac{1 - \rho_r^{2k}}{1 - \rho_r^2}}_{+} \sigma_r^2 - \underbrace{\frac{\left(1 + \phi_p\right)\left(1 - \rho_i\right)}{\left(1 + \phi_p - \rho_i\right)^2} \frac{1 - \rho_i^{2k}}{1 - \rho_i^2}}_{-} \sigma_i^2.$$

This expression shows that even under a price level-targeting regime shocks to the natural rate, taken in isolation (i.e., here setting $\sigma_i^2 = 0$), would generate Gibson's paradox. The presence of the process $i_t^*$, however, weakens the correlation between prices and nominal interest rates, since the coefficient on $\sigma_i^2$ in the previous expression is negative. Finally, as $\phi_p \rightarrow +\infty$, $E[(p_{t+k} - E_t p_{t+k})(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})] \rightarrow 0$: if the central bank reacts with infinite strength to deviations of the price level from target, Gibson's paradox vanishes from the raw data.

### 4.3.3 An exogenous path for the money stock

Finally, under a regime in which the central bank implements its policy by setting a specific *exogenous* path for the *money supply*, the relevant asset pricing condition for determining the value of money is equation (5), which can be written as

$$\frac{1}{P_t} = \frac{1}{P_t} \frac{\theta_t^m}{\theta_t^c} \frac{L_m\left(\frac{M_t}{P_t}\right)}{U_c(Y_t)} + E_t \left\{ \frac{1}{R_{t,t+1}} \frac{1}{P_{t+1}} \right\}. \tag{30}$$

This expression exhibits some similarities with (11), with the current value of money depending on the discounted expected value of money in the future. Crucially, within this expression the discount factor is once again $R_{t,t+1}$, whose expected value is the source of variation of the natural real rate of interest. The key differences compared to the Gold Standard are that here (*i*) the money supply is exogenous, as opposed to endogenous, and (*ii*) what matters for the determination of the current value of money—and therefore the price level—are the real liquidity benefits provided by money, as opposed to such benefits *compared* to the utility benefits of gold.

Taking a log-linear approximation of (30) we obtain

$$p_t = \frac{(1-\beta)}{\phi_m}\left[(\hat{\theta}_t^c - \hat{\theta}_t^m) + \epsilon_m^{-1}\hat{M}_t - \sigma^{-1}\hat{Y}_t\right] + \frac{\beta}{\phi_m}(r_t^n + E_t p_{t+1}), \tag{31}$$

for positive parameters $\epsilon_m$ and $\phi_m$, with $\phi_m > \beta$. This equation can be solved forward in order to obtain the price level as a function of the exogenous processes, including that for the money supply. Working as in the previous sub-sections we can then compute the covariance between $(p_{t+k} - E_t p_{t+k})$ and $(\hat{\imath}_{t+k} - E_t \hat{\imath}_{t+k})$ conditional on shocks to $r_t^n$ and $\hat{\theta}_t^c$, which is *identical* to the corresponding expression (21) for the Gold Standard with $\phi = \phi_m$. Our discussion in Section 4.2.1 therefore also applies here. In particular, as long as the variation in the additional stochastic processes beyond the natural rate is not large enough to blur the correlation between prices and the nominal rate induced by the natural rate, Gibson's paradox should appear in the raw data. In fact, our analysis in Section 6 suggest that under certain rules for the evolution of the money supply this is indeed the case. Specifically, we will show that under a regime that keeps the level of the money stock constant the impulse-response functions (IRFs) to shocks to the natural rate generate a positive long-horizon correlation between prices and long-term nominal interest rates.

### 4.3.4 Gibson's paradox and the Lucas critique

Our analysis of Gibson's paradox under four monetary regimes suggests that *whether* the paradox appears in the raw data, and the *extent* to which it appears (if it does), hinges to a crucial, although not exclusive degree on the nature of the monetary regime. This is nothing but an illustration of the key tenet of the Lucas critique: the reduced-form stochastic properties of macroeconomic data are shaped to an important, although not exclusive extent by the nature of the policy rules.

We now turn to the empirical evidence for the Gold Standard and inflation-targeting regimes.

# 5 Evidence from Structural VAR Methods

In this section we perform, for either the Gold Standard or inflation targeting regimes, an exercise conceptually in line with Kurmann and Otrok's (2013), in which we compare (the IRFs produced by) two structural disturbances that we *independently* identify based on *two alternative rotations*: a shock driving long-horizon variation in the natural rate of interest (which for the sake of simplicity we label as 'natural rate shock'), and a disturbance maximizing the long-horizon covariance between the price level and a long-term nominal interest rate (which we label as a 'Gibson's paradox shock'). Our main result is that, for either monetary regime, and for all of the countries we analyze, the two shocks are virtually the same, as (1) they exhibit a remarkably strong correlation, and (2) the IRFs' credible sets they produce are near-uniformly numerically very close. In line with the previous discussion, the natural interpretation of this evidence is that (I) under the Gold Standard Gibson's paradox had originated from highly persistent fluctuations in the natural rate of interest, and (II) the paradox has nothing to do with the Gold Standard per se, and in fact it is hidden in the data generated by inflation targeting regimes. This suggests that, in principle, it should be possible to recover it from the raw data generated by these regimes. As we will show in Section 7 this is indeed the case.

We start by discussing details pertaining to Bayesian estimation of the reduced-form VARs and the identification of the natural rate and Gibson's paradox shocks, and we then turn to the empirical evidence.

## 5.1 Methodology

### 5.1.1 Bayesian estimation of the reduced-form VARs

We estimate Bayesian VARs based on the methodology proposed by Giannone, Lenza, and Primiceri (2015). Let the VAR($p$) model be

$$Y_t = B_0 + B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t \tag{32}$$

with $Y_t$ and $u_t$ being $N \times 1$, and $u_t \sim N(0, \Sigma)$. By defining $\beta \equiv$vec$([B_0, B_1, ..., B_p]')$ and $x_t \equiv [1, Y'_{t-1}, ..., Y'_{t-p}]'$, equation (32) can be rewritten as

$$Y_t = X_t \beta + u_t, \tag{33}$$

where $X_t \equiv I_N \otimes x'_t$. The prior distribution for the VAR coefficients is postulated to belong to the Normal-Wishart family, i.e.

$$\Sigma \sim IW(\Psi; d) \tag{34}$$

$$\beta | \Sigma \sim N(b; \Sigma \otimes \Omega), \tag{35}$$

where the elements of $\Psi$, $d$, $b$ and $\Omega$ are functions of a lower-dimensional vector of hyperparameters. The degree of freedom of the Inverse-Wishart distribution is set to $d$=$N$+2, which is the minimum value that guarantees the existence of the prior mean of $\Sigma$. $\Psi$ is postulated to be a diagonal matrix with the $N \times 1$ vector of hyperparameters $\psi$ on the main diagonal.

The conditional Gaussian prior for $\beta$ is of the Minnesota type, with the only difference that instead of imposing Litterman's 'random-walk prior', we postulate the following first moment:

$$E\left[(B_s)_{ij}|\Sigma\right] = \begin{cases} \mu & \text{if } i = j \text{ and } s = 1 \\ 0 & \text{otherwise} \end{cases} . \tag{36}$$

In Sections 5.2.1 and 5.2.2 we discuss how we set $\mu$ for the Gold Standard and inflation targeting regimes, respectively. As for the second moment, as in Giannone *et al.* (2015) we postulate that

$$Cov\left[(B_s)_{ij}, (B_r)_{hm}|\Sigma\right] = \begin{cases} \lambda^2 \frac{1}{s^2} \frac{\Sigma_{ih}}{\psi_j/(d-N-1)} & \text{if } m = j \text{ and } r = s \\ 0 & \text{otherwise} \end{cases} . \tag{37}$$

where the hyperparameter $\lambda$ controls the scale of the variances and covariances, thus determining the prior's overall tightness. We set the hyperpriors for $\lambda$ and $\psi$ as in Giannone *et al.* (2015), and we estimate the VAR as discussed there.[27]

### 5.1.2 Identification

We identify the natural rate and Gibson's paradox shocks as follows. As for the shock driving long-horizon variation in the natural rate of interest, under either the Gold Standard or inflation targeting regimes we identify it via Uhlig's (2004) procedure, as the disturbance explaining the maximum fraction of the forecast error variance (FEV) of the long-term nominal interest rate at a long but finite horizon, which we set to 25 years. Since, as previously discussed, under either monetary regime inflation had, and has been very strongly mean-reverting, and in fact statistically (nearly) indistinguishable from a white noise process with a zero and a positive mean, respectively, this is the natural identifying restriction. The reason for this is straightforward: with inflation strongly mean-reverting, the bulk of long-horizon variation of long-term nominal interest rates can *only* be driven by long-horizon fluctuations in the natural rate.

As for Gibson's paradox shocks we identify them as the disturbances generating the largest positive covariance between the price level and the long-term nominal interest rate at the same long but finite horizon we use for identifying the natural rate shock.[28] This identifying restriction is the *natural* one, since it is in fact a *definition* of the shocks generating Gibson's paradox.

[27] We use the MATLAB codes found at Giorgio Primiceri's web page.

[28] We implement this restriction via the methodology proposed by Benati (2014), which is based on the notion of working with the entire set of available rotation matrices, maximizing the relevant criterion function over the set of the corresponding rotation angles via numerical methods. Monte Carlo evidence there illustrates its extremely robust performance.

## 5.2 Evidence

### 5.2.1 The Gold Standard

For both the United Kingdom and the United States we estimate the VARs based on monthly data for the logarithm of the price level, a long-term nominal interest rate, the spread between the long-term and a short-term nominal rate, and a cyclical indicator of real economic activity (the unemployment rate for the former country, and a 'real activity' indicator from the NBER Historical Database for the latter one). For Norway and Germany, for which we could not find a cyclical indicator, we only estimate the VARs based on the long rate, the spread, and the logarithm of the price level. The sample periods are January 1855-June 1914 for the United Kingdom, January 1879-November 1913 for the United States,[29] November 1851-June 1914 for Norway, and March 1879-December 1902 for Germany. A detailed description of the data can be found in Online Appendix A.1. We set the lag order to either six or twelve. Since the evidence produced by the two lag orders is qualitatively the same, in what follows we uniquely present and discuss results based on six lags.

We set $\mu$ in expression (36) as follows. For both the spread and the cyclical real activity indicator we set it to 0.75, reflecting the non-negligible extent of persistence of either series, and their ultimately mean-reverting nature. As for the long rate we set $\mu$ = 1-0.5/$T$, where $T$ is the sample length, corresponding to the parameterization for a near unit root process. This reflects our view that, although very highly persistent, this series ultimately ought to be mean-reverting.[30] As for prices we also set $\mu$ = 1-0.5/$T$. This near unit root specification is motivated by the fact that under metallic standards prices had remained extraordinarily stable over periods of centuries.[31]

Figures 3*a* and 3*b* report the medians and the 68 per cent-coverage credible sets of the IRFs to the two shocks, whereas Figures 4*a* and 4*b* show scatterplots of the medians of the posterior distributions of the two shocks, together with the distributions of their contemporaneous correlation coefficient. The evidence in the two figures speaks for itself, and it very strongly suggests that the two shocks, which we *independently* identified based on two *alternative rotations*, are in fact one and the same. In particular, for any of the four countries both the medians and the credible sets of the IRFs are near-uniformly numerically very close, thus showing that Gibson's paradox and

[29] We end the sample period in the month preceding the creation of the Federal Reserve. Over our entire sample the role of monetary authority was therefore performed by the Treasury.

[30] Under the Gold Standard, with inflation near-indistinguishable from white noise, the long-horizon components of nominal interest rates had uniquely been driven by the corresponding component of the natural rate of interest. As a matter of logic, the natural rate cannot take literally *any* value between minus and plus infinity, and it therefore ought to be mean-reverting.

[31] For example, for the United Kingdom Elisabeth Schumpeter's (1938) index for the prices of consumer goods had been equal to 109 at the beginning of the sample, in 1661, and to 104.9 at the end, in 1823. By the same token, the Sauerbeck-Statist price index had been equal to 89 in 1846, an to 85 in 1913 (this price index is plotted in the third panel of Figure 1.*a*, and is discussed in Online Appendix A.1). Qualitatively the same evidence holds for all countries under metallic standards.

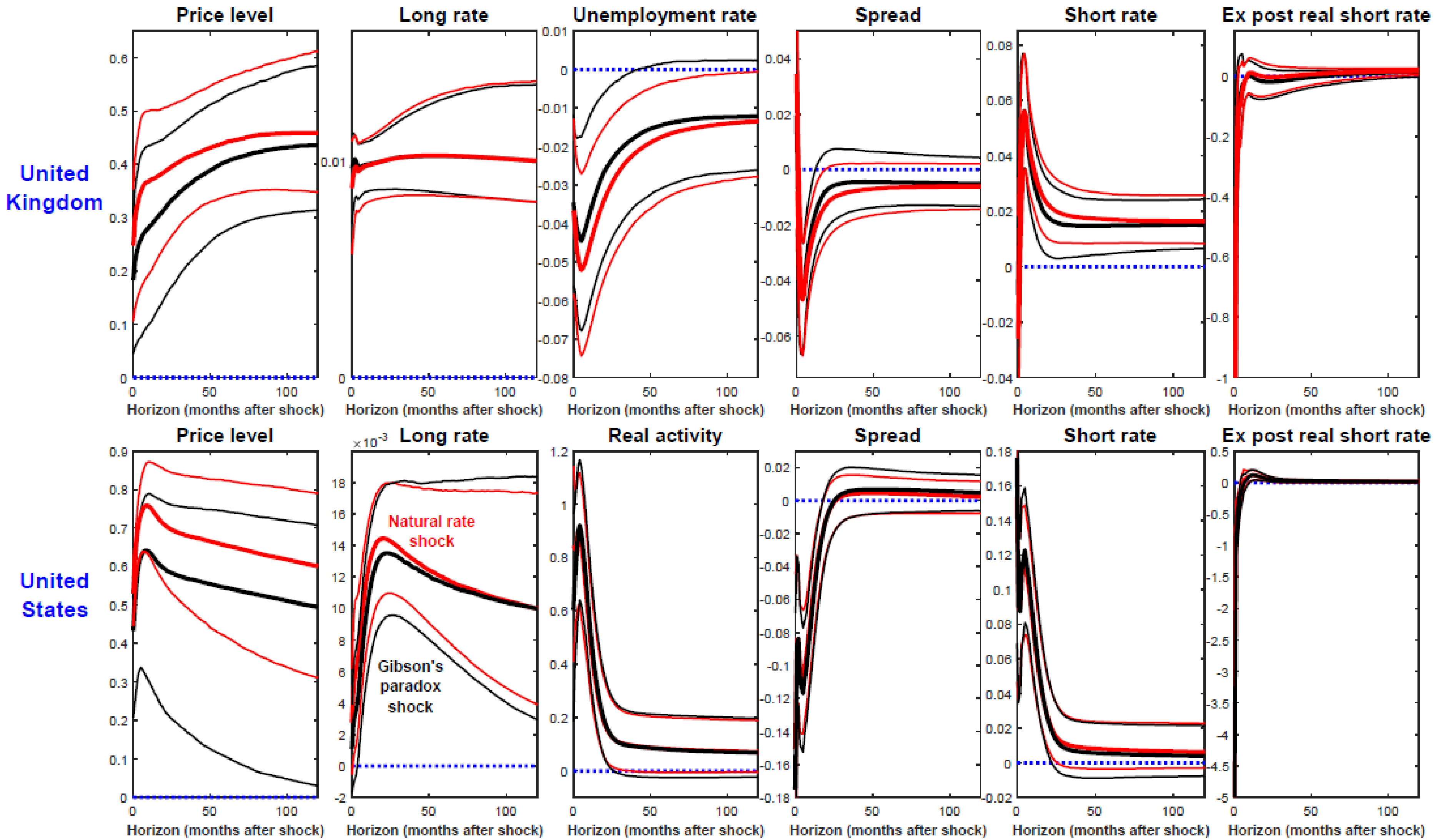


Figure 3a United Kingdom and United States under the Gold Standard: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets)

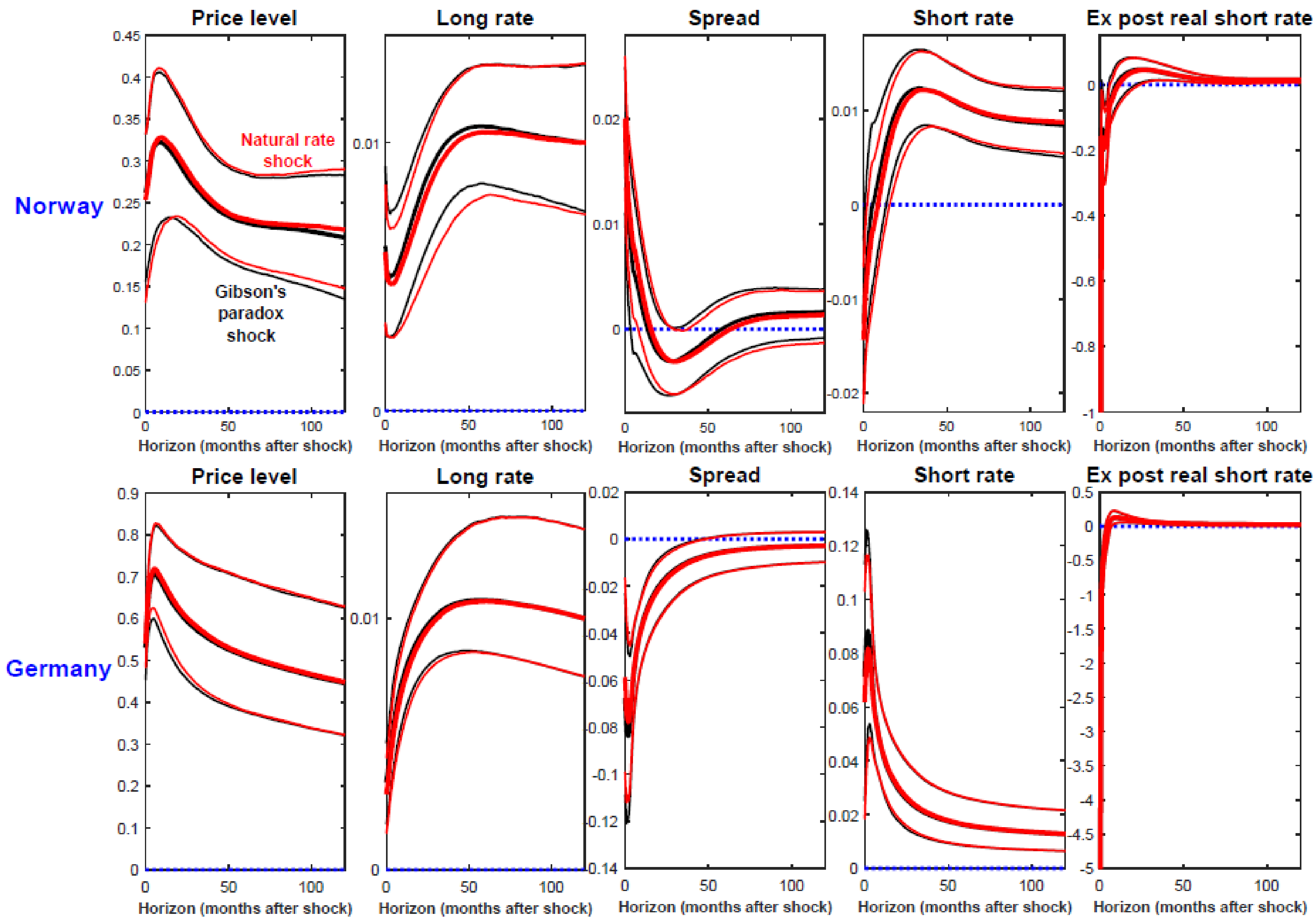


Figure 3b Norway and Germany under the Gold Standard: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets)

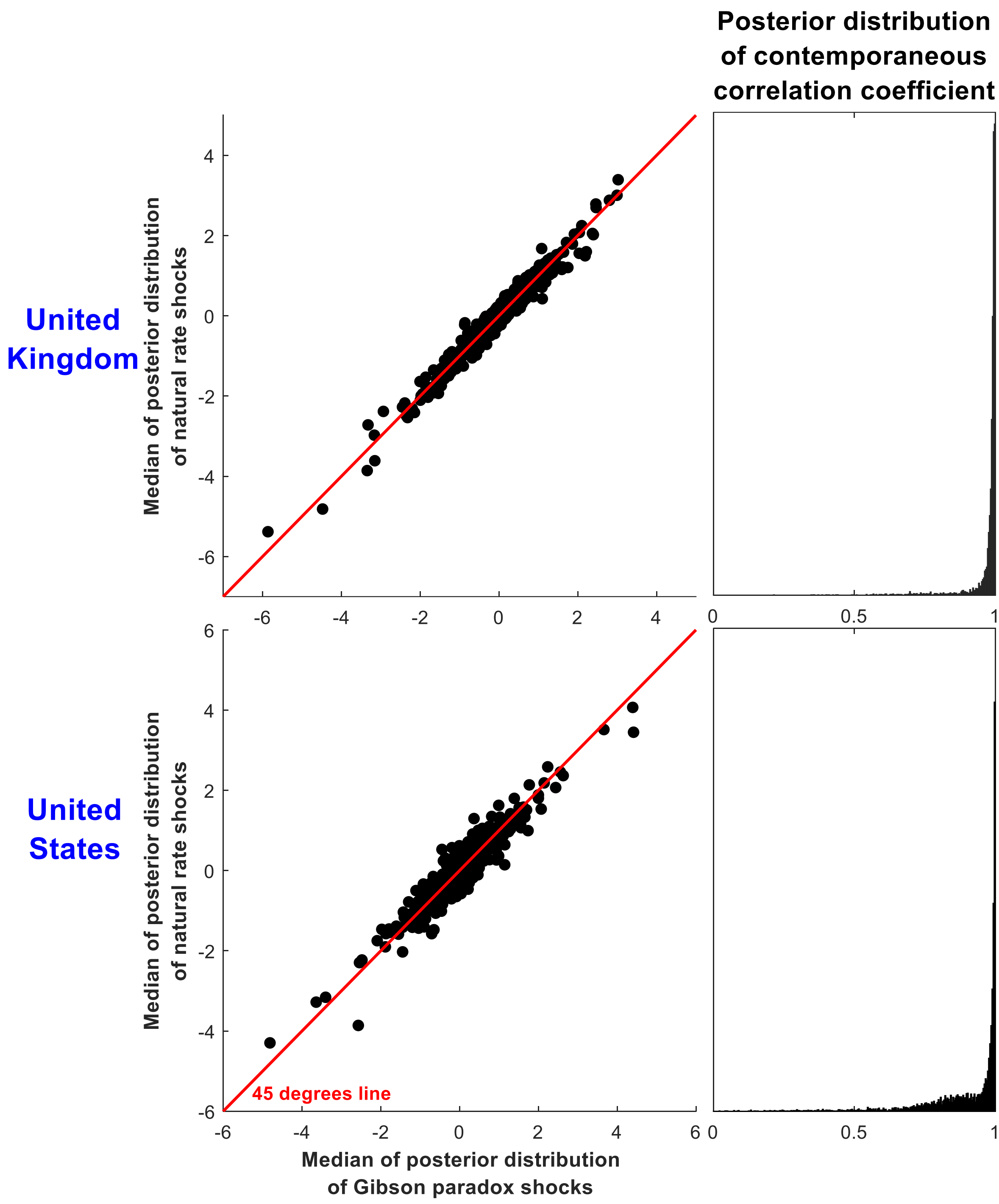


Figure 4a United Kingdom and United States under the Gold Standard: Comparing Gibson's paradox and natural rate shocks

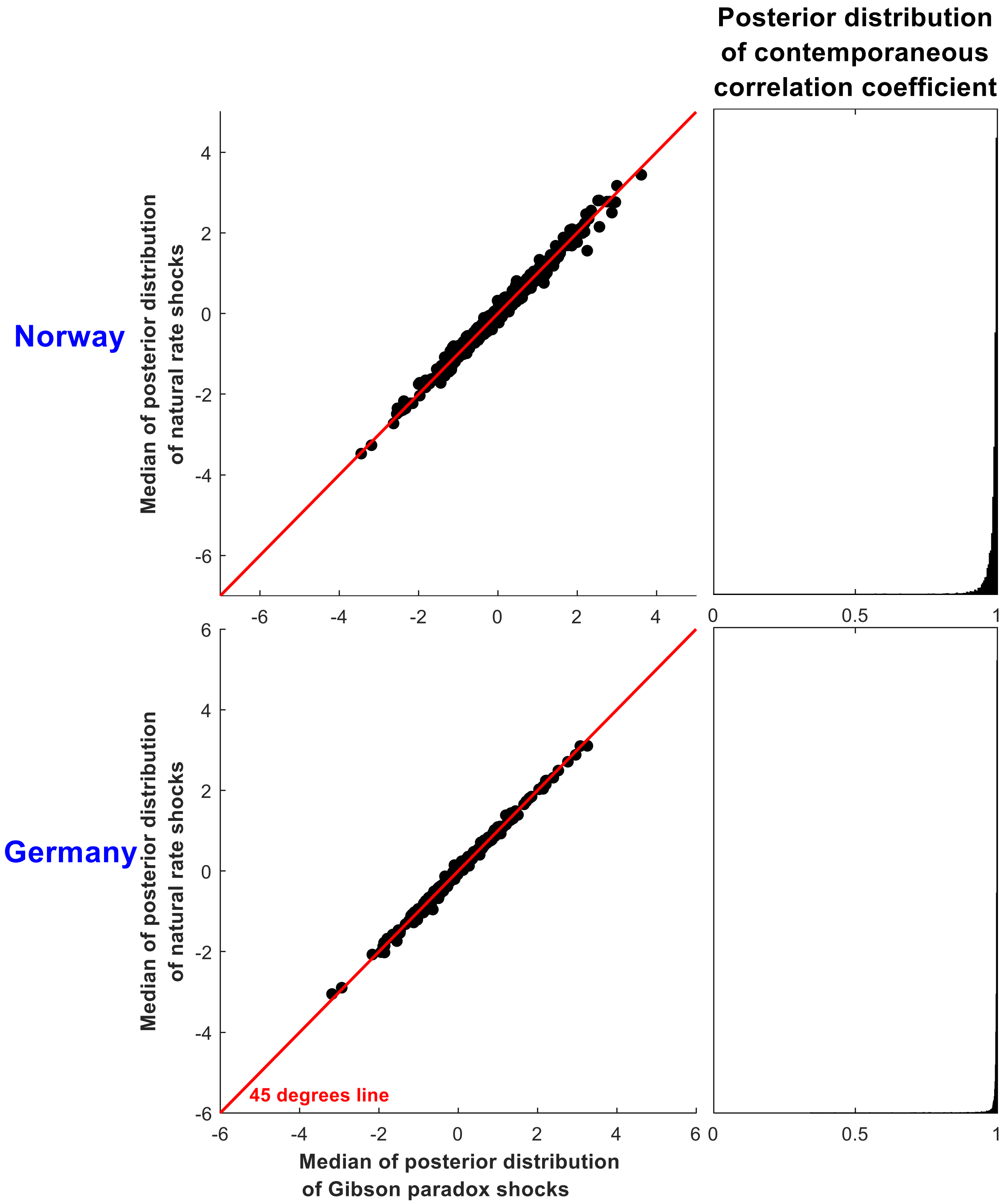


Figure 4b Norway and Germany under the Gold Standard: Comparing Gibson's paradox and natural rate shocks

natural rate shocks generate the very same response in the economy. By the same token, the scatterplots of the shocks are tightly clustered around the 45 degrees line, and the posterior distributions of their contemporaneous correlation coefficient are tightly clustered towards one. In particular, the mode of the posterior distribution of the correlation coefficient is equal to one for all countries, whereas the median ranges between 0.955 for the United States to 0.998 for Germany.

It is to be noticed that our evidence for the correlation coefficient is significantly stronger than that which led Kurmann and Otrok (2013) to argue that disturbances driving the bulk of fluctuations of the slope of the term structure of interest rates are in fact TFP news shocks. Indeed, Kurmann and Otrok (2013, pp. 2625 and 2628) estimated a contemporaneous correlation between slope shocks and TFP news shocks equal to either 0.84 or 0.86.

The IRFs paint a near-uniformly consistent pattern across countries, with the two identified shocks causing a transitory economic expansion in either the United Kingdom or the United States; a sharp, temporary fall in the ex post real short-term rate in all countries; and a decrease in the spread (with the exception of Norway, for which the response is not clear-cut).

We now turn to inflation targeting regimes.

### 5.2.2 Inflation targeting regimes

We estimate VARs for CPI inflation,[32] a long rate, the spread between the long and a short-term rate, and the difference between the logarithms of investment and consumption for Australia, New Zealand, Sweden, the United Kingdom, the Euro area, Switzerland, Denmark, and West Germany. Strictly speaking, only the first four countries are *de jure* inflation targeters. Over the sample periods considered herein (1999Q1-2008Q3), however, the European Central Bank's (ECB) monetary policy strategy had aimed at keeping inflation 'below but close to 2 per cent', so that for all practical purposes it should be regarded as a *de facto* inflation targeter.[33] The same holds for the Swiss National Bank, which following the introduction of its 'new monetary policy concept' in January 2000 has defined price stability as an inflation rate between 0 and 2 per cent. As for Denmark, since the start of European Monetary Union its monetary policy has consistently targeted the exchange rate of the Danish *krona* vis-a-vis the Euro, thus importing the Euro area's monetary policy. Finally, as stressed by Bernanke, Laubach, Mishkin, and Posen (1999, Chapter 4), Germany's monetary policy had been a precursor of inflation targeting.[34] So, although only Australia, New Zealand, Sweden, and the United Kingdom are in fact *de jure*

[32] Evidence based on the personal consumption expenditure (PCE) deflator is qualitatively the same, and it is available upon request.

[33] The ECB moved to a 2 per cent target in July 2021.

[34] We focus on West Germany because of the discontinuities in the data (in particular for consumption and investment) introduced by reunification.

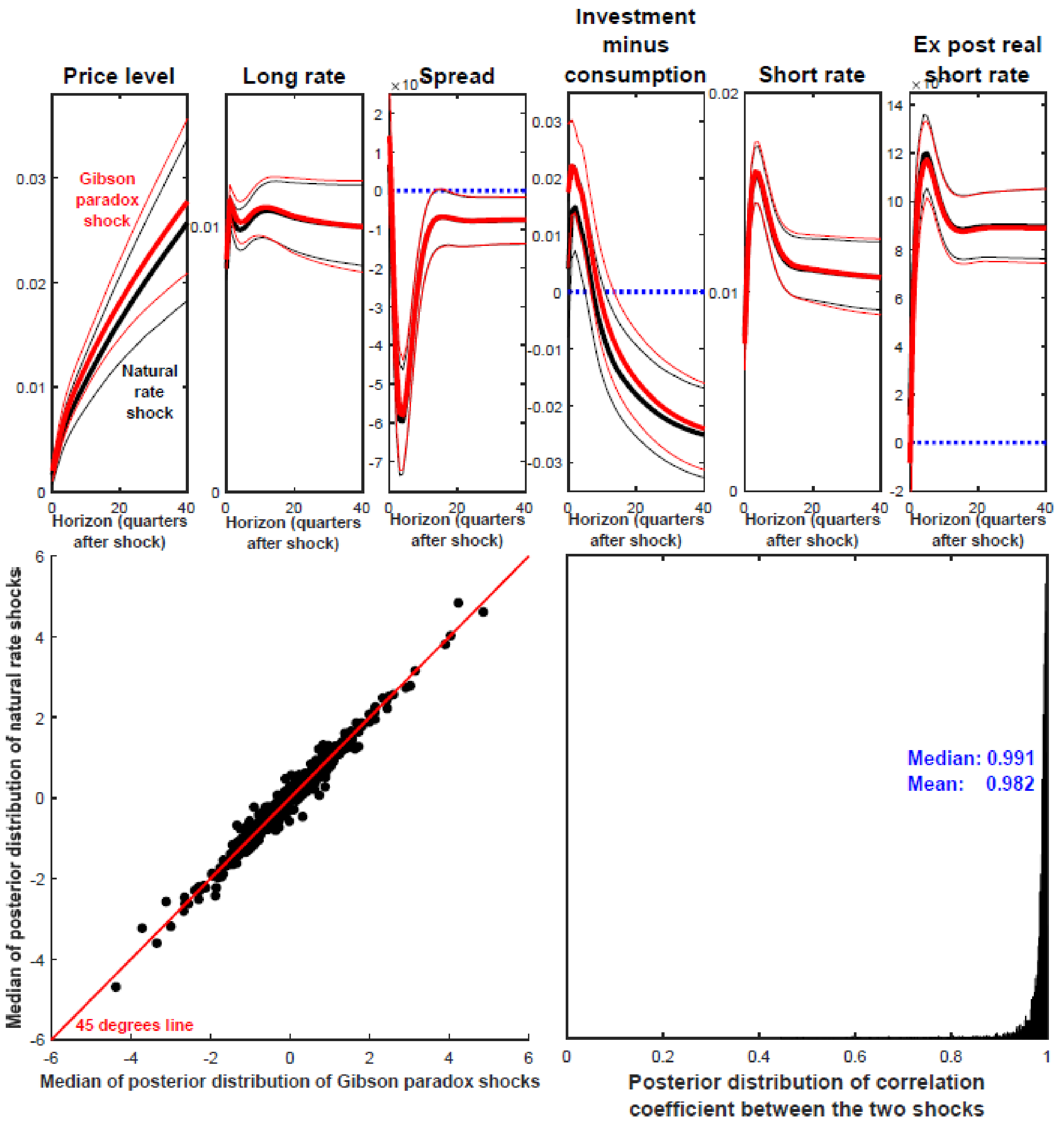


Figure 5 Evidence from joint estimation for inflation-targeting countries: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the shocks

inflation targeters, for the sake of simplicity in what follows we will refer to all of these eight countries as 'inflation targeting regimes'.

There are two main reasons for entering the logarithm of the investment/consumption ratio in the VAR. First, standard Neoclassical growth theory predicts such a ratio to depend on the natural rate of interest. Within the context of Ramsey's optimal growth model, Online Appendix G shows that an increase in the natural rate due to an increase in either the rate of time preference ($\rho$) or the growth rate of consumption per capita ($g$) causes an increase in the consumption/GDP ratio and a decrease in the investment/GDP ratio, thus causing a decrease in the investment/consumption ratio. An increase in the natural rate due to an increase in population growth ($n$), on the other hand, has the opposite effect. To the extent that the decrease in the natural rate of interest that has taken place (at least) since the early 1990s has been mostly driven by decreases in $\rho$ broadly interpreted[35] and $g$, we should therefore expect that very highly persistent shocks to the natural rate should cause a long-horizon decrease in the logarithm of the investment/consumption ratio. As we will see, this is indeed the case.

Second, investment and consumption (and GDP) are cointegrated[36] but whereas consumption is quite close to the common unit root in the system[37] investment features a very large transitory component. As a result, the logarithm of the investment/consumption ratio captures the reaction of transitory real economic activity to economic shocks.

For inflation we set $\mu$ in expression (36) to $\mu = 0$, reflecting the fact that under inflation targeting this series has been near-uniformly indistinguishable from a white noise process with a positive mean.[38] For the long rate and the spread, for the reasons discussed in the previous section, we set $\mu = 1 - 0.5/T$ and $\mu = 0.75$ respectively. Finally, for the logarithm of the investment/consumption ratio we set $\mu = 1 - 0.5/T$, reflecting the fact that as a matter of logic this ratio, although very highly persistent, cannot feature an exact unit root.

Figures A.1$a$-A.1$h$ in the Online Appendix report the medians and the 68 per cent coverage credible sets of the IRFs to the two shocks for any of the eight countries, whereas the top panels of Figure 5 report the same objects obtained from *jointly* estimating the VAR for all of the countries together, allowing for a minimal extent of heterogeneity across countries in the form of *country-specific intercepts*.[39] The two bottom panels of Figure 5 and Figures A.1$a$-A.1$h$ show a scatterplot of the medians of the posterior distributions of the estimated Gibson's paradox and natural rate shocks and, respectively, the posterior distribution of the contemporaneous correlation

[35] See Del Negro, Giannone, Giannoni, and Tambalotti (2017).

[36] See e.g. King, Plosser, Stock, and Watson (1991).

[37] See e.g. Cochrane (1994).

[38] See Benati (2008).

[39] So, to be clear both the VAR matrices $B_1$, $B_2$, ...$B_p$, and the covariance matrix $\Sigma$ are postulated to be common across countries, whereas the vector $B_0$ is country-specific.

coefficient between the two shocks.

Exactly as for the Gold Standard, the evidence in Figure 5 and in Figures A.1$a$-A.1$h$ speaks for itself, and it suggests once again that the two independently-identified shocks are in fact one and the same. First, for all series the 68 per cent coverage credible sets of the IRFs produced by the two shocks are very close, and often near-indistinguishable. Second, the medians of the posterior distributions of the two identified shocks are tightly clustered around the 45 degrees line. Third, the posterior distribution of their contemporaneous correlation coefficient is tightly clustered towards one. Once again, this evidence is significantly stronger than that produced by Kurmann and Otrok (2013): in particular, the median of the posterior distribution of the correlation coefficient in Figure 5 is equal to 0.991 whereas, as mentioned, Kurmann and Otrok's (2013) estimates of the correlation coefficient between slope and TFP news shocks were equal to either 0.84 or 0.86. The corresponding evidence for individual countries is qualitatively the same, with the median of the posterior distribution of the correlation coefficient ranging between 0.9895 and 0.9996 for Australia, Denmark, New Zealand, Sweden, and Switzerland, and being equal to 0.9772 for the Euro area, 0.9582 for West Germany, and 0.9169 for the United Kingdom.

Consistent with the evidence for the Gold Standard, the shocks generate a transitory economic expansion, captured by the increase in the logarithm of the investment/consumption ratio at short horizons, and an insignificant reaction on impact of the spread, followed by a sustained decrease. The long rate jumps on impact and exhibits little variation after that (at least, up to the 10 years horizon considered herein), whereas the response of prices is sluggish and drawn-out. Finally, at long horizons the investment-to-consumption ratio decreases. This suggests that, in the sample periods considered herein, long-horizon variation in the natural rate has mainly originated from changes in the rate of time preference (broadly interpreted) and in the growth rate of consumption per capita.

### 5.2.3 A comparison with the evidence from estimated New Keynesian models

The evidence produced by SVAR methods is especially convincing because it is predicated on a minimal set of assumptions: a plausible time-series representation of the data, and a minimal set of identifying restrictions for the structural disturbances. An alternative approach is to estimate fully-specified DSGE models. The drawback of this approach is that it requires making several high-level assumptions on the exact details of the entire structure of the economy, which might affect the inference in a material way. Because of this, our own preference goes to the previously discussed SVAR-based evidence.

In spite of this, in Online Appendix B we report the evidence obtained by estimating New Keynesian models for either the U.S. or the U.K. under the Gold Standard, and for any of the eight inflation targeting regimes. Two main findings clearly emerge from this body of evidence. First, consistent with the SVAR-based evidence, *only* highly persistent disturbances to the natural rate of interest could have generated Gibson's paradox under the Gold Standard. Second, under inflation targeting such disturbances generate the *very same* conditional correlation between prices and the long rate observed under the Gold Standard.

#### 5.2.4 Considerations suggested by the evidence for inflation targeting regimes

The evidence for inflation targeting regimes from either SVAR methods or estimated DSGE models naturally suggests two considerations:

*first*, it raises the obvious question of why, exactly, Gibson's paradox is nowhere nearly apparent in the raw data generated under inflation targeting. In principle one possible explanation could be that disturbances other than shocks to the natural rate of interest 'blur' the positive long-horizon correlation induced by natural rate shocks, thus making it *disappear* from the raw data. In fact, as the evidence in Section 7 shows, this does not seem to be the case: once controlling for the deterministic component of the drift in the price level induced by the presence of a positive inflation target, a positive long-horizon correlation between the two series can indeed be easily recovered from the data generated by these regimes. This suggests that the main reason why Gibson's paradox is not apparent in the raw data generated under inflation targeting is simply that the presence of a positive inflation target, by introducing a positive drift in the price level, causes the long-horizon correlation to become *hidden* in the raw data.

*Second*, the evidence for these regimes suggests that Gibson's paradox had *nothing* to do with the Gold Standard per se. In particular, the fact that under inflation targeting disturbances to the natural rate generate a positive long-horizon correlation between prices and the long rate naturally suggests that, in principle, other monetary regimes might also be able to generate the paradox.

We therefore now turn to discussing how Gibson's paradox may, or may not arise under alternative monetary policy regimes.

# 6 Long-Term Nominal Interest Rates and Prices Under Alternative Monetary Policy Rules

Figure 6 shows results from the following exercise. We take the posterior distribution of the structural parameters of the New Keynesian model estimated for the United Kingdom under inflation targeting (i.e., the estimated model that produced the IRFs shown in Figure A.4 in the Online Appendix), and for each draw from the posterior we replace the estimated Taylor rule with either the price level targeting rule

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\phi E_t p_{t+1}; \tag{38}$$

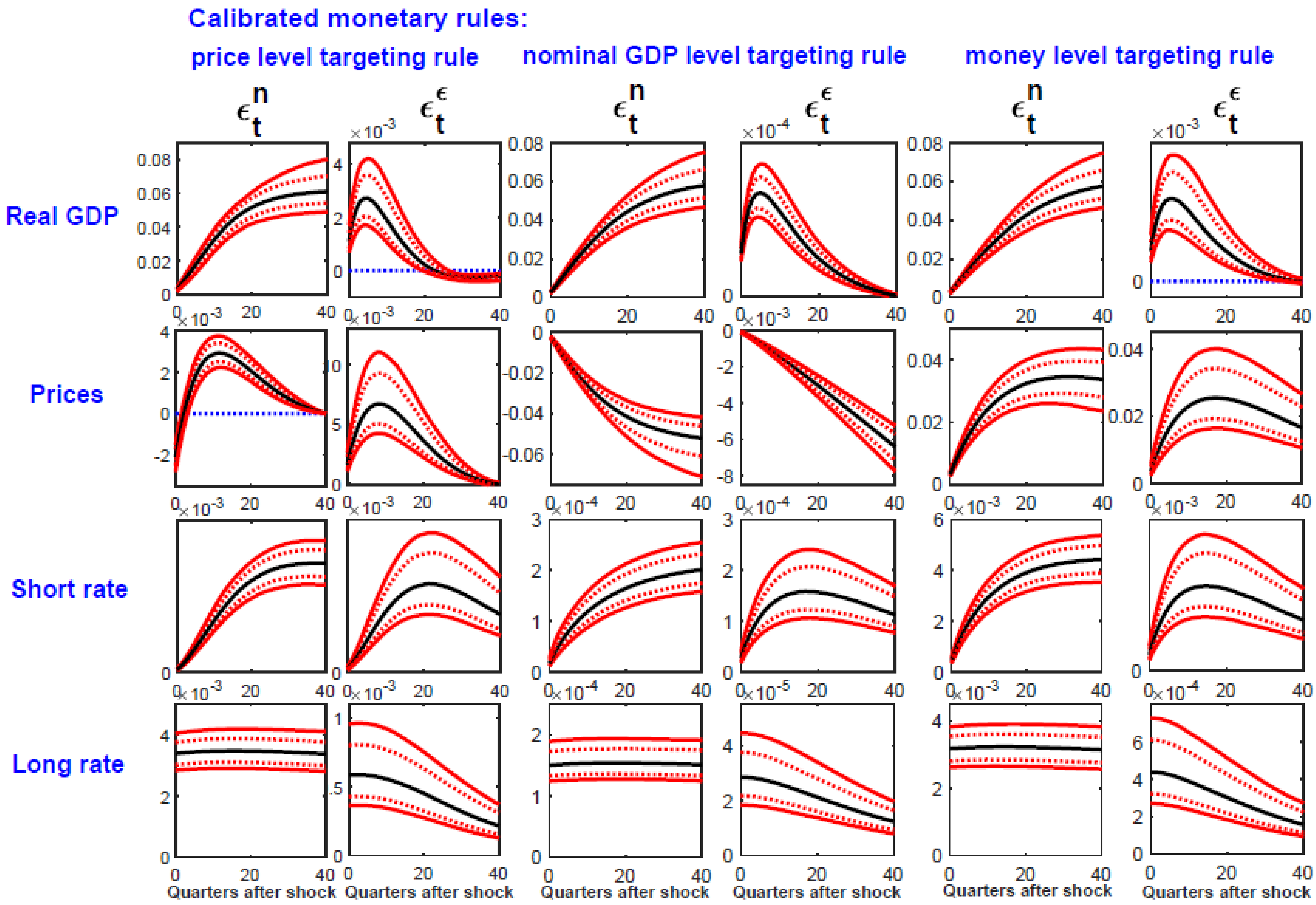


Figure 6 Impulse-response functions of the estimated New Keynesian model to natural rate shocks under alternative monetary policy rules (medians of the posterior distributions, and 68 and 90 per cent coverage credible sets)

the nominal GDP level targeting rule

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\phi[y_t + p_t] = \rho_i i_{t-1} + (1 - \rho_i)\phi[y_t^N + \hat{y}_t + p_t]; \tag{39}$$

or a monetary rule keeping the logarithm of the money stock, $m_t$, constant,

$$m_t = \bar{m} \tag{40}$$

together with the money demand equation

$$m_t - p_t = y_t - \alpha i_t \tag{41}$$

We set the policy parameters as follows. As for $\rho_i$ in (38) and (39), for each draw from the posterior distribution we keep the estimate of $\rho_i$ corresponding to that draw. We set $\phi$ in (38) and (39) to 1.5. As for the money level targeting rule we set $\bar{m} = 0$ and $\alpha = 10$.

Figure 6 reports the IRFs to the two shocks driving the natural rate of interest, $\epsilon_t^n$ and $\epsilon_t^\epsilon$. The former disturbance is an innovation to the trend growth rate of natural outut, whereas the latter one is the corresponding shock to the residual component of the natural rate.[40] We report the IRFs for all the key variables, excluding money velocity which is here of secondary interest. The full set of IRFs is however available upon request.

The evidence confirms that the long-horizon correlation between prices and the long-term rate crucially depends on the nature of the monetary regime. In particular, under regimes targeting either the price level or the level of nominal GDP Gibson's paradox would *not* appear and, different from an inflation targeting regime, it would not even be 'hidden' in the raw data. The reasons for this are straightforward. By its very nature a price level targeting regime, by making prices strongly mean-reverting (as illustrated by their IRFs in Figure 6), rules out as a matter of logic *any* long-horizon correlation between the long rate and the price level. Under nominal GDP targeting, on the other hand, the responses of prices to either $\epsilon_t^n$ or $\epsilon_t^\epsilon$ are *negative*, whereas those of the long rate are once again uniformly positive. The implication is that in response to either shock the correlation between prices and the long rate has the wrong sign at all horizons. Finally, a regime targeting the level of the money stock would generate a positive long-horizon correlation between the two series conditional on either shock. Specifically, $\epsilon_t^n$ would generate a very strong correlation, since both series' IRFs exhibit very high persistence, whereas the correlation produced by $\epsilon_t^\epsilon$ would be somewhat weaker, as the series' responses exhibit a non-negligible extent of mean-reversion.

We now turn to showing that Gibson's paradox can in fact be recovered from the data generated by inflation targeting regimes.

[40] See equations (B.4), (B.11) and (B.12) in the Online Appendix.

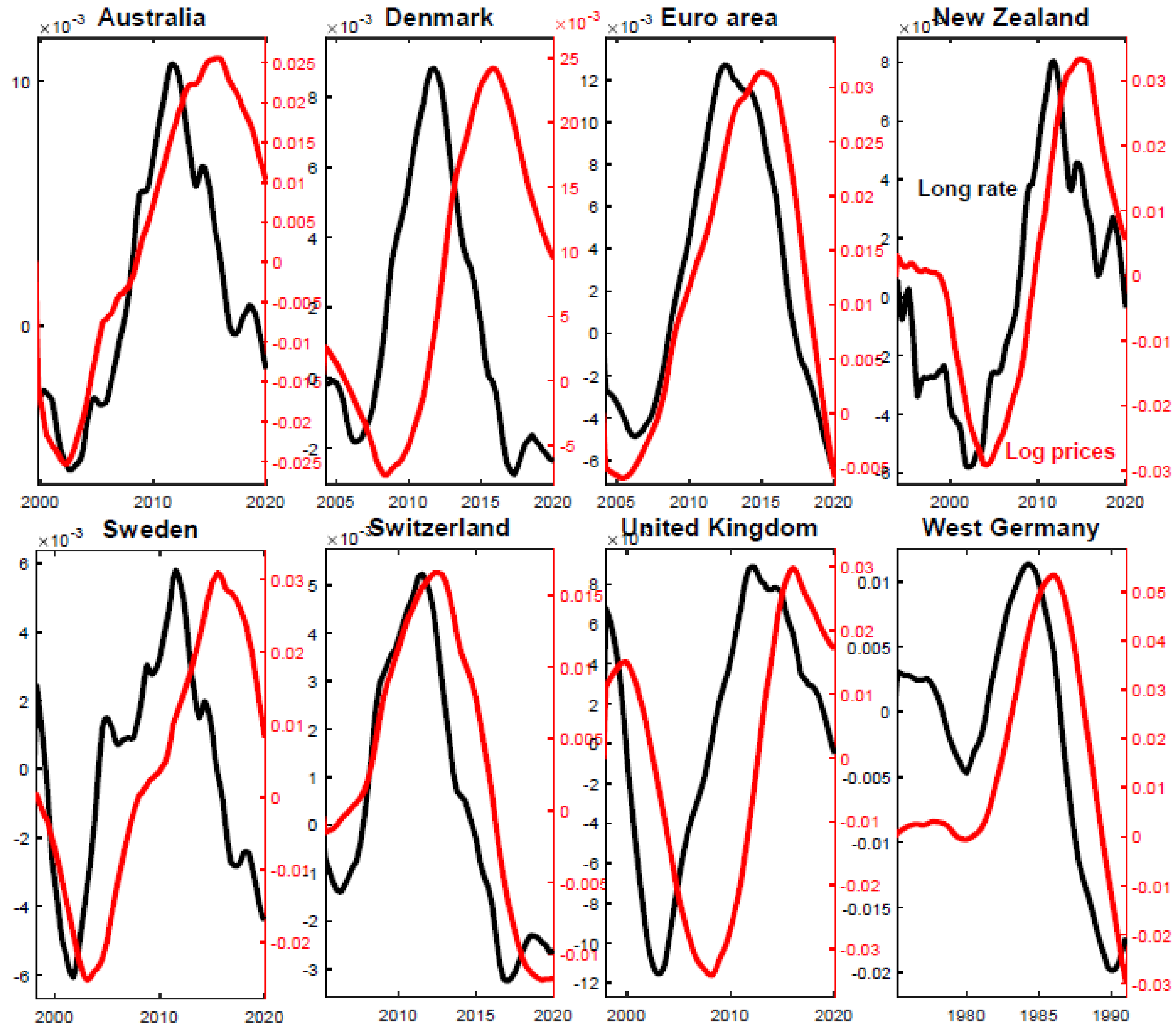


Figure 7 Recovering Gibson's paradox under inflation targeting, European Monetary Union, Switzerland's 'New Monetary Policy Concept', and Denmark's exchange rate targeting regime

# 7 Recovering Gibson's Paradox in the Raw Data Under Inflation Targeting Regimes

As we discussed in Section 4.3, a plausible explanation for the fact that under inflation targeting Gibson's paradox is not apparent in the raw data, as it had been under the Gold Standard, is that (1) the presence of a positive inflation target introduces a positive deterministic drift in the price level,[41] and (2) since (at least) the early 1990s, when inflation targeting regimes started being introduced, the natural rate of interest has exhibited a negative secular drift.

Figure 7 shows evidence in support of this conjecture. The figure reports results from the following exercise. For any of the eight inflation targeting countries we estimate via OLS the VARs in levels

$$Y_t = B_0 + B_1 Y_{t-1} + ... + B_p Y_{t-p} + u_t \tag{42}$$

featuring the same five quarterly series as in Section 5.2.[42] We set the lag order to four. Based on the estimated VARs we then re-run history by (1) setting the estimated VAR intercept, $B_0^{OLS}$, to zero, and (2) feeding to the VAR the *reduced-form* residuals, i.e. the $u_t^{OLS}$. Whereas (1) removes the deterministic drifts in the series—in particular, in the price level and the long-term nominal interest rate—(2) makes sure that the counterfactual we are running is conditional on *exactly* the same shocks that have historically driven the economy. Finally, in order to focus on the long horizons, we smooth the resulting counterfactual series by taking 5-year rolling averages.

Evidence of a positive long-horizon correlation between prices and the long-term nominal rate is very clear. Interestingly, there is clear evidence of a low-frequency lead of long rates onto prices. This is compatible with the SVAR evidence in Figure 5, where on impact the long rate essentially jumps to its new long-run equilibrium level, whereas the response of the price level is delayed and drawn-out.

# 8 Assessing the Sub-Optimality of Monetary Policy

The appearance of Gibson's paradox under either the Gold Standard or inflation targeting regimes is clear indication of the *sub-optimality* of the monetary policies

[41] Notice that since the drift is *deterministic*, it has *nothing* to do with our notion of 'long run', which involves the stochastic portion of a process at either ($i$) long horizons or ($ii$) low frequencies. Although ($i$) and ($ii$) refer to the time and, respectively, the frequency domain, for our own purposes they capture the same notion.

[42] CPI inflation, a long-term nominal interest rate, the spread between a long- and a short-term nominal interest rate, and the logarithms of real consumption and real investment.

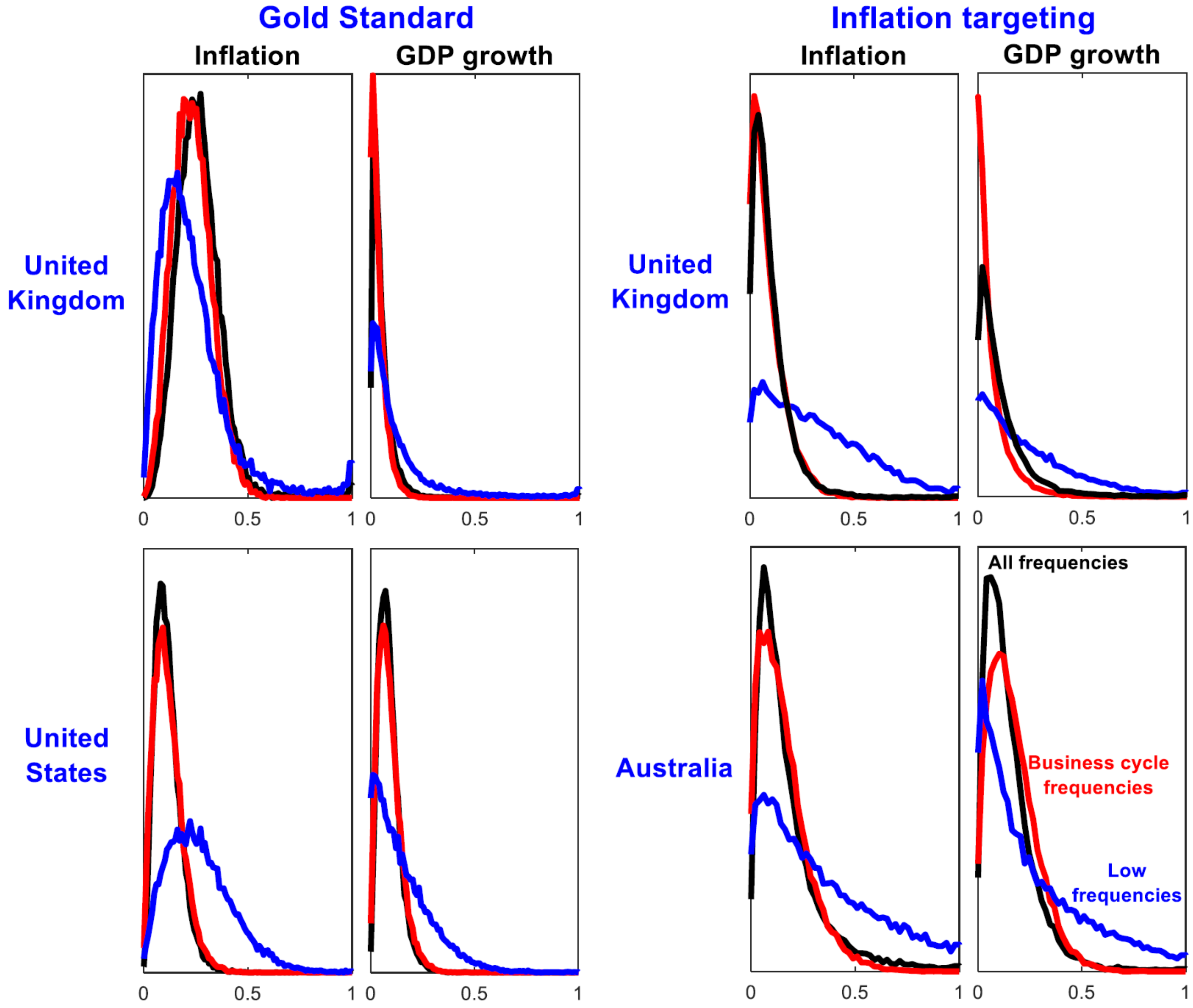


Figure 8 Posterior distributions of the fractions of variance of inflation and real GDP growth explained by shocks to the natural rate of interest by frequency band

that had, and have been followed under these monetary frameworks.[43] Intuitively, if the central bank were able to

(1) precisely track fluctuations in the natural rate of interest and

(2) neutralize their impact on the economy

Gibson's paradox would *never* appear in the data. To the extent that instead either it appears in the raw data (as under the Gold Standard), or it can be easily recovered (as under inflation-targeting regimes), this implies that the central bank either ($i$) is unable to precisely track fluctuations in the natural rate or ($ii$) if it is able to do so, it somehow fails to neutralize their impact on the economy. Under either set of circumstances, monetary policy is in fact sub-optimal.

A natural metric for assessing the sub-optimality of monetary policy is therefore the fraction of the variance of macroeconomic time series that is explained by shocks to the natural rate. Figure 8 reports evidence from the following exercise. For the United Kingdom and the United States under the Gold Standard, and for the United Kingdom and Australia under inflation targeting,[44] we estimate Bayesian VARs for CPI inflation,[45] real GDP growth, and a short- and a long-term nominal interest rate.

We estimate the VARs as in Section 5, based on the methodology proposed by Giannone, Lenza, and Primiceri (2015), with the only difference that in estimation we impose stationarity upon the VAR as in e.g. Cogley and Sargent (2002, 2005) and Primiceri (2005).[46] This reflects our prior view that all of these series are ultimately mean-reverting.[47]

In setting up the Minnesota-type prior we proceed as follows. For inflation we set $\mu$ in (36) to zero, reflecting the fact that under inflation targeting regimes this series has been near-uniformly indistinguishable from a white noise process with a positive mean (Benati, 2008). For real GDP growth we set it $\mu = 0.5$, reflecting the mild extent of serial correlation that is typical of this series. For the short- and the long-term nominal rate, on the other hand, we set $\mu$ to 0.75 and to 1-0.5/$T$, respectively, where $T$ is the sample length.[48] For the short rate this reflects its non-negligible extent of persistence, whereas for the long rate the fact that this series is best thought of as near unit root process.

For each draw from the posterior distribution we then identify the shock to the

[43] Woodford (2023) discusses the conditions under which optimal policy results in the nominal interest rate mimicking the natural rate of interest.

[44] Under inflation targeting we exclude the period following the collapse of Lehman Brothers.

[45] Evidence based on the PCE deflator is qualitatively the same, and it is available upon request.

[46] So, to be clear, in the MCMC algorithm used for estimation, we move to iteration $i$+1 if and only if the draw for the VAR's parameters associated with iteration $i$ is stationary. Otherwise, we redraw the parameters for iteration $i$.

[47] Both nominal interest rates, and to a lesser extent real GDP growth have exhibited broad downward trends over the sample period. As a matter of logic, however, neither variable can feature a unit root, because this would imply that they could take literally *any* value between minus and plus infinity.

[48] As discussed in Section 5.2.1, setting $\mu$ = 1-0.5/$T$ corresponds to the parameterization for a near unit root process.

natural rate of interest as before, i.e. as the disturbance explaining the maximum fraction of the variance of the long rate at the 25 years horizon. We then Fourier-transform the identified SVAR, and for each series we compute the fraction of its variance that is explained by natural rate shocks across either all frequencies, the business-cycle frequencies, or the low frequencies.[49] In doing so we exploit the fact that the variance of a stationary series within a specific frequency band is equal to the integral of its spectral density within that band.

Figure 8 shows the posterior distributions of the fractions of variance of inflation and real GDP growth explained by shocks to the natural rate of interest.[50] The main finding is that for all countries, and under either monetary regime, shocks to the natural rate had, and have explained non-negligible fractions of the variance of the two series. In the light of the long-standing criticism of the inflexibility of the Gold Standard,[51] the evidence for this regime should come as no surprise. More interesting is the fact that evidence for inflation targeting regimes is qualitatively the same, as this points towards the sub-optimality of the way inflation targeting has been implemented in practice. Once again it is important to stress that, by itself, this result is silent on the ultimate cause(s) of such failure on the part of monetary authorities to neutralize the impact on the economy of shocks to the natural rate. However, be it because they could not precisely track its fluctuations, or because they somehow failed to act upon such knowledge, it is a fact that under both regimes monetary policies had and have been sub-optimal.

By the same token, based on a sample of ten inflation-targeting countries since the 1990s, Benati (2025) shows that in recent years shocks to the natural rate of interest have played a large, or even dominant role in driving the dynamics of credit leverage[52] and real asset prices. In particular, these shocks have driven about 70-80% of long-horizon fluctuations in real house prices, and smaller, but still sizeable fractions of long-horizon variation of credit leverage and oil and gold prices.

# 9 Conclusions

For more than a century a vast literature has studied Gibson's paradox, without reaching any consensus on what, exactly, had originated it. Following Friedman and Schwartz (1982), the only broad agreement appears to be that, since the paradox had only appeared under the Gold Standard, and it has instead been absent from post-WWII raw data, it had likely originated from the peculiar workings of monetary regimes based on commodity money.

[49] Following standard conventions we define the business cycle and the low frequencies as those pertaining to fluctuations between 6 and 32 quarters, and beyond 32 quarters, respectively.

[50] The full set of results is reported in Figures A.5*a*-A.5*b* in the Online Appendix.

[51] See in particular Keynes (1925) and Eichengreen (1996).

[52] Defined as the ratio between credit from domestic commercial banks to the domestic private non-financial sector and nominal GDP.

In this paper we have advanced the view that Gibson's paradox has nothing to do with the Gold Standard or commodity-money regimes per se, and it rather originates from long-horizon variation in the natural rate of interest under certain types of monetary regimes that make inflation Strongly mean-reverting. The implication is that Gibson's paradox is, in principle, a feature of a potentially wide array of monetary arrangements. Consistent with this, we have shown that Gibson's paradox can be recovered from the data generated under inflation targeting regimes once removing the deterministic component of the drift in the price level induced by the presence of a positive inflation target.

Intuitively, the mechanism underlying the emergence of Gibson's paradox under specific types of monetary regimes hinges on the interaction between the Fisher equation and an asset pricing condition determining the current value of money. The Fisher equation implies that long-horizon fluctuations in the natural rate of interest automatically map, one-for-one, into corresponding fluctuations in nominal interest rates at all maturities. The asset pricing condition, on the other hand, implies that increases in the natural rate—which is the discount factor for the determination of the current value of money—map into decreases in the expected value of future money, and therefore in the current value of money, which are obtained via corresponding increases in the price level. Long-horizon increases (decreases) in the natural rate of interest therefore map into corresponding long-horizon increases (decreases) in the price level via the asset pricing condition, and in nominal interest rates via the Fisher equation. Although this mechanism works for nominal interest rates at all maturities, in practice it is especially apparent for long rates, which behave as the long-horizon components of short rates.

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# Online Appendix for: Gibson's Paradox and the Natural Rate of Interest

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May 9, 2025

## A The Data

Here follows a detailed description of the dataset.

### A.1 The gold standard

**Canada** An annual series for the consumer price index (CPI) is from Rolnick and Weber's (1995, 1997) dataset. An annual series for a nominal long-term interest rate has been kindly provided by Warren Weber.

**Denmark** Annual series for the CPI and a nominal long-term interest rate are both from the dataset assembled by Kim Abildgren, which is described in detail in Abildgren (2006), and is available from his web page at: https://sites.google.com/view/kim-abildgren/historical-statistics.

**France** An annual series for the price level is from Rolnick and Weber's (1995, 1997) dataset. An annual series for a nominal long-term interest rate ('taux long') is from Section 7 ('Evaluation des taux de l'interet'), pp 93-96, of Saint Marc (1983).

**Germany** Monthly series for the price level ('Wholesale Price Index for Hamburg, Germany, Index 1879-1888=100, Monthly, Not Seasonally Adjusted'), a nominal long-term interest rate ('Bond Yields for Germany'), and a nominal short-term interest rate ('Private Discount Rate, Prime Banker's Acceptance, Open Market for Berlin, Germany, Percent, Monthly, Not Seasonally Adjusted') are all from the NBER Historical database. The FRED II acronyms are M04054DE00HAMM314NNBR, M1328ADEM193NNBR, and

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M13018DE00BERM156NNBR, respectively. The wholesale price index series has been seasonally adjusted via ARIMA X-12 as implemented in EViews.

**Norway** A monthly series for a nominal long-term interest rate is from Klovland (2004). A monthly series for Norges Bank's discount rate, end-of-month data, is from Norges Bank. A monthly series for the wholesale price index available for the period January 1777-December 1919 is from Norway's long-run historical statistics database, which is available at the website of Norges Bank (Norway's central bank). The data are documented in Grytten (2014). The wholesale price index series has been seasonally adjusted via ARIMA X-12 as implemented in EViews.

**United Kingdom** All U.K. data are from version 3.1 of the Excel spreadsheet "A millennium of macroeconomic data" which is available from the Bank of England's website at: http://www.bankofengland.co.uk/statistics/research-datasets. The first version of the dataset (which was called "Three centuries of macroeconomic data") was discussed in detail in Hills and Dimsdale (2010). Details for the series plotted in Figure 1.$a$ in the main text are as follows. In the first panel, an annual series for the CPI is from sheet A.47 ('Wages and prices'), whereas a series for a consol rate is from sheet A.31 ('Interest rates'). In the second panel, a monthly seasonally unadjusted series for the prices of domestic commodities from sheet M.6 ('Monthly prices and wages'), whereas a series for the yield on 3% consols is from sheet M.10 ('Monthly long-term interest rates'). The series for the prices of domestic commodities is originally from Gayer, Rostow, and Schwartz (1953). In the third panel the monthly seasonally unadjusted series for the 'Sauerbeck Statist price index including duty' if from sheet M.6 ('Monthly prices and wages'), whereas in the fourth panel the monthly seasonally unadjusted series for the 'Sauerbeck Statist index for all commodities' if from sheet M.6 ('Monthly prices and wages'). In the third and fourth panels, a series for the 'Yield on consols corrected for Goschen's conversion issues' is from sheet M.10 ('Monthly long-term interest rates'). Then, a seasonally unadjusted weekly series for 'Gold Coin and Bullion in the Bank of England's balance sheet' is from sheet W.1 ('Issue Department'), and it has been converted to the monthly frequency by taking averages within the month. Monthly series for the Bank of England's discount rate and a consol yield are from sheet M.9 ('Monthly short-term interest rates 1694-2016') and M.10 ('Monthly long-term interest rates 1753-2016'), respectively. Turning to the quarterly data used to estimate the New Keynesian model in Section 4, series for the Bank of England's discount rate, a consol yield, and the Bank Of England stock of gold have been obtained by converting to the quarterly frequency the previously mentioned corresponding monthly series, by taking averages within the quarter. As for the price level we took the monthly seasonally unadjusted series for the 'Spliced wholesale/producer price index, 1790-2015' from sheet M.6 ('Monthly prices and wages'), we seasonally adjusted it via ARIMA X-12 as implemented in *EViews*, and we converted it to the quarterly frequency by taking averages within the quarter. Finally, as for real GDP we interpolated to the quarterly frequency the annual real GDP series from sheet A.8 ('National Accounts')—i.e. the series 'Real UK GDP at market prices, geographically-consistent estimate based on post-1922 borders, £mn, Chained Volume measure, 2013 prices'—as in Bernanke, Gertler, and Watson (1997), using as the quarterly interpolator series capturing the state of the business cycle the monthly unemployment rate series (converted to the quarterly frequency by taking averages within the quarter) from sheet M.5 ('Monthly activity'). For the purpose of performing econometric work (but not for plotting purposes), all seasonally unadjusted series have been seasonally adjusted via ARIMA X-12 as implemented in *EViews*.

**United States** Details for the series plotted in Figure 1.*b* in the main text are as follows. The monthly seasonally unadjusted series for the wholesale price index is from Warren and Pearson (1933), whereas the quarterly series for a long-term nominal interest rate is the series for the 'Yield on corporate bonds' from Table 2 of Balke and Gordon (1986). Turning to the quarterly series used to estimate the New Keynesian model, the long rate is the just mentioned series for the yield on corporate bonds, whereas a seasonally adjusted series for 'Real GNP in 1972 dollars' is also from Table 2 of Balke and Gordon (1986). The series for the price index has been obtained by seasonally adjusting the previously mentioned series from Warren and Pearson (1933) via ARIMA X-12 as implemented in *EViews*, and then converting the resulting series to the quarterly frequency by taking averages within the quarter. Monthly series for call money rate ('Call Money Rates, Mixed Collateral for United States, Percent, Monthly'), the stock of gold held by the monetary authority ('Gold Held in the Treasury and Federal Reserve Banks for United States, Millions of Dollars, Monthly'), and the remaining stock of gold in the economy ('Gold Outside the Treasury and Federal Reserve Banks for United States, Millions of Dollars, Monthly') are all from the NBER Historical database (the FRED II acronyms are M13001USM156NNBR, M1437AUSM144NNBR, and M1431AUSM144NNBR respectively). All of the three series have been converted to the quarterly frequency by taking averages within the quarter. Since the original gold stock series are seasonally unadjusted, we seasonally adjusted them via ARIMA X-12 as implemented in *EViews*.

## A.2 Interwar period

Details for the series plotted in Figure 1.$c$ in the main text are as follows.

**Denmark** Monthly series for the CPI and a nominal long-term interest rate ('Yield on long-term mortgage bonds') are both from the dataset assembled by Kim Abildgren, which is described in detail in Abildgren (2006), and is available from his web page at: https://sites.google.com/view/kim-abildgren/historical-statistics.

**Norway** A monthly series for the CPI available since January 1920 is from Norway's long-run historical statistics database, which is available at the website of Norges Bank (Norway's central bank). The data are documented in Klovland (2013). A monthly series for a nominal long-term interest rate is also from Norway's long-run historical statistics database. Details about the series are as follows. Until December 1920 it is the series for 'Norwegian long-term government bonds, monthly (1820-1920)', whereas since January 1921 it is the series ST10 from the sheet p1_c4_table_A3_Monthly ('Norwegian bond yields by maturity (average life), monthly (1921-2005)') in the spreadsheet bond_yields.xls. Both series are documented in Klovland (2004).

**United Kingdom** Again, the data are from version 3.1 of the Excel spreadsheet "A millennium of macroeconomic data". Specifically, a series for the wholesale price index ('Spliced wholesale/producer price index') is from sheet M.6 ('Monthly prices and wages'), whereas a series for a long-term nominal interest rate ('Long-term consols yield 1753-2015, corrected for Goschen's conversion issues') is from sheet M.10 ('Monthly long-term rates').

**United States** The monthly seasonally unadjusted series for the wholesale price index is from Warren and Pearson (1933). The long-term nominal interest rate series ('Yield On Long-Term United States Bonds for United States') is from the NBER Historical database (the FRED II acronym is M1333AUSM156NNBR).

## A.3 Post-WWII period

**Australia** Quarterly seasonally adjusted series for nominal and real GDP, real consumption and real investment expenditure, the personal consumption expenditure (PCE) deflator, and the CPI are all from the Australian Bureau of Statistics. The short rate ('3-month BABs/NCDs, Bank Accepted Bills/Negotiable Certificates of Deposit-3 months; monthly average, Quarterly average, Per cent, ASX, 42767, FIRMMBAB90') is from the Reserve Bank of Australia. M1 ('M1: Seasonally adjusted, $ Millions') is from the Reserve Bank of Australia. A series for a nominal long-term interest rate has been constructed as follows. Until 2013Q1 it is the series for the 10-year yield on Australian government bonds, which is available at the Reserve Bank of Australia's website. Since then it is the series for the 10-year yield on Commonwealth government bonds.

**Denmark** Quarterly seasonally adjusted series for nominal and real GDP, real consumption and real investment expenditure, the PCE deflator, and the CPI are all from Statistics Denmark. A monthly seasonally adjusted series for M1 ('Money stock M1, end of period, Units: DKK bn.') is from Denmark's central bank, Danmarks Nationalbank, and it has been converted to the quarterly frequency by taking averages within the quarter. A series for the central bank's discount rate is from Danmarks Nationalbank. A series for the 'Yield on long-term Danish government bonds' is from Kim Abildgren's database.

**Euro area** All of the quarterly data are from the European Central Bank's statistical database, which is accessible at its website.

**New Zealand** Quarterly seasonally adjusted series for nominal and real GDP, real consumption and real investment expenditure, the PCE deflator, and the CPI are all from Statistics New Zealand. A quarterly seasonally adjusted series for M1 is from the Reserve Bank of New Zealand. Monthly series for the 'Overnight interbank cash rate' and the 10-year yield on government bonds traded on the secondary market are both from the Reserve Bank of New Zealand, and they have been converted to the quarterly frequency by taking averages within the quarter.

**Sweden** Quarterly seasonally adjusted series for nominal and real GDP, real consumption and real investment expenditure, the PCE deflator, and the CPI are all from Statistics Sweden. A monthly series for the 3-month interbank rate is from FRED II (the acronym is IR3TIB01SEM156N). A monthly series for the 10-year yield on government bonds is from Statistics Sweden. Both series have been converted to the quarterly frequency by taking averages within the quarter.

**Switzerland** Both M1 and the short rate ('Monetary aggregate M1, Level' and 'Switzerland - CHF - Call money rate (Tomorrow next)', respectively) are from the Swiss National Bank's internet data portal. Quarterly seasonally adjusted series for nominal and real GDP, real consumption and real investment expenditure, the PCE deflator, and the CPI are all from the State Secretariat for Economic Affairs (SECO) at https://www.seco.admin.ch/seco/en/home. A series for the 10-year yield on government bonds is from FRED II, the internet data portal at the Federal Reserve Bank of St. Louis website (the acronym is IRLTLT01CHM156N).

**United Kingdom** Quarterly seasonally adjusted series for nominal and real GDP, real consumption and real investment expenditure, the PCE deflator, and the CPI are all from the Office for National Statistics. A break-adjusted stock of M1 is from version 3.1 of the Excel spreadsheet “A millennium of macroeconomic data” (specifically, it is from sheet Q.3, ‘Quarterly break-adjusted and seasonally-adjusted monetary aggregates 1870-2016’). Quarterly series for a nominal long-term rate (‘Medium-term/10 year bond yield’) and the Bank of England’s monetary policy rate (‘Bank Rate’) are both from sheet M.1 (‘Monthly headline series, Quarterly average of monthly series’) of the spreadsheet “A millennium of macroeconomic data”.

**West Germany** Seasonally adjusted series for the CPI, a short-term nominal interest rate, and real chain-linked private consumption and gross fixed capital formation are all from the Bundesbank. A series for a long-term nominal interest rate is from the International Monetary Fund’s International Financial Statistics.

### A.4 The sample periods

For all countries results are based on samples excluding the period following the collapse of Lehman Brothers. The main reasons for doing so are (1) the extraordinary turbulence of most of that period; and (2) the fact that for several countries it was characterized by the introduction of unconventional monetary policies. Although in estimation we use, whenever possible, a shadow rate instead of the standard monetary policy rate, it is an open question how much the combination of (1) and (2) could have distorted the inference. As for the starting dates, they are the following. For New Zealand and the United Kingdom, which introduced inflation targeting in February 1990 and October 1992 respectively, they are 1990Q2 and 1993Q1. As for Australia, which never explicitly announced the introduction of the new regime, we follow Benati and Goodhart (2011) in taking 1994Q3 as the starting date. The rationale is that, based on the central bank’s communication, during those months it became apparent that the central bank had indeed started following an inflation-targeting strategy. For the Euro area, where European Monetary Union started in January 1999, we start the sample in 1999Q1. By the same token for Switzerland, for which the Swiss National Bank introduced a new ‘monetary policy concept’ in January 2000, we take 2000Q1 as the starting date. As for Denmark, which has consistently followed a policy of pegging the Krone first to the Deutsche Mark and then to the Euro, thus importing the strong anti-inflationary stance of the Bundesbank, and then of the European Central Bank, we start the sample in 1999Q1. Finally, as for West Germany we start the sample in 1970Q1 due to data availability, and we end it at the quarter preceding German reunification, i.e. in 1990Q4.

# B Details about the New Keynesian Models

## B.1 The model for the Gold Standard

We start by discussing the structure of the model, and we then turn to the stochastic properties of the key driving processes.

### B.1.1 The model

In order to estimate the model of Section 4.1 we make some additional assumptions that are motivated by the need to have a realistic empirical characterization of the data. Specifically, we add habit formation in consumption in order to capture the high inertia that characterizes empirical measures of the output gap, and price rigidities in order to provide a realistic description of the inflationary process.[1]

To model habit formation in consumption we consider a utility function of the form

$$U(C_t - hC_{t-1}) = \frac{(C_t - hC_{t-1})^{1-\sigma_x^{-1}}}{1-\sigma_x^{-1}} \tag{B.1}$$

Denoting by $x_t = C_t - hC_{t-1}$ we have that $\hat{x}_t = (1-h)^{-1}(\hat{C}_t - h\hat{C}_{t-1})$ and $\sigma_x^{-1} \equiv -(xU_{xx}/U_x)$. A first-order approximation of the Euler equation implies that

$$\hat{x}_t = E_t\hat{x}_{t+1} - \sigma_x(\hat{\imath}_t - E_t\pi_{t+1} - \epsilon_t), \tag{B.2}$$

where $\epsilon_t$ is a reparameterization of preference shocks and $\hat{\imath}_t$ is the short-term nominal interest rate in log-deviations with respect to the steady state. Assuming consumption is equal to output, and detrending output by the natural rate of output, $y_t^n$ we obtain

$$\tilde{y}_t = \gamma E_t\tilde{y}_{t+1} + (1-\gamma)\tilde{y}_{t-1} - \sigma(\hat{\imath}_t - E_t\pi_{t+1} - r_t^n) + v_t, \text{ with } \epsilon_t^v \sim N(0, \sigma_v^2), \tag{B.3}$$

in which $\tilde{y}_t$ is the output gap; $\sigma = \sigma_x(1-h)/(1+h), \gamma = 1/(1+h)$; and the natural rate of interest is

$$r_t^n = \epsilon_t + \sigma^{-1}\gamma E_t\Delta y_{t+1}^n - \sigma^{-1}(1-\gamma)\Delta y_t^n. \tag{B.4}$$

As in Calvo's model, the Phillips curve is given by

$$\pi_t = k(\eta y_t + \sigma_x^{-1}\hat{x}_t) + \beta E_t\pi_{t+1} + u_t, \text{ with } u_t \sim N(0, \sigma_u^2)$$

[1] Notice that empirical evidence (see in particular Kackmeister, 2007) suggests that under the Gold Standard prices were markedly *stickier* than after World War II. In fact this is what one should expect under a monetary regime in which average inflation had been essentially zero, as opposed to the positive values of the post-WWII period. On this see also Levy and Young (2004).

with $\pi_t = p_t - p_{t-1}$, where $p_t$ is the logarithm of the price level; $k$ is a positive parameter; $\eta$ is the inverse of the Frisch elasticity of labor supply; and $\beta$ is the rate of time preference. Detrending the previous expression we obtain

$$\pi_t = \kappa \tilde{y}_t + \kappa_1 \Delta \tilde{y}_t + \kappa_1 \Delta y_t^n + \beta E_t \pi_{t+1} + u_t, \tag{B.5}$$

in which $\kappa = k(\eta + \sigma_x^{-1})$ and $\kappa_1 = k\sigma^{-1}(1-\gamma)$.

To complete the model under the Gold standard we take a first-order approximation of equation (9) in the main text obtaining a further restriction for the inflation process as

$$\pi_t = -\vartheta_g \Delta \hat{g}_t^c + \vartheta_G \Delta \hat{g}_t - \vartheta_\theta (\Delta \hat{\theta}_t^g - \Delta \hat{\theta}_t^m), \tag{B.6}$$

where $\vartheta_g, \vartheta_G$ and $\vartheta_\theta$ are all positive parameters detailed in Online Appendix D.

A first-order approximation of the equilibrium on the money market implies an equilibrium relationship involving the levels of the central bank's gold stock, prices, output and interest rates:

$$\hat{g}_t^c - p_t = q_y \hat{x}_t - q_i \hat{\imath}_t - q_\theta (\hat{\theta}_t^c - \hat{\theta}_t^m), \tag{B.7}$$

for positive parameters $q_y$, $q_i$ and $q_\theta$ detailed in Online Appendix D. By taking first differences of (B.7) we obtain

$$\begin{aligned} \Delta \hat{g}_t^c - \pi_t \quad = \quad & q_y(1+\rho)(\Delta y_t + \Delta y_t^n) - q_y \rho (\Delta y_{t-1} + \Delta y_{t-1}^n) + \\ & -q_i(\Delta \tilde{\imath}_t + \Delta r_t^n) - q_\theta (\Delta \hat{\theta}_t^c - \Delta \hat{\theta}_t^m), \end{aligned}$$

where $\rho = (1-\gamma)\sigma_x/\sigma$.

To characterize the long-term interest rate, $\hat{\imath}_{L,t}$, we postulate a bond with a decaying coupon structure 1, $\delta$, $\delta^2$, $\delta^3$..., where $\delta > 0$, and such that:

$$\delta = (1+i)\left(1 - \frac{1}{m}\right), \tag{B.8}$$

in which $m$ is the maturity of the long-term bond expressed in quarters. Note that, in a first-order approximation, the price of the long-term bond $Q_t$ is related to the short-term rate and to the expected future bond price as

$$\hat{Q}_t = -\hat{\imath}_t + \frac{\delta}{1+i} E_t \hat{Q}_{t+1},$$

where $1+i$ is the steady-state gross nominal rate. In Online Appendix E we show that this implies that the long-term interest rate is related to the short-term rate through

$$\hat{\imath}_{L,t} = \frac{1+i-\delta}{1+i} \hat{\imath}_t + \frac{\delta}{1+i} E_t \hat{\imath}_{L,t+1}. \tag{B.9}$$

Finally, note that that in a steady state $i_t^L = i_t = i$ for all $t$.

### B.1.2 The driving processes

We assume that $\hat{\theta}_t^g$, $\hat{\theta}_t^c$, and $\hat{\theta}_t^m$ evolve according to the stationary AR(1) processes[2]

$$\hat{\theta}_t^x = \rho_x \hat{\theta}_{t-1}^x + \theta_t^x, \text{ with } \theta_t^x \sim N(0, \sigma_{\theta^x}^2) \tag{B.10}$$

for $x = g$, $c$, $m$, and $\mid \rho_x \mid <1$.

Turning to the two drivers of the natural rate of interest in expression (B.4), $\epsilon_t$ and $\Delta y_t^n$, although they are routinely modelled as random walks,[3] in what follows we postulate that they evolve according to the zero-mean[4] AR(1) processes

$$\epsilon_t = \rho_\epsilon \epsilon_{t-1} + \epsilon_t^\epsilon, \text{ with } \epsilon_t^\epsilon \sim N(0, \sigma_\epsilon^2) \tag{B.11}$$

$$\Delta y_t^n = \rho_n \Delta y_{t-1}^n + \epsilon_t^n, \text{ with } \epsilon_t^n \sim N(0, \sigma_n^2). \tag{B.12}$$

Although a random-walk specification for both $\epsilon_t$ and $\Delta y_t^n$ is standard in the literature, strictly speaking it cannot be correct, because it would imply that both trend output growth and the natural rate of interest could take any value between plus and minus infinity. These assumptions imply that the expression for the natural rate of interest becomes

$$r_t^n = \epsilon_t + \sigma^{-1}[\gamma \rho_n - (1-\gamma)] \Delta y_t^n. \tag{B.13}$$

Notice that, for reasonable values of the habit persistence parameter $h$, $[\gamma \rho_n - (1-\gamma)] > 0$, thus implying that, in line with the conventional wisdom encoded (e.g.) in Ramsey's optimal growth model, an increase in trend output growth maps into a corresponding increase in the natural rate of interest. In fact, although in estimation we impose no constraint on $[\gamma \rho_n - (1-\gamma)]$, this object is consistently estimated to be positive for all countries and sample periods.

Finally, we postulate that the logarithm of the overall stock of gold, $g_t$, evolves as a random-walk with drift, i.e.

$$g_t = g_{t-1} + \mu_g + \epsilon_t^g, \text{ with } \epsilon_t^g \sim N(0, \sigma_g^2). \tag{B.14}$$

This assumption requires some discussion. Taken literally, expression (B.14) implies indeed that the evolution of $g_t$ had been exogenous, and entirely unrelated to macroeconomic developments.[5] This, however, cannot be literally true: e.g. gold scarcity, which characterized

[2] Relaxing this assumption by making the three processes evolve as random walks produces the same qualitative evidence, and in fact makes our results even stronger. This evidence is available upon request.

[3] See in particular Laubach and Williams (2003) and Holston, Laubach and Williams (2017).

[4] Since, as discussed below, in what follows we model deviations of both the short- and the long-term nominal interest rate from their sample averages, the fact that both (B.11) and (B.12) are zero-mean entails no loss of generality, and it is rather the appropriate way to specify the two processes.

[5] This was indeed the assumption made by Lee and Petruzzi (1986).

the period between the 1860s and the early 1890s,[6] and manifested itself in the guise of deflation[7]—and therefore an increase in the real price of gold in terms of goods—most likely stimulated the search for both new gold fields and more efficient extraction methods. At the margin this should have had an impact, so that both subsequent gold fields discoveries, and subsequent advances in the extraction process might have been spurred, at least in part, by previous gold scarcity. The implication is that expression (B.14) should not be taken literally. Rather, it should be regarded as an approximation to a more complex model of the evolution of $g_t$ in which current gold scarcity or abundance, reflected in the real price of gold in terms of goods, has some marginal, non-zero impact on the future evolution of the overall stock of gold.

A more complex model in which gold mining is endogenous, and reacts to fluctuations in the real price of gold, could be developed along the lines of Barsky and Summers (1985). The key reason why we do not do so, and in what follows we work with specification (B.14), is that for strictly *logical* reasons this cannot make *any* material difference to our analysis—exactly as it did for Barsky and Summers (1988).[8]

**Why endogeneity of gold makes no material difference** The reason for this is straightforward. As discussed, the mechanism underlying the appearance of Gibson's paradox under the Gold Standard hinges on the interaction between the Fisher equation and the asset pricing condition determining the current value of money. As for the Fisher equation, endogeneity of gold makes no difference: an $x\%$ change in the natural rate of interest still causes an $x\%$ corresponding change in the long-term nominal rate. As for the asset pricing condition, a decrease (say) in the natural rate causes an increase in the current value of money, i.e. a fall in the price level, and therefore an increase in the real price of gold. This leads to more mining and therefore to an increase in the overall stock of gold. In turn, this causes an increase the price level, thus counteracting the initial decrease due to the fall in the natural rate. However—and this is the crucial point—this mechanism *cannot fully counteract* the forces originating from the initial fall in the natural rate, because in equilibrium the price level *ought to decrease.* If endogeneity of gold fully counteracted these forces the price level would return to its initial value, thus violating the asset pricing condition. The implication is that, as a simple matter of logic, endogeneity of gold cannot make any

[6] See the discussion in Chapter 3 of Friedman and Schwartz (1963).

[7] See Figures 1*a*-1*b* in the main text. This is especially apparent in the last two panels of Figure 1*a*, and in the first two panels of Figure 1*b*.

[8] See the discussion in Barsky and Summers (1988, footnote 8).

material difference to our analysis.

We now turn to the New Keynesian model under monetary policy regimes different from the Gold Standard.

## B.2 The New Keynesian model under alternative monetary regimes

The corresponding New Keynesian model under alternative monetary policy regimes is described by

$$(\pi_t - \bar{\pi}) = \kappa \tilde{y}_t + \kappa_1 \Delta \tilde{y}_t + \kappa_1 \Delta y_t^n + \beta E_t(\pi_{t+1} - \bar{\pi}) + u_t, \tag{B.15}$$

where $\bar{\pi}$ is the central bank's inflation target, and by expressions (B.3) and (B.9). Equilibrium on the money market is characterized by a standard expression describing the demand for real money balances as a fraction of GDP as a function of the short-term nominal interest rate. Following Benati et al. (2021) we take M1 as the relevant monetary aggregate (so that we focus on the demand for money for transaction purposes), and we assume that the demand for real money balances takes the 'Selden-Latané'[9] functional form, which is linear in money velocity and the short-term rate, so that the inverse of the demand for real money balances as a fraction of GDP is

$$V_t \equiv \frac{Y_t}{M_t} = \psi + \alpha i_t + \lambda_t \tag{B.16}$$

where $V_t$ is money velocity, defined as the ratio between nominal GDP and the nominal money stock. As shown in Benati et al. (2021), for low-inflation, and therefore low-interest rate countries such as those analyzed herein the data clearly prefer the 'Selden-Latané' specification to either of the two money-demand specifications proposed by Cagan (1956) and Meltzer (1963), which have dominated post-WWII research on money demand. We assume that $\lambda_t$ follows a stationary AR(1) process, $\lambda_t = \rho_\lambda \lambda_{t-1} + \epsilon_{\lambda,t}$, with $\epsilon_{\lambda,t} \sim N(0, \sigma_\lambda^2)$. The model can be closed, e.g., with a money supply rule or an interest rate rule. For estimation purposes we will close it with a standard forward-looking Taylor rule with smoothing,

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1 - \rho_i)[\phi_\pi E_t(\pi_{t+1} - \bar{\pi}) + \phi_y E_t \hat{y}_{t+1}] + \epsilon_{i,t}, \text{ with } \epsilon_{i,t} \sim N(0, \sigma_i^2). \tag{B.17}$$

We now turn to the empirical evidence for the Gold Standard and inflation-targeting regimes.

## B.3 Evidence from estimated New Keynesian models

In this section we discuss the evidence obtained by estimating the New Keynesian models for either the Gold Standard or inflation-targeting regimes. In order to explore the models'

[9] From Selden (1956) and Latané (1960).

ability to replicate Gibson's paradox we focus on their impulse-response functions (IRFs) to the structural shocks. We start by discussing details of the Bayesian estimation procedure, and we then turn to the evidence.

### B.3.1 The data

*Gold Standard* For the United Kingdom, for which we only have a series for the stock of gold held at the Bank of England, whereas we have no data on the remaining stock of gold in the economy, we estimate the model of Section B.1 based on data for the Bank of England's discount rate, a consol yield, and the logarithms of real GDP, the wholesale price index, and the stock of gold held at the Bank of England. The sample period is 1856Q1-1914Q2. For the United States, for which we also have data on the stock of gold held outside the monetary authority,[10] we estimate the model based on a call money rate, a corporate bond yield, and the logarithms of real GNP, Warren and Pearson's (1933) wholesale price index, the overall stock of gold, and the stock of gold held by the monetary authority. The sample period is 1879Q1-1913Q3. For details on the data see Online Appendix A.1.

*Inflation-targeting regimes* For all eight countries[11] we estimate the model of Section B.2 based on data for a short- and a long-term nominal interest rate, the velocity of M1 (computed as the ratio between nominal GDP and nominal M1), and the logarithms of real GDP and the CPI.[12] For details on the data see Online Appendix A.3.

### B.3.2 Bayesian estimation

We estimate the New Keynesian models described in Sections B.1 and B.2 via standard Bayesian methods.

**Calibrated parameters** We calibrate $\beta = 0.9975$, and we set $\mu_y$ and the steady-state nominal interest rate, $i$, to the average values taken by $\Delta y_t$ and the short-term nominal interest rate over the sample period. As for $\mu_g$, for the United States, for which we have data on the stock of gold held both by the monetary authority and in the rest of the economy, we set it to the sample average of $\Delta g_t$, i.e. 0.0164. As for the United Kingdom, for which we only have data on the stock of gold held at the Bank of England (i.e. $g_t^c$), since $\Delta g_t$ is an unobserved state variable the value of $\mu_g$ is not needed for estimation purposes. On the

[10] Since we end the sample period in 1913Q3 (see below), before the founding of the Federal Reserve in December 1913, the monetary authority had been the Treasury.

[11] Australia, Denmark, the Euro area, New Zealand, Sweden, Switzerland, the United Kingdom, and West Germany.

[12] Evidence based on the PCE deflator is qualitatively the same, and it is available upon request.

other hand, since $g_t^c$ exhibits a clear upward trend over the sample period, in the extended state-space form of the model featuring also $g_t^c$ and the log-levels of real GDP ($y_t$) and prices ($p_t$), we include in the equation for $g_t^c$ an intercept $\mu_{g^c}$, which we calibrate to the sample average of $\Delta g_t^c$, i.e. $5.36\times10^{-3}$.

In the light of the evidence from estimating long-run money demand curves, which suggests that a unitary elasticity of money demand with respect to output is the empirically relevant case (see e.g. Benati et al. 2021), we set $\epsilon_m = \sigma$, which implies indeed that $q_y = 1$. Finally, we set $\epsilon_g = \epsilon_m$.

For the United States we calibrate the share of non-monetary gold, $s_g$, to its average value over the sample period (0.2403) based on the series for the stock of gold held by the monetary authority, and the remaining stock of gold in the economy, detailed in Online Appendix A.1. For the United Kingdom we set $s_g$ to the same value as for for the United States. Although this choice is clearly arbitrary, we have explored the robustness of our results to alternative assumptions about $s_g$, estimating the model conditional on $s_g$ equal to 0.1, 0.2, 0.3, 0.4 and 0.5, obtaining results that are qualitatively the same as those based on the calibration $s_g$=0.2403 (this evidence is available upon request).

**Prior distributions** In performing Bayesian estimation we postulate prior distributions only for those parameters for which we have sufficiently reliable prior information. For all other parameters (specifically, all innovation variances, and all autoregressive coefficients) we do not postulate any prior, so that in fact they are estimated simply based on the likelihood of the data.

Table B.1 reports the prior distributions for the New Keynesian models of Sections B.1 and B.2. In line with the literature, for both models we assume that the elasticity of intertemporal substitution, $\sigma_x$, is smaller than 1, and we postulate a Beta prior centered at 0.5, and with a standard deviation of 0.15. Although this prior has a non-negligible curvature, in fact it allows for essentially any value between 0 and 1. We also postulate the same prior for the extent of forward-lookingness in the IS curve, $\gamma$. We postulate a comparatively flat slope of the Phillips curve, $\kappa$, which we encode in a quite informative Gamma prior with a mode of 0.05 and a standard deviation of 0.01. In line with previous estimates in the literature the prior for the inverse of the Frisch elasticity of labor supply $\eta$ is centered at 2.5, but the standard deviation of 0.25 allows for essentially any value between 1.5 and 3.5.

Turning to $\delta$, for the United Kingdom under the Gold Standard, for which in estimation we use a consol (i.e., a perpetuity) as the long rate, we set $\delta = (1 + i)$, which obtains from expression (B.8) for $m \to \infty$. On the other hand, for the United States under the Gold Standard, and for Denmark under inflation-targeting, for which we do not have precise information about the maturity of the long-term rate we use in estimation, we postulate for (1+$i$)-$\delta$ a Gamma prior distribution with mode $2.6\times10^{-3}$ and standard deviation $5.0\times10^{-4}$. In terms of the corresponding maturity of the long-term bond, this prior implies that $m$ in expression (B.8) is greater than approximately ten years, but beyond that horizon it imposes little information. Finally, for all other countries under inflation-targeting, for which (as detailed in Online Appendix A.3) we take as the long-term rate the 10-year yield on government bonds, we calibrate $m$=40 and we compute the corresponding value for $\delta$ based on expression (B.8).

**Table B.1 Prior distributions of the structural parameters for the New Keynesian models**

| Parameter | Domain | Density | Mode | Standard deviation |
|---|---|---|---|---|
| $\kappa$ | $\mathbb{R}^+$ | Gamma | 0.05 | 0.01 |
| $\sigma_x$ | [0, 1] | Beta | 0.5 | 0.15 |
| $\gamma$ | [0, 1] | Beta | 0.5 | 0.15 |
| $\eta$ | $\mathbb{R}^+$ | Gamma | 2.5 | 0.25 |
| (1+$i$)-$\delta$ [a] | $\mathbb{R}^+$ | Gamma | $2.6\times10^{-3}$ | $5.0\times10^{-4}$ |
| *Gold Standard* | | | | |
| Corr$(i_{L,t}, p_t)_{25\text{Y}}$ | [0, 1] | Beta | 0.9 | 0.1 |
| *Inflation targeting regimes* | | | | |
| $\phi_\pi$ | $\mathbb{R}^+$ | Gamma | 1.5 | 0.15 |
| $\phi_y$ | $\mathbb{R}^+$ | Gamma | 0.25 | 0.1 |
| $-\psi$ | $\mathbb{R}^+$ | Gamma | 1 | 0.1 |
| $\alpha$ | $\mathbb{R}^+$ | Gamma | 10 | 1 |

[a] Only for the United States under the Gold Standard and Denmark under inflation targeting. For the United Kingdom under the Gold Standard we set $\delta$=(1+$i$). For other countries under inflation targeting (see text) the maturity of the long-term rate is 10 years, and so we calibrate $\delta$ accordingly.

The priors for the parameters of the Taylor rule ($\rho_r$, $\phi_\pi$, and $\phi_y$) are standard. The priors for $\psi$ for $\alpha$ are in line with the values in the literature on estimating long-run Selden-Latané money demand curves for M1 (see Benati et al., 2021).

Finally, for the Gold Standard (but not for inflation targeting regimes) we postulate a Beta prior with a mode of 0.9 for Corr$(i_{L,t}, p_t)_{25\text{Y}}$, the coefficient of correlation between forecast errors for the long rate and the price level at the 25 years ahead horizon. The rationale for doing so is in order to force the estimation algorithm towards regions of the parameter space that can effectively capture Gibson's paradox. The comparatively large standard deviation, which we set at 0.1, allows however for essentially any value between 0.3 and 1.

**Numerical maximization of the log posterior** We numerically maximise the log posterior—defined as ln $L(\theta|Y)$ + ln $P(\theta)$, where $\theta$ is the vector collecting the model's structural parameters, $L(\theta|Y)$ is the likelihood of $\theta$ conditional on the data, and $P(\theta)$ is the prior—via simulated annealing. Following Goffe, Ferrier, and Rogers (1994) we implement simulated annealing via the algorithm proposed by Corana, Marchesi, Martini and Ridella (1987), setting the key parameters to $T_0$=100,000, $r_T$=0.9, $N_t$=5, $N_s$=20, $\epsilon$=$10^{-6}$, $N_\epsilon$=4, where $T_0$ is the initial temperature, $r_T$ is the temperature reduction factor, $N_t$ is the number of times the algorithm goes through the $N_s$ loops before the temperature starts being reduced, $N_s$ is the number of times the algorithm goes through the function before adjusting the stepsize, $\epsilon$ is the convergence (tolerance) criterion, and $N_\epsilon$ is number of times convergence is achieved before the algorithm stops. Finally, initial conditions were chosen stochastically by the algorithm itself, while the maximum number of functions evaluations, set to 1,000,000, was never achieved.

**Drawing from the posterior distribution** We draw from the posterior distribution via Random Walk Metropolis (RWM) exactly as in An and Schorfheide (2006, Section 4.1). In implementing RWM we exactly follow An and Schorfheide, with the single exception of the method we use to calibrate the covariance matrix's scale factor (the parameter $c$ below) for which we follow the methodology described in the next paragraph. Let then $\hat{\theta}$ and $\hat{\Sigma}$ be the mode of the maximised log posterior and its estimated Hessian, respectively (we compute $\hat{\Sigma}$ numerically as in An and Schorfheide, 2006). We start the Markov chain of the RWM algorithm by drawing $\theta^{(0)}$ from $N(\hat{\theta},\, c^2\hat{\Sigma})$. For $s$ = 1, 2, ..., $N$ we then draw $\tilde{\theta}$ from the proposal distribution $N(\theta^{(s-1)},\, c^2\hat{\Sigma})$, accepting the jump (i.e., $\theta^{(s)} = \tilde{\theta}$) with probability min $\{1,\, r(\theta^{(s-1)},\, \theta|Y)\}$, and rejecting it (i.e., $\theta^{(s)} = \theta^{(s-1)}$) otherwise, where

$$r(\theta^{(s-1)}, \theta|Y) = \frac{L(\theta|Y)\ P(\theta)}{L(\theta^{(s-1)}|Y)\ P(\theta^{(s-1)})}$$

A key issue in implementing Metropolis algorithms is how to calibrate the covariance matrix's scale factor in order to achieve an acceptance rate of the draws close to the ideal one (in high dimensions) of 0.23—see Gelman, Carlin, Stern and Rubin (1995). Our approach for calibrating $c$ is based on the idea of estimating a reasonably good approximation to the inverse relationship between $c$ and the acceptance rate by running a pre-burn-in sample. Specifically, let $C$ be a grid of possible values for $c$ (in what follows, we consider a grid over the interval [0.1, 1] with increments equal to 0.05). For each value of $c$ in the grid (call it $c_j$) we run $n$ draws of the RWM algorithm, storing, for each $c_j$, the corresponding fraction of accepted draws, $f_j$. We then fit a third-order polynomial to the $f_j$'s via least squares, and letting $\hat{a}_0$, $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ be the estimated coefficients, we choose $c$ by solving numerically the equation $\hat{a}_0+\hat{a}_1 c+\hat{a}_2 c^2+\hat{a}_3 c^3=0.23$.

We check convergence of the Markov chain to the ergodic distribution based on Geweke's (1992) inefficiency factors of the draws for each individual parameter. The inefficiency factors are defined as the inverse of the relative numerical efficiency measure of Geweke (1992),

$$RNE = (2\pi)^{-1}\frac{1}{S(0)}\int_{-\pi}^{\pi} S(\omega)d\omega,$$

where $S(\omega)$ is the spectral density of the sequence of draws from RWM for the parameter of interest at frequency $\omega$. We estimate the spectral densities via the lag-window estimator as described in chapter 10 of Hamilton (1994). We also considered an estimator based on the fast-Fourier transform, and results were very similar. For all parameters the IFs are equal to at most 10, that is, well below the values of 20-25 which are typically taken to indicate problems in the convergence of the Markov chain.

## B.4 Evidence

### B.4.1 The Gold Standard

Figure A.2 in the Online Appendix shows, for the United States during the Classical Gold Standard period, the medians of the posterior distributions of the IRFs to the structural shocks, together with the 68 and 90 per cent coverage credible sets, whereas Figure A.3 shows the corresponding evidence for the United Kingdom.[13] In order to meaningfully interpret this evidence it is important to recall that, as discussed both in the Introduction and in Section 4.2, what is needed in order to replicate Gibson's paradox is a shock (or a set of shocks) that has a very highly persistent impact of the same sign on the price level and the long-term nominal interest rate. In Figures A.2 and A.3 the *only* shock that accomplishes this is $\epsilon_t^{\epsilon}$—i.e., the shock to the component of the natural rate of interest that is unrelated to fluctuations in trend GDP growth—whereas all of the other shocks are incapable of

[13] In order not to clutter the figures we do not report evidence for the overall stock of gold, $g_t$, since based on (B.14) this evidence is trivial.

generating such long-horizon correlation. In particular, $\epsilon_t^n$ has a *negative* long-horizon impact on prices, so that as a matter of logic it cannot produce Gibson's paradox.

Turning to $\theta_t^c$ and the shocks to the overall stock of gold, and to the IS and Phillips curves shocks, all of them have a very short-lived impact on the long rate, which mean-reverts to zero either one period after impact (this is the case for $\theta_t^c$ and the IS curve shock, $v_t$), or within about five years. On the other hand, $\theta_t^g$ and $\theta_t^m$ induce persistent *in*creases and *de*creases, respectively, in the long rate and the price level, so that the correlation they generate is the opposite of what is needed.

### B.4.2 Inflation-targeting regimes

Turning to inflation-targeting regimes, the evidence for the United Kingdom in the first two columns of Figure A.4, which is qualitatively the same as that for all of the other seven countries,[14] shows that different from the Gold Standard both $\epsilon_t^n$ and $\epsilon_t^\epsilon$ generate a positive long-horizon correlation between prices and the long rate. The shapes of the responses of the long rate to the two disturbances—very highly persistent for $\epsilon_t^\epsilon$, and mean-reverting to a non-negligible extent for $\epsilon_t^\epsilon$—suggest however that the former shock produces a stronger correlation than the latter one. Finally, the IRFs of prices and the long rate to the other shocks show that, exactly as for the Gold Standard, none of them can generate the paradox.

The evidence for inflation targeting regimes suggests two considerations.

First, the evidence for the IRFs in the first two columns of Figure A.4 begs the obvious question of why, exactly, Gibson's paradox is nowhere nearly apparent in the raw data generated under inflation targeting. In principle one possible explanation could be that shocks to the IS and Phillips curve, and monetary policy shocks 'blur' the positive long-horizon correlation induced by $\epsilon_t^n$ and $\epsilon_t^\epsilon$, thus making it *disappear* from the raw data. In fact, as the evidence in Section 8 in the main text of the paper shows, this does not seem to be the case: once controlling for the deterministic component of the drift in the price level induced by the presence of a positive inflation target, a positive long-horizon correlation between the two series can indeed be recovered from the data generated by inflation-targeting regimes. This suggests that the main reason why Gibson's paradox is not apparent in the raw data generated under inflation targeting is simply that the presence of a positive inflation target, by introducing a positive drift in the price level, causes the long-horizon correlation to become *hidden* in the raw data.

Second, the evidence for inflation targeting regimes suggests that Gibson's paradox had

[14] We do not report this evidence for reasons of space, but it is available upon request.

*nothing* to do with the Gold Standard per se. In particular, the fact that under inflation targeting both disturbances to the natural rate generate a positive long-horizon correlation between prices and the long rate naturally suggests that, in principle, other monetary regimes might also be able to generate the paradox.

# C Details About the Approximations for the Model of the Gold Standard

The expression

$$\frac{\theta_t^m}{\theta_t^C}\frac{L_m\left(\frac{g_t^c}{P_t}\right)}{U_c(Y_t)} = \frac{i_t}{1+i_t}$$

is approximated as follows. We have that

$$\begin{aligned} &\frac{L_{mm}}{U_c}\left(\frac{g_t^c - g^c}{P} - \frac{g^c}{P^2}(P_t - P)\right) - \frac{L_m}{U_c^2}U_{cc}(Y_t - Y) + \frac{L_m}{U_c}\left(\theta_t^m - 1 - (\theta_t^c - 1)\right) \\ = \; &\frac{1}{(1+i)^2}(1 + i_t - (1+i)) \end{aligned}$$

and

$$\frac{L_m}{U_c}\frac{L_{mm}}{L_m}\frac{g^c}{P}(\hat{g}_t^c - \hat{P}_t) - \frac{L_m}{U_c}\left(\frac{U_{cc}Y}{U_c}\hat{Y}_t - \hat{\theta}_t^m + \hat{\theta}_t^c\right) = \frac{1}{(1+i)}\hat{\imath}_t$$

Therefore

$$-\epsilon_m^{-1}(\hat{g}_t^c - \hat{P}_t) + \sigma^{-1}\hat{Y}_t = \frac{1}{i}\hat{\imath}_t + \hat{\theta}_t^c - \hat{\theta}_t^m$$

and so

$$(\hat{g}_t^c - \hat{P}_t) = q_y\hat{Y}_t - q_i\hat{\imath}_t - q_\theta(\hat{\theta}_t^c - \hat{\theta}_t^m)$$

$$q_y = \frac{\sigma^{-1}}{\epsilon_m^{-1}} \qquad q_i = \frac{1}{\epsilon_m^{-1} i} \qquad q_\theta = \frac{1}{\epsilon_m^{-1}}$$

Turning to the expression

$$\frac{1}{P_t}\theta_t^m L_m\left(\frac{g_t^c}{P_t}\right) = \theta_t^g V_g(G_t - g_t^c)$$

it is approximated as follows. We have that

$$\begin{aligned} &-\left(\frac{1}{P}\right)^2 L_m(P_t - P) + \left(\frac{1}{P}\right)L_{mm}\left(\frac{g_t^c - g}{P} - \frac{g}{P^2}(P - P\right) \\ = \; &V_{gg}(G_t - G - (g_t^c - g^c)) + V_g(\theta_t^g - 1 - (\theta_t^m - 1)) \end{aligned}$$

$$-\frac{1}{P}L_m\hat{P}_t + \frac{1}{P}L_{mm}\frac{g^c}{P}(\hat{g}_t^c - \hat{P}_t) = V_{gg}G(\hat{G}_t - s_g\hat{g}_t^c) + V_g(\hat{\theta}_t^g - \hat{\theta}_t^m)$$

$$-\hat{P}_t - \epsilon_m^{-1}(\hat{g}_t^c - \hat{P}_t) = -\epsilon_g^{-1}(\hat{G}_t - s_g\hat{g}_t^c) + \hat{\theta}_t^g - \hat{\theta}_t^m$$

in which

$$\epsilon_m = -\frac{L_m}{L_{mm}m} \qquad \epsilon_g = -\frac{V_g}{V_{gg}g} \qquad s_g = g^c/G$$

Therefore we have that

$$(\epsilon_m^{-1} - 1)\hat{P}_t = (\epsilon_m^{.1} + \epsilon_g^{-1}s_g)\hat{g}_t^c - \epsilon_g^{-1}\hat{G}_t + \hat{\theta}_t^g - \hat{\theta}_t^m$$

$$\hat{P}_t = -\vartheta_g \hat{g}_t^c + \vartheta_G \hat{G}_t - \vartheta_\theta(\hat{\theta}_t^g - \hat{\theta}_t^m)$$

where

$$\vartheta_g = \frac{(\epsilon_m^{.1} + \epsilon_g^{-1}s_g)}{(1-\epsilon_m^{-1})} \qquad \vartheta_G = \frac{\epsilon_g^{-1}}{(1-\epsilon_m^{-1})} \qquad \vartheta_\theta = \frac{1}{(1-\epsilon_m^{-1})}$$

# D Derivation of the Expression for the Long-Term Nominal Interest Rate

In this Appendix we detail the derivation of the expression for the long-term nominal interest rate, $i_{L,t}$, in Section 6.1.1 in the main text. As mentioned there, we assume a decaying coupon structure 1, $\delta$, $\delta^2$, $\delta^3$..., where

$$\delta = (1+i)\left(1 - \frac{1}{m}\right)$$

where $m$ is the maturity of the long-term bond expressed in quarters. Note that

$$i_{L,t} = -Q_t + \frac{\delta}{1+i}Q_t$$

and

$$Q_t = -i_t + \frac{\delta}{1+i}E_t Q_{t+1}$$

where $Q_t$ is the price of the long-term bond. Therefore

$$i_{L,t} = i_t + \frac{\delta}{1+i}(Q_t - E_t Q_{t+1})$$

Since

$$Q_t - E_t Q_{t+1} = -i_t + \frac{\delta}{1+i}E_t Q_{t+1} - E_t Q_{t+1} = -i_t + E_t i_{L,t+1}$$

we have that

$$i_{L,t} = \frac{1+i-\delta}{1+i}i_t + \frac{\delta}{1+i}E_t i_{L,t+1}$$

Finally, note that that in a steady state $i_t^L = i_t = i$ for all $t$.

# E The State-Space Forms of the New Keynesian Models

Here follows a description of the state-space forms of the New Keynesian models for the Gold Standard and the inflation targeting regimes.

## E.1 Gold Standard

The model for the Gold Standard is described by the following equations, where the notation is the same as in the main text of the paper:

$$\Delta y_t^n = \rho_n \Delta y_{t-1}^n + \epsilon_t^n, \text{ with } \epsilon_t^n \sim N(0, \sigma_n^2). \tag{E.1.1}$$

$$\Delta g_t = \mu_g + \epsilon_t^g, \text{ with } \epsilon_t^g \sim N(0, \sigma_g^2). \tag{E.1.2}$$

$$r_t^n = \epsilon_t + \sigma^{-1}[\gamma \rho_n - (1-\gamma)]\Delta y_t^n. \tag{E.1.3}$$

$$\epsilon_t = \rho_\epsilon \epsilon_{t-1} + \epsilon_t^\epsilon, \text{ with } \epsilon_t^\epsilon \sim N(0, \sigma_\epsilon^2) \tag{E.1.4}$$

$$\pi_t = \kappa \tilde{y}_t + \kappa_1 \Delta \tilde{y}_t + \kappa_1 \Delta y_t^n + \beta E_t \pi_{t+1} + u_t, \text{ with } u_t \sim N(0, \sigma_u^2) \tag{E.1.5}$$

$$\tilde{y}_t = \gamma E_t \tilde{y}_{t+1} + (1-\gamma)\tilde{y}_{t-1} - \sigma(\hat{\imath}_t - E_t \pi_{t+1} - r_t^n) + v_t, \text{ with } \epsilon_t^v \sim N(0, \sigma_v^2), \tag{E.1.6}$$

$$\pi_t = -\vartheta_g \Delta \hat{g}_t^c + \vartheta_G \Delta \hat{g}_t - \vartheta_\theta (\Delta \hat{\theta}_t^g - \Delta \hat{\theta}_t^m), \tag{E.1.7}$$

$$\begin{aligned} \Delta \hat{g}_t^c - \pi_t \quad = \quad & q_y(1+\rho)(\Delta y_t + \Delta y_t^n) - q_y \rho (\Delta y_{t-1} + \Delta y_{t-1}^n) + \\ & -q_i(\Delta \tilde{\imath}_t + \Delta r_t^n) - q_\theta (\Delta \hat{\theta}_t^c - \Delta \hat{\theta}_t^m), \end{aligned} \tag{E.1.8}$$

$$\hat{\imath}_{L,t} = \frac{1+i-\delta}{1+i} \hat{\imath}_t + \frac{\delta}{1+i} E_t \hat{\imath}_{L,t+1}. \tag{E.1.9}$$

$$\hat{\theta}_t^x = \rho_x \hat{\theta}_{t-1}^x + \theta_t^x, \text{ with } x = g,\ c,\ m,\ \mid \rho_x \mid < 1, \text{ and } \theta_t^x \sim N(0, \sigma_{\theta^x}^2) \tag{E.1.10}$$

By defining the state vector $\xi_t$, the vector of forecast errors $\eta_t$, and the vector of the structural shocks $\epsilon_t$ as

$$\xi_t = [\Delta y_t^n, \Delta g_t, \Delta g_t^c, r_t^n, \epsilon_t, \pi_t, \tilde{y}_t, \hat{\imath}_t, \hat{\imath}_{L,t}, E_t \pi_{t+1}, E_t y_{t+1}, E_t \tilde{\imath}_{L,t+1}, \hat{\theta}_t^g, \hat{\theta}_t^c, \hat{\theta}_t^m, \tilde{y}_{t-1}]', \tag{E.1.11}$$

$$\eta_t = [\eta_t^\pi \eta_t^y \eta_t^{i_L}]', \tag{E.1.12}$$

and

$$\epsilon_t = [\epsilon_t^{\Delta y^n}\ \epsilon_t^g\ \epsilon_t^\epsilon\ u_t\ v_t\ \hat{\theta}_t^g\ \hat{\theta}_t^c\ \hat{\theta}_t^m]', \tag{E.1.13}$$

and augmenting the model with the identity

$$\tilde{y}_{t-1} = \tilde{y}_{t-1} \tag{E.1.14}$$

and the definition of the three forecast errors

$$\eta_t^{\pi} = \pi_t - E_{t-1}\pi_t \tag{E.1.15}$$

$$\eta_t^{y} = y_t - E_{t-1}y_t \tag{E.1.16}$$

$$\eta_t^{\tilde{\imath}_L} = \tilde{\imath}_{L,t} - E_{t-1}\tilde{\imath}_{L,t} \tag{E.1.17}$$

the model can be put into the Sims (2000) canonical form

$$\Gamma_0\xi_t = \Gamma_1\xi_{t-1} + \Psi\epsilon_t + \Pi\eta_t \tag{E.1.18}$$

where the matrices $\Gamma_0$, $\Gamma_1$, $\Psi$, and $\Pi$ feature (convolutions of) the structural parameters.

Solving the model as in Sims (2000) produces the following representation for the state vector $\xi_t$:

$$\xi_t = \tilde{F}\xi_{t-1} + \tilde{A}_0\epsilon_t \tag{E.1.19}$$

It is important to stress that all of the state variables in $\xi_t$ are stationary.

For the United States we estimate model (E.1.1)-(E.1.10) based on the six observed variables we discussed in the text: a call money rate ($\hat{\imath}_t$), a corporate bond yield ($\hat{\imath}_{L,t}$), and the logarithms of real GNP ($y_t$), Warren and Pearson's (1933) wholesale price index ($p_t$), the overall stock of gold ($\hat{g}_t$), and the stock of gold held by the monetary authority ($\hat{g}_t^c$). In order to do this we define the following state-space form, with state equation

$$\underbrace{\left[\begin{array}{c} y_t^n \\ \hat{g}_t \\ \hat{g}_t^c \\ p_t \\ \hline \xi_t \end{array}\right]}_{S_t} = \underbrace{\left[\begin{array}{c} \mu_y \\ \mu_g \\ \mu_g^c \\ 0 \\ \hline 0_{16\times 1} \end{array}\right]}_{h} + \underbrace{\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \text{Row of } \Delta y_t^n \text{ in } \tilde{F} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \text{Row of } \Delta \hat{g}_t^c \text{ in } \tilde{F} \\ 0 & 0 & 0 & 1 & \text{Row of } \pi_t \text{ in } \tilde{F} \\ \hline \multicolumn{4}{c|}{0_{16\times 4}} & \tilde{F} \end{array}\right]}_{F} \underbrace{\left[\begin{array}{c} y_{t-1}^n \\ \hat{g}_{t-1} \\ \hat{g}_{t-1}^c \\ p_{t-1} \\ \hline \xi_{t-1} \end{array}\right]}_{S_{t-1}} +$$

$$+ \underbrace{\left[\begin{array}{c} \text{Row of } \Delta y_t^n \text{ in } \tilde{A}_0 \\ 0 \quad 1 \qquad 0_{1\times 6} \\ \text{Row of } \Delta \hat{g}_t^c \text{ in } \tilde{A}_0 \\ \text{Row of } \pi_t \text{ in } \tilde{A}_0 \\ \hline \tilde{A}_0 \end{array}\right]}_{A_0} \underbrace{\left[\begin{array}{c} \epsilon_t^{\Delta y^n} \\ \epsilon_t^{g} \\ \epsilon_t^{\epsilon} \\ u_t \\ v_t \\ \hat{\theta}_t^{g} \\ \hat{\theta}_t^{c} \\ \hat{\theta}_t^{m} \end{array}\right]}_{\epsilon_t} \tag{E.1.20}$$

where $\xi_t$, $\tilde{F}$, and $\tilde{A}_0$ are the same objects as in equation (E.1.19), and observation equation

$$\underbrace{\begin{bmatrix} y_t \\ p_t \\ \hat{g}_t \\ \hat{g}_t^c \\ \hat{\imath}_t \\ \hat{\imath}_{L,t} \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mu_i \\ \mu_{i_L} \end{bmatrix}}_{c} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & ... & 1 & 0 & 0 & ... \\ 0 & 1 & 0 & 0 & ... & 0 & 0 & 0 & ... \\ 0 & 0 & 0 & 1 & ... & 0 & 0 & 0 & ... \\ & & & & ... & 0 & 1 & 0 & ... \\ & & & & ... & 0 & 0 & 1 & ... \\ 0 & 0 & 1 & 0 & ... & 0 & 0 & 0 & ... \end{bmatrix}}_{H'} \underbrace{\begin{bmatrix} y_t^n \\ \hat{g}_t \\ \hat{g}_t^c \\ p_t \\ \xi_t \end{bmatrix}}_{S_t} \tag{E.1.21}$$

We calibrate $\mu_y$, $\mu_g$, and $\mu_g^c$ to the average values taken by the log-differences of real GNP, the overall stock of gold, and the stock of monetary gold over the sample period, and we calibrate $\mu_i$, and $\mu_{i_L}$ to the average values taken by the short- and the long-term nominal interest rates over the sample period.

For the United Kingdom we proceed in the same way, with the only difference that we only use the five observed variables discussed in the text: the Bank of England's discount rate ($\hat{\imath}_t$), a consol yield ($\hat{\imath}_{L,t}$), and the logarithms of real GDP ($y_t$), the wholesale price index ($p_t$), and the stock of gold held at the Bank of England ($\hat{g}_t^c$), so that equations (E.1.20) and (E.1.21) are modified accordingly.

## E.2 Inflation targeting regimes

The model for inflation targeting regimes is described by equations (E.1.1), (E.1.3), (E.1.4), (E.1.6), (E.1.9), (E.1.15)-(E.1.17), and

$$(\pi_t - \bar{\pi}) = \kappa\tilde{y}_t + \kappa_1\Delta\tilde{y}_t + \kappa_1\Delta y_t^n + \beta E_t(\pi_{t+1} - \bar{\pi}) + u_t, \text{ with } u_t \sim N(0, \sigma_u^2) \tag{E.2.1}$$

$$V_t = \psi + \alpha\hat{\imath}_t + \lambda_t \tag{E.2.2}$$

$$\lambda_t = \rho_\lambda\lambda_{t-1} + \epsilon_{\lambda,t}, \text{ with } \epsilon_{\lambda,t} \sim N(0, \sigma_\lambda^2) \tag{E.2.3}$$

$$\hat{\imath}_t = \rho_i\hat{\imath}_{t-1} + (1-\rho_i)[\phi_\pi E_t(\pi_{t+1} - \bar{\pi}) + \phi_y E_t\hat{y}_{t+1}] + \epsilon_{i,t}, \text{ with } \epsilon_{i,t} \sim N(0, \sigma_i^2). \tag{E.2.4}$$

By defining the state vector $\xi_t$, the vector of forecast errors $\eta_t$, and the vector of the structural shocks $\epsilon_t$ as

$$\xi_t = [\Delta y_t^n, r_t^n, \epsilon_t, \pi_t, \tilde{y}_t, \hat{\imath}_t, \hat{\imath}_{L,t}, E_t\pi_{t+1}, E_t y_{t+1}, E_t\tilde{\imath}_{L,t+1}, \lambda_t]', \tag{E.2.5}$$

$$\eta_t = [\eta_t^\pi \eta_t^y \eta_t^{i_L}]', \tag{E.2.6}$$

and

$$\epsilon_t = [\epsilon_t^{\Delta y^n} \; \epsilon_t^\epsilon \; u_t \; v_t \; \epsilon_{i,t} \; \epsilon_{\lambda,t}]', \tag{E.2.7}$$

the model can be put into the Sims (2000) canonical form and can be solved as for the Gold Standard, producing the representation (E.1.19) for the dynamics of the state vector $\xi_t$.

For all countries we estimate the model based on data for a short- and a long-term nominal interest rate ($\hat{\imath}_t$ and $\hat{\imath}_{L,t}$), the velocity of M1 ($V_t$, computed as the ratio between nominal GDP and nominal M1), and the logarithms of real GDP and the GDP deflator ($y_t$ and $p_t$). In order to do this we define the following state-space form, with state equation

$$\underbrace{\left[\begin{array}{c} y_t^n \\ p_t \\ \hline \xi_t \end{array}\right]}_{S_t} = \underbrace{\left[\begin{array}{c} \mu_y \\ \bar{\pi} \\ \hline 0_{11\times1} \end{array}\right]}_{h} + \underbrace{\left[\begin{array}{cc|c} 1 & 0 & \text{Row of } \Delta y_t^n \text{ in } \tilde{F} \\ 0 & 1 & \text{Row of } \pi_t \text{ in } \tilde{F} \\ \hline \multicolumn{2}{c|}{0_{11\times4}} & \tilde{F} \end{array}\right]}_{F} \underbrace{\left[\begin{array}{c} y_{t-1}^n \\ p_{t-1} \\ \hline \xi_{t-1} \end{array}\right]}_{S_{t-1}} +$$

$$+ \underbrace{\left[\begin{array}{c} \text{Row of } \Delta y_t^n \text{ in } \tilde{A}_0 \\ \text{Row of } \pi_t \text{ in } \tilde{A}_0 \\ \hline \tilde{A}_0 \end{array}\right]}_{A_0} \underbrace{\left[\begin{array}{c} \epsilon_t^{\Delta y^n} \\ \epsilon_t^{\epsilon} \\ u_t \\ v_t \\ \epsilon_{i,t} \\ \epsilon_{\lambda,t} \end{array}\right]}_{\epsilon_t} \tag{E.2.8}$$

and observation equation

$$\underbrace{\left[\begin{array}{c} y_t \\ p_t \\ \hat{\imath}_t \\ \hat{\imath}_{L,t} \\ V_t \end{array}\right]}_{Y_t} = \underbrace{\left[\begin{array}{c} 0 \\ 0 \\ \mu_i \\ \mu_{i_L} \\ \psi \end{array}\right]}_{c} + \underbrace{\left[\begin{array}{ccccccccc} 1 & 0 & ... & 1 & 0 & 0 & ... & 0 \\ 0 & 1 & ... & 0 & 0 & 0 & ... & 0 \\ 0 & 0 & ... & 0 & 1 & 0 & ... & 0 \\ & & ... & 0 & 0 & 1 & ... & 0 \\ 0 & 0 & ... & 0 & \alpha & 0 & ... & 1 \end{array}\right]}_{H'} \underbrace{\left[\begin{array}{c} y_t^n \\ p_t \\ \xi_t \end{array}\right]}_{S_t} \tag{E.2.9}$$

We calibrate $\mu_y$ and $\bar{\pi}$ to the average value taken by the log-difference of real GDP over the sample period, and to the inflation target, respectively. We also calibrate $\mu_i$ and $\mu_{i_L}$ to the average values taken by the short- and the long-term nominal interest rates over the sample period.

# F Relationship with Barsky and Summers (1988)

Barsky and Summers (1988)'s model is similar to the one we use in Section 3.1. They assume perfect foresight and stationarity. Under these assumptions equation (4) in Section 3.1.1 implies that

$$1 + i_t = (1 + r)\frac{P_{t+1}}{P_t}.$$

Since

$$1 + r_t = \frac{1}{\beta}\frac{U_c(Y_t)}{U_c(Y_{t+1})}$$

$1 + r_t = \beta^{-1}$. Equation (6) in the main text implies that

$$\frac{1}{P_t} = \frac{1}{P} = \frac{V_g(G - g^c)}{U_c(Y)} + \frac{1}{1+r}\frac{1}{P}.$$

Therefore the price level is constant and equal to

$$P = \frac{U_c(Y)}{V_g(G - g^c)}\frac{r}{1+r} \tag{F.1}$$

Since inflation is equal to zero,

$$1 + i = 1 + r.$$

On the other hand, equilibrium on the money market implies that

$$\frac{L_m\left(\frac{g^c}{P}\right)}{U_c(Y)} = \frac{r}{1+r} \tag{F.2}$$

Equations (F.1) and (F.2) are those used by Barsky and Summers (1988) in order to explain the price of gold and the relationship between the real interest rate and the price level.

# G The Natural Rate of Interest and the Investment/Consumption Ratio Within the Ramsey Model

## G.1 The problem

At time $t_0$=0 an infinitely-lived household maximizes its discounted flow of utility from $t$=$t_0$ until $t$=$\infty$:

$$\underset{c_t,k_t}{\text{Max}}\ U_0 \equiv \int_0^\infty e^{-\rho t}\frac{c_t^{1-\gamma}}{1-\gamma}dt \tag{G.1}$$

where $c_t$ and $k_t$ are per capita consumption and the per capital capital stock, i.e. $c_t$=$C_t/N_t$ and $k_t$=$K_t/N_t$, with with $C_t$, $K_t$, and $N_t$ being overall consumption, the capital stock, and population, respectively, and $\rho$ is the household's rate of intertemporal preference. $K_t$ evolves according to

$$\frac{dK_t}{dt} = I_t = Y_t - C_t - \delta K_t \tag{G.2}$$

where $Y_t$ is output, $I_t$ is investment, and $\delta$ is the depreciation rate of capital. Output is produced via a production function $Y_t = F(K_t, N_t)$, which possesses the usual properties (e.g., positive first derivatives). E.g., it could be the Cobb-Douglas production function

$$Y_t = K_t^\alpha N_t^{1-\alpha}, \tag{G.3}$$

with 0<$\alpha$<1. Finally, population is assumed to grow at the rate $n$, i.e.

$$\frac{1}{N_t}\frac{dN_t}{dt} = n \tag{G.4}$$

The production function (G.3) and the equation of motion for the capital stock (G.2) can be reformulated in per capita terms as follows. By defining output per capita $y_t{=}Y_t/N_t$, we have $y_t{=}f(k_t)$, i.e. with the Cobb-Douglas

$$y_t = f(k_t) = k_t^{\alpha}, \tag{G.5}$$

so that $f'(k_t) = \alpha k_t^{\alpha-1} > 0$, and $f''(k_t) = \alpha(\alpha-1)k_t^{\alpha-2} < 0$. As for (G.2), we divide it by $N_t$, after some math we obtain

$$\frac{dk_t}{dt} = f(k_t) - c_t - (n+\delta)k_t \tag{G.6}$$

The problem therefore boils down to maximizing (G.1) subject to (G.6).

## G.2 The steady-state

We define the Hamiltonian function

$$H_t \equiv e^{-\rho t}U_t(c_t) + \mu_t\frac{dk_t}{dt} = e^{-\rho t}\frac{c_t^{1-\gamma}}{1-\gamma} + \mu_t[f(k_t) - c_t - (n+\delta)k_t] \tag{G.7}$$

where $\mu_t$ is the costate variable associated with the state variable $k_t$, i.e. within the present context, the shadow price of capital. The solution is defined by the following two first-order conditions (FOCs),

$$\frac{\partial H_t}{\partial c_t} = 0 \tag{G.8}$$

$$\frac{d\mu_t}{dt} = -\frac{\partial H_t}{\partial k_t} \tag{G.9}$$

The first FOC, (G.8), boils down to

$$\mu_t = e^{-\rho t}c_t^{-\gamma} \tag{G.10}$$

from which we have

$$\lambda_t \equiv \mu_t e^{\rho t} = c_t^{-\gamma} \tag{G.11}$$

with the multiplier $\lambda_t$ being the marginal utility of consumption. Taking derivatives with respect to time of $\lambda_t = \mu_t e^{\rho t}$ we obtain

$$\frac{d\mu_t}{dt} = e^{-\rho t}\frac{d\lambda_t}{dt} - \rho\lambda_t e^{-\rho t} \tag{G.12}$$

Putting this expression together with the second FOC (G.9),

$$\frac{d\mu_t}{dt} = -\mu_t[f'(k_t) - (n+\delta)] \tag{G.13}$$

we obtain

$$\frac{1}{\lambda_t}\frac{d\lambda_t}{dt} = -[r_t - (n + \delta + \rho)] \tag{G.14}$$

Finally, since $\lambda_t = c_t^{-\gamma}$, we obtain the Euler equation

$$\frac{1}{c_t}\frac{dc_t}{dt} = \frac{1}{\gamma}[r_t - (n + \delta + \rho)] \tag{G.15}$$

In the steady-state

$$\frac{1}{c_t}\frac{dc_t}{dt} = \frac{1}{y_t}\frac{dy_t}{dt} = g \tag{G.16}$$

The steady-state is chacterized by the following four equations, where for simplicity we have eliminated the time index:

$$r = \gamma g + n + \delta + \rho \tag{G.17}$$

$$f(k) = c + (n + \delta)k \tag{G.18}$$

$$r = f'(k) \tag{G.19}$$

$$w = f(k) - kf'(k). \tag{G.20}$$

where $w$ is the real wage. Expression (G.17) states that the natural rate depends on population growth ($n$), the depreciation rate of capital ($\delta$), the rate of time preference ($\rho$), and the rate of growth of real GDP per capita ($g$), multiplied by the utility parameter $\gamma$.

We now turn to characterizing the impact of changes in the determinants of the natural rate.

## G.3 The impact of changes in the determinants of the natural rate

By differentiating (G.16) and (G.19) with respect to $g$ we obtain

$$\frac{dk}{dg} = \frac{\gamma}{f''(k)} < 0 \tag{G.21}$$

Differentiating (G.18) we obtain

$$f'(k)dk = dc + (n + \delta)dk \tag{G.22}$$

which together with (G.21) implies that

$$\frac{dc}{dg} = \frac{\gamma(\gamma g + \rho)}{f''(k)} < 0 \tag{G.23}$$

Let us now turn to the shares of consumption and investment in GDP, defined as

$$\theta_C = \frac{C_t}{Y_t} = \frac{c_t}{y_t} \qquad \text{and} \qquad \theta_I = \frac{I_t}{Y_t} = \frac{i_t}{y_t} \tag{G.24}$$

By taking derivative of $\theta_C = c_t/y_t$ with respect to $g$ we obtain

$$\frac{d\theta_C}{dg} = -\frac{\gamma(n+\delta)w}{y^2 f''(k)} > 0 \tag{G.25}$$

The implication is that an increase in $g$ causes an increase in the share of consumption in GDP, and therefore a decrease in the share of investment $\theta_I$, and consequently a decrease in the investment/consumption ratio. By the same token, it can be shown that

$$\frac{d\theta_C}{d\rho} = -\frac{(n+\delta)w}{y^2 f''(k)} > 0 \tag{G.26}$$

and

$$\frac{d\theta_C}{dn} = \underbrace{\frac{1}{yf''(k)}}_{-}\left[\underbrace{f'(k)(1-\theta_C)-n}_{?}\underbrace{-kf''(k)}_{+}\right] \tag{G.27}$$

For $d\theta_C/dn$ to be negative it ought to be the case that[15]

$$f'(k)(1-\theta_C) - n = r(n+\delta)\frac{k}{y} - n > 0 \tag{G.28}$$

For plausible parameterizations this is indeed the case.[16] Further, for the Cobb-Douglas (G.3) we have, after some math,

$$\frac{d\theta_C}{dn} = \frac{k^{2(1-\alpha)}}{\alpha}\left[\frac{\alpha\delta}{\alpha-1} - (\gamma g + \rho + \delta)\right] \tag{G.29}$$

which is unambiguously negative.

Summing up, increases in $g$ and $\rho$ cause an increase in $\theta_C$, and therefore a decrease in $\theta_I$ and in the investment/consumption ratio, whereas increases in $n$ have the opposite effect. We ignore the impact of changes in $\delta$ because it is most likely negligible.

[15] Going from the left to the right hand-side of (G.28) makes use of the fact that $1-\theta_C = (n+\delta)k/y$. In turn, this comes from the fact that, in the steady-state, $f(k) = y = c + (n+\delta)k$.

[16] Specifically, $\delta$ should be around 0.1-0.12; the average log-difference of working-age population in the United States since the early 1990s has been around $3.5\times10^{-3}$; the capital/output ratio estimate of King and Levine (1994) is equal, for the United States, to about 3; and $r$ should be around 0.02.

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# Figures for the Online Appendix

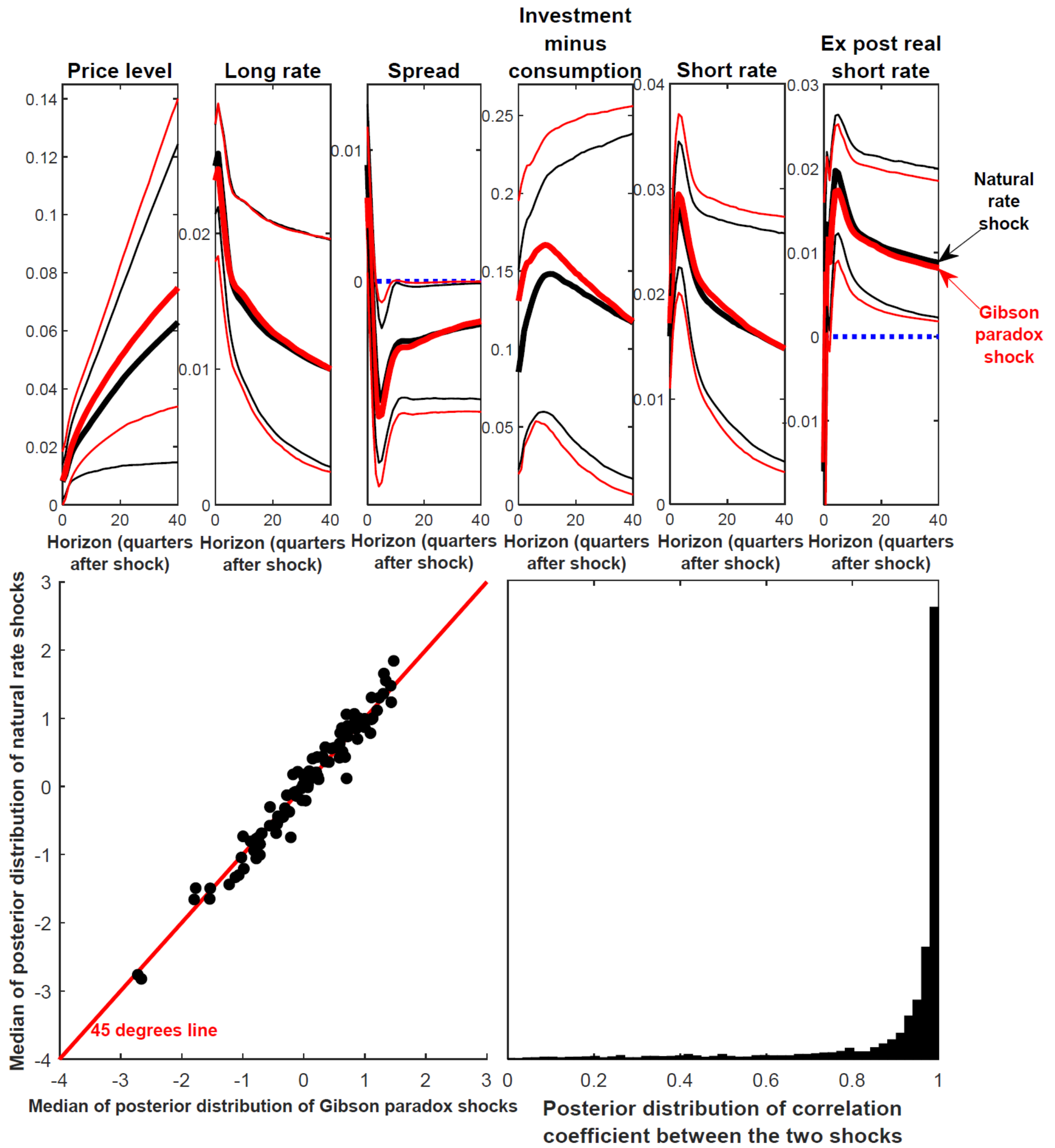


Figure A.1a Evidence for Australia under inflation-targeting: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the two shocks

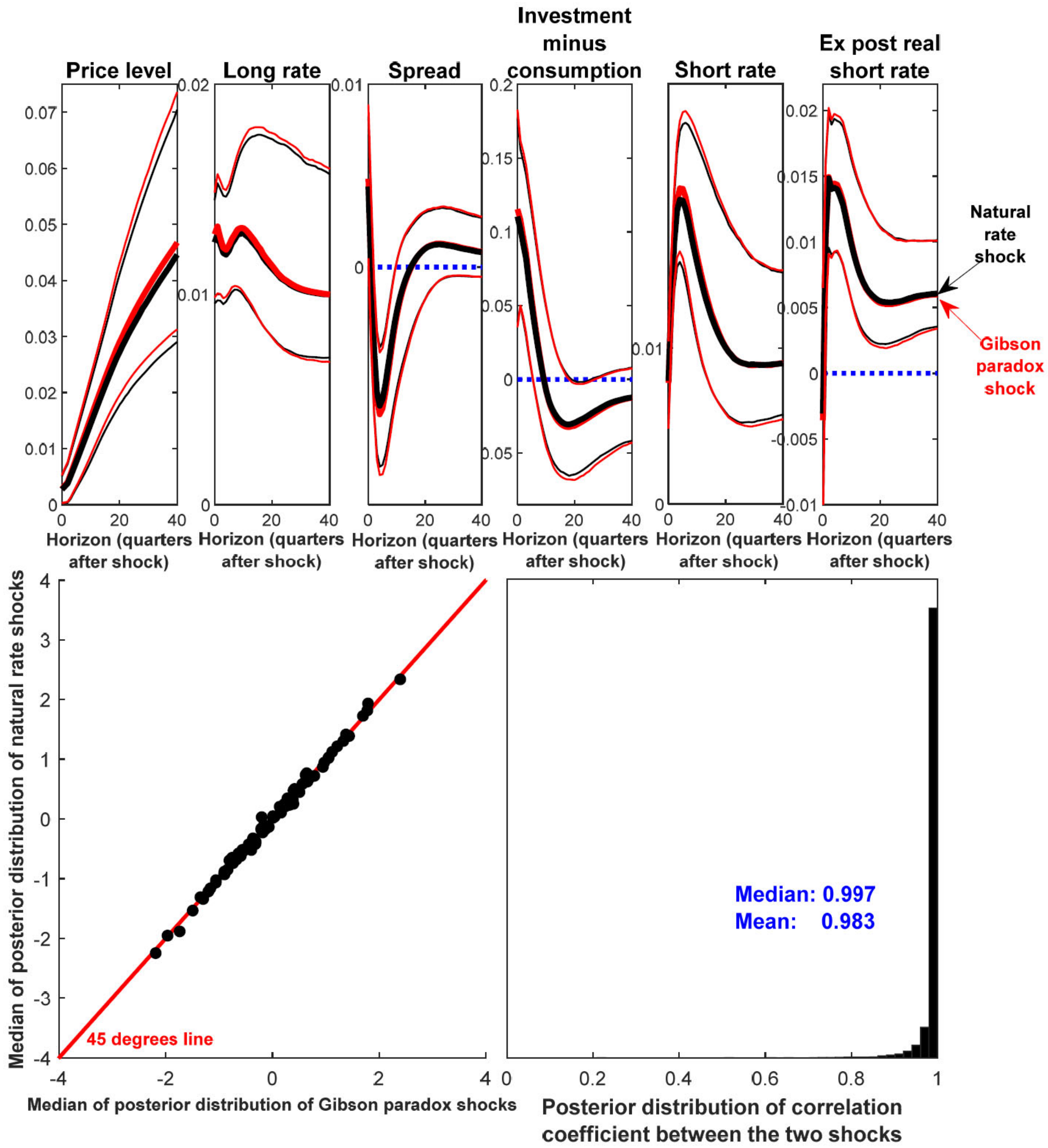


Figure A.1b Evidence for Denmark under inflation-targeting: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the shocks

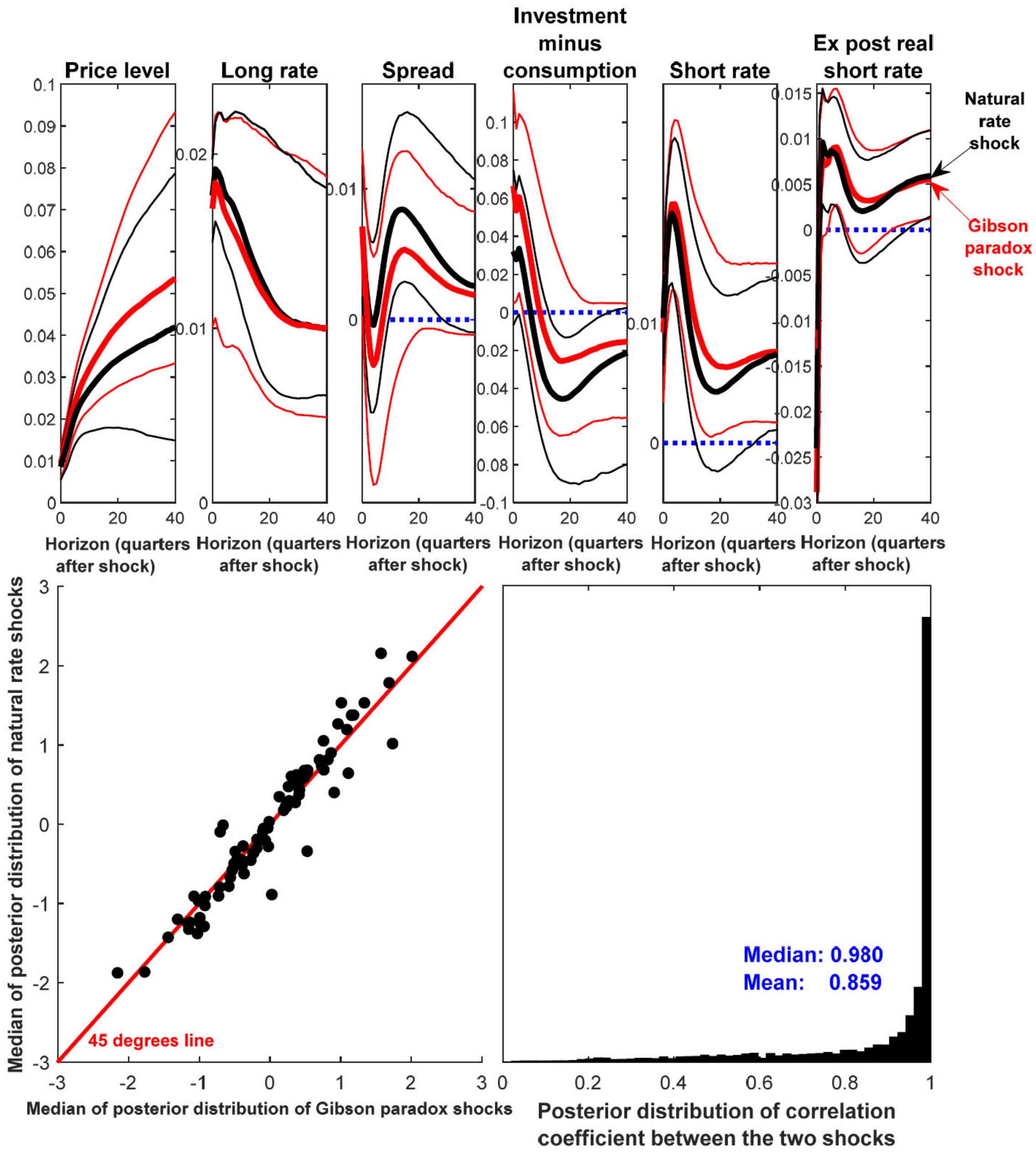


Figure A.1c Evidence for the Euro area under inflation-targeting: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the shocks

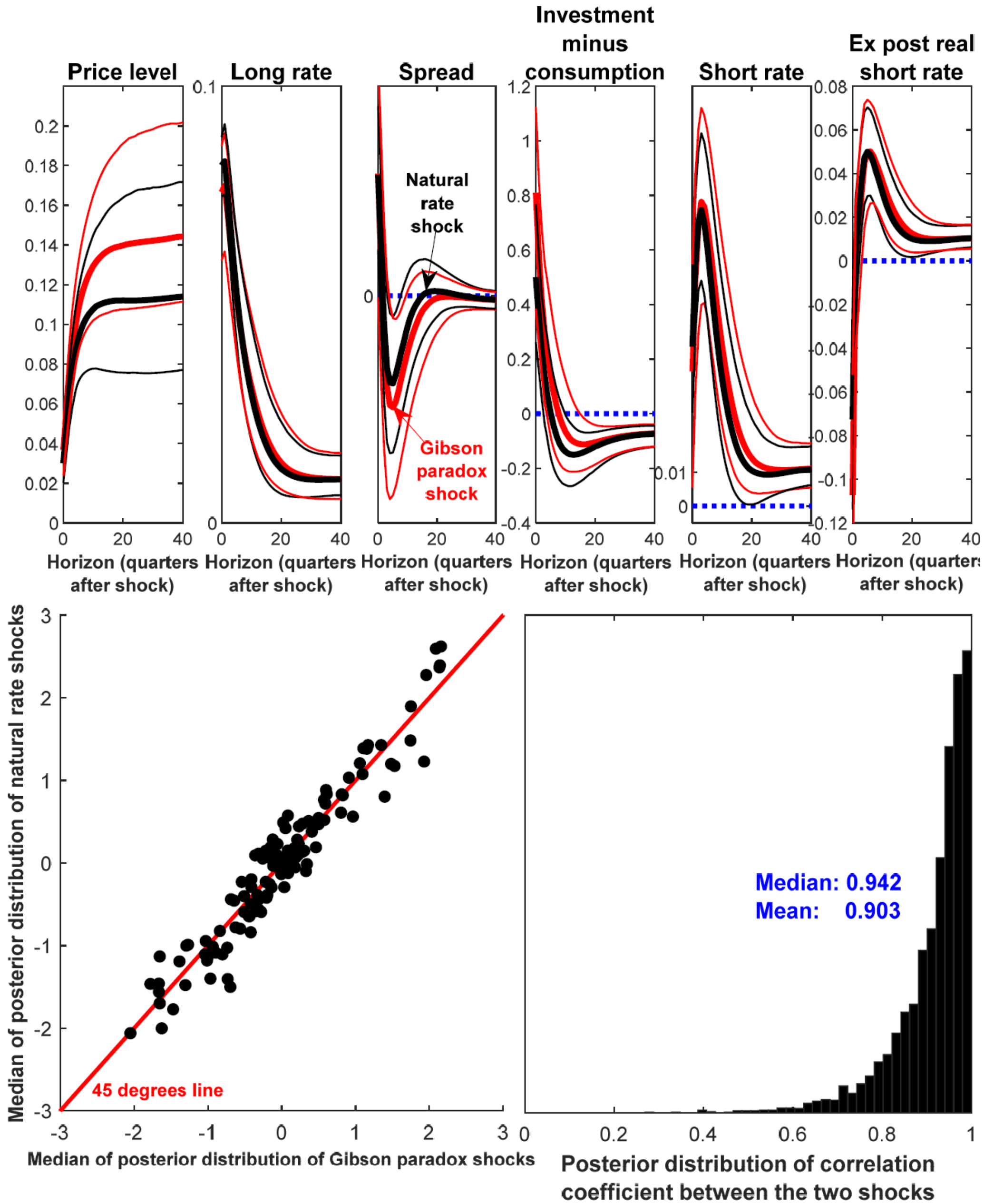


Figure A.1d Evidence for New Zealand under inflation-targeting: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the shocks

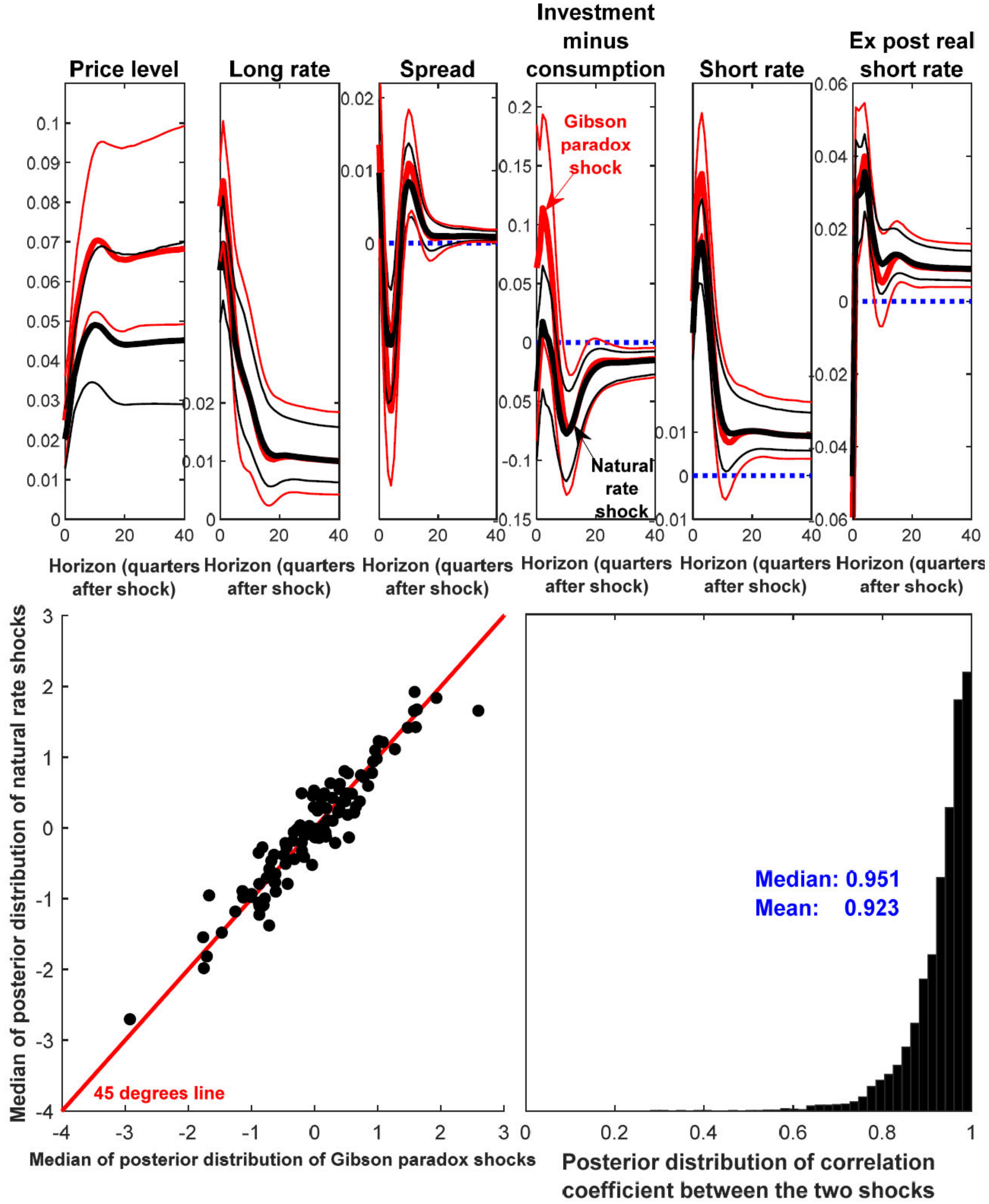


Figure A.1e Evidence for Sweden under inflation-targeting: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the shocks

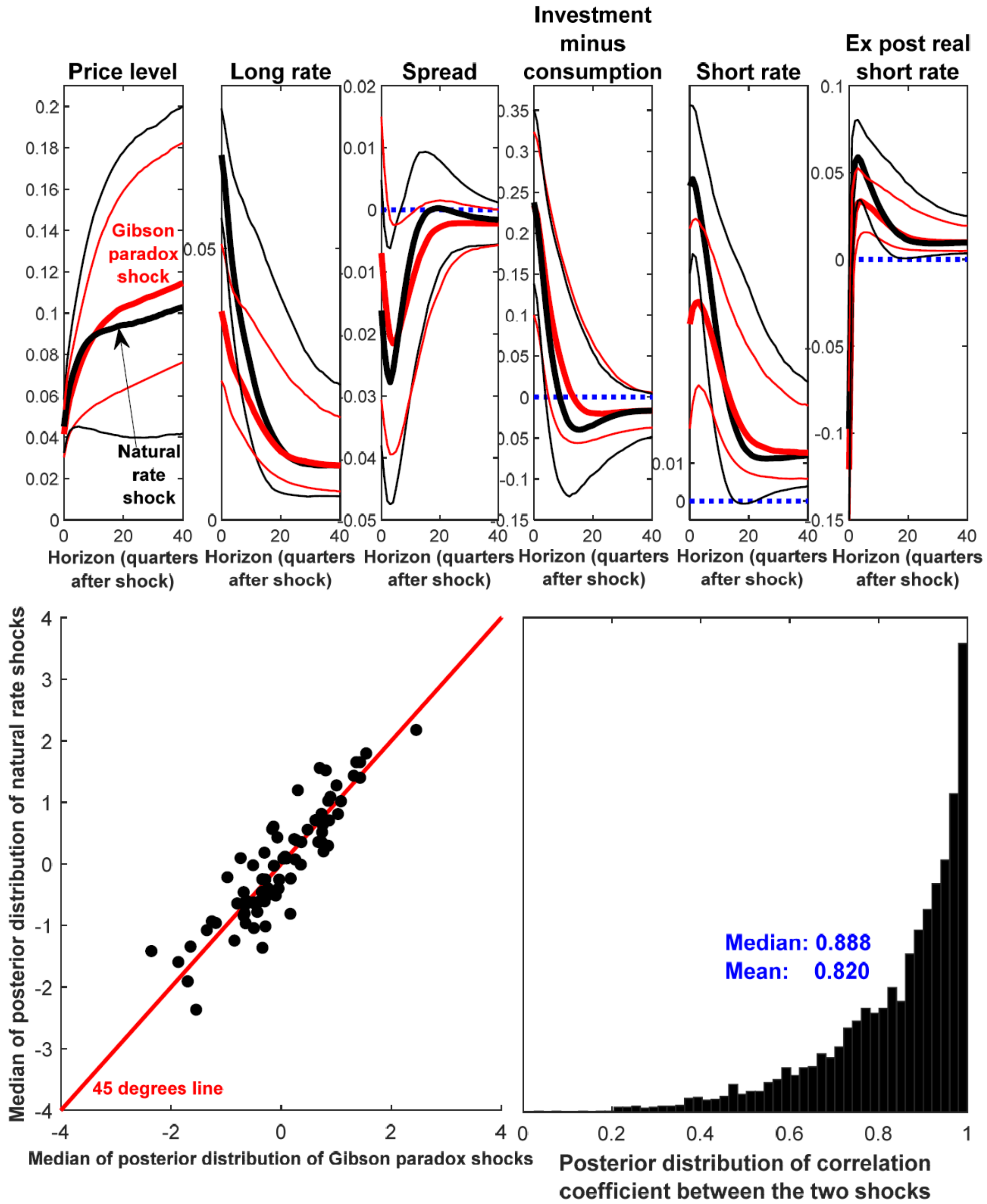


Figure A.1f Evidence for Switzerland under inflation-targeting: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the shocks

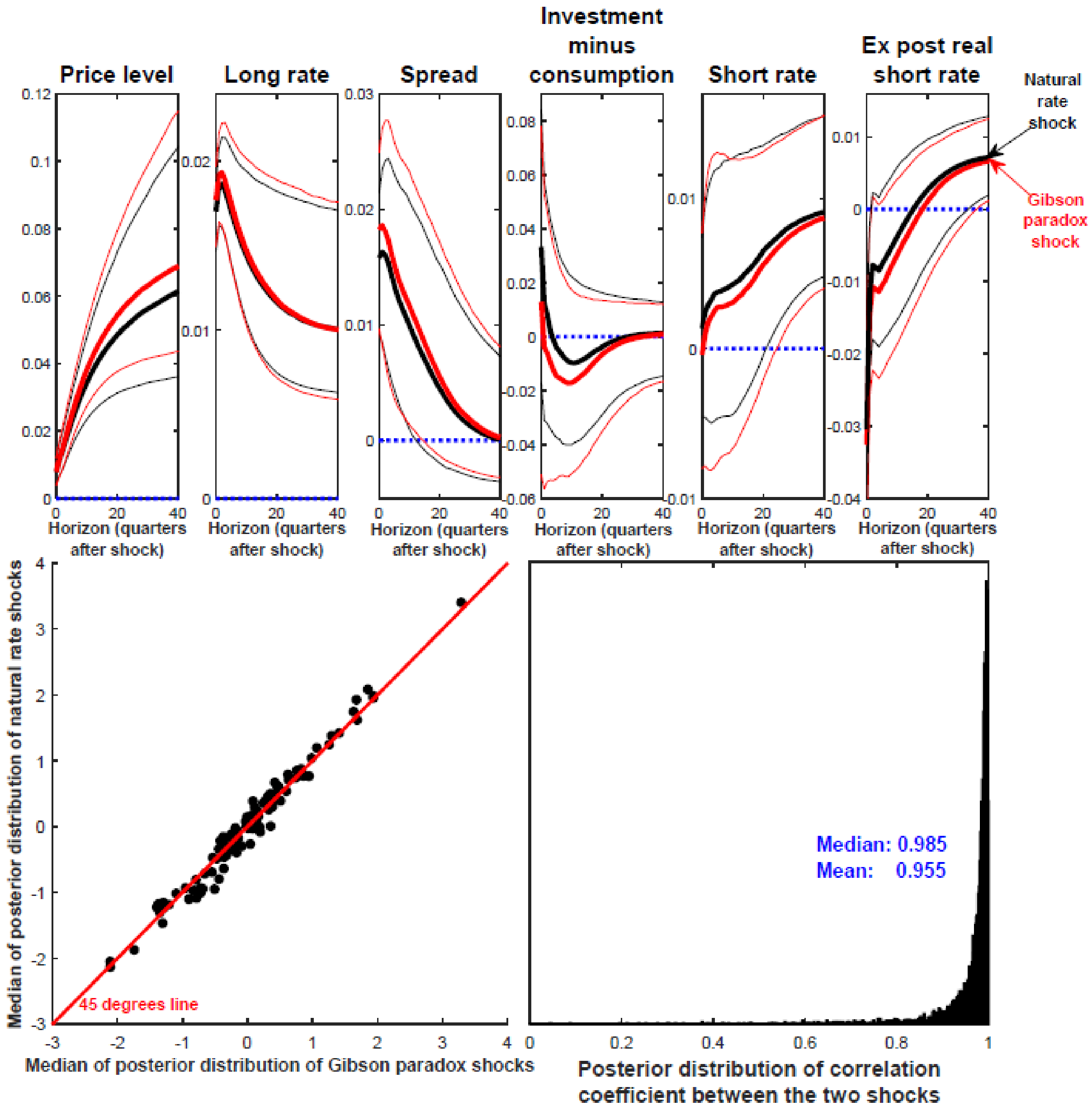


Figure A.1g Evidence for the United Kingdom under inflation-targeting: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the two shocks

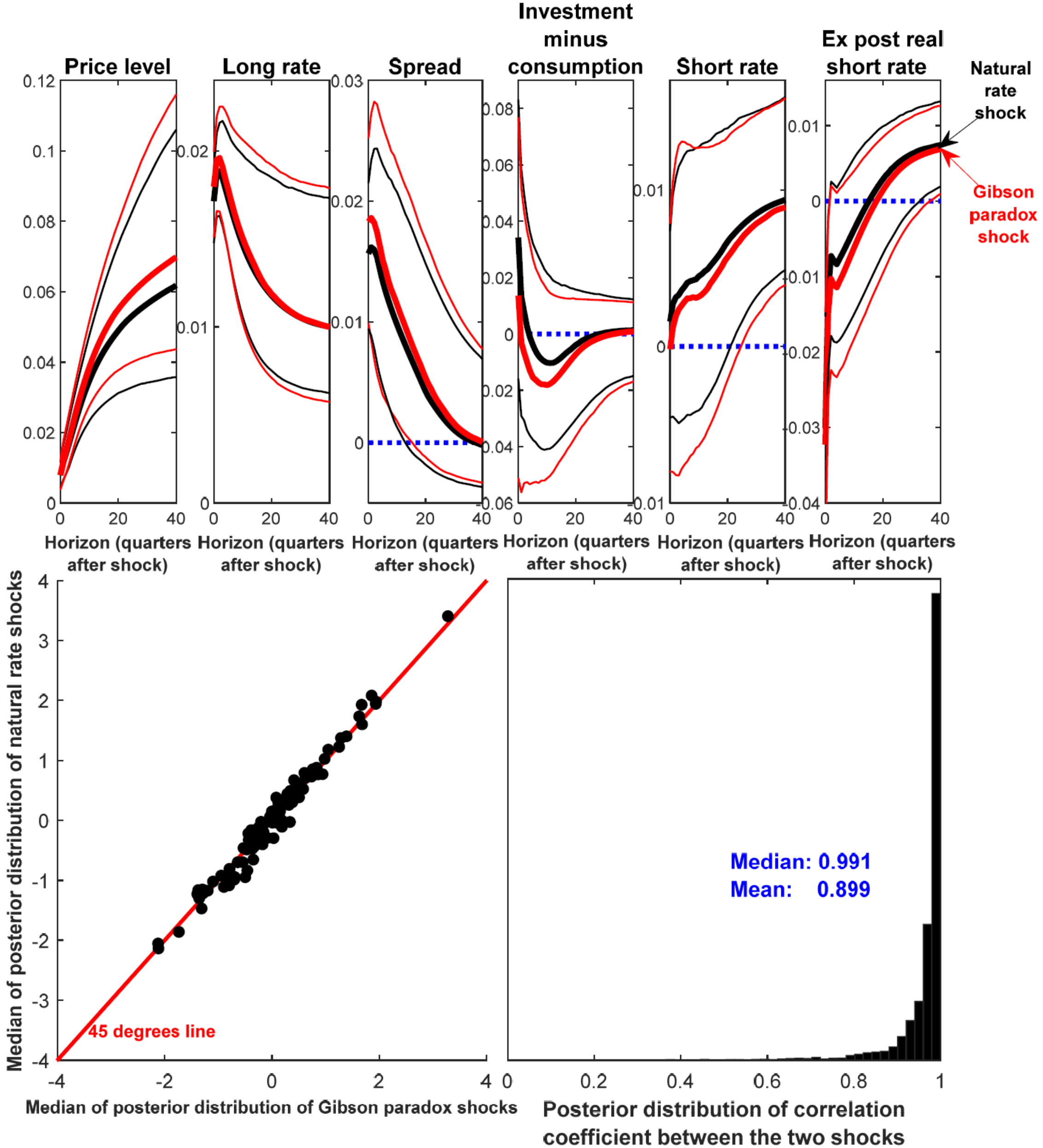


Figure A.1h Evidence for West Germany: Impulse-response functions to Gibson's paradox and natural rate shocks (median, and 68 per cent coverage credible sets), median identified shocks, and posterior distribution of contemporaneous correlation coefficient between the shocks

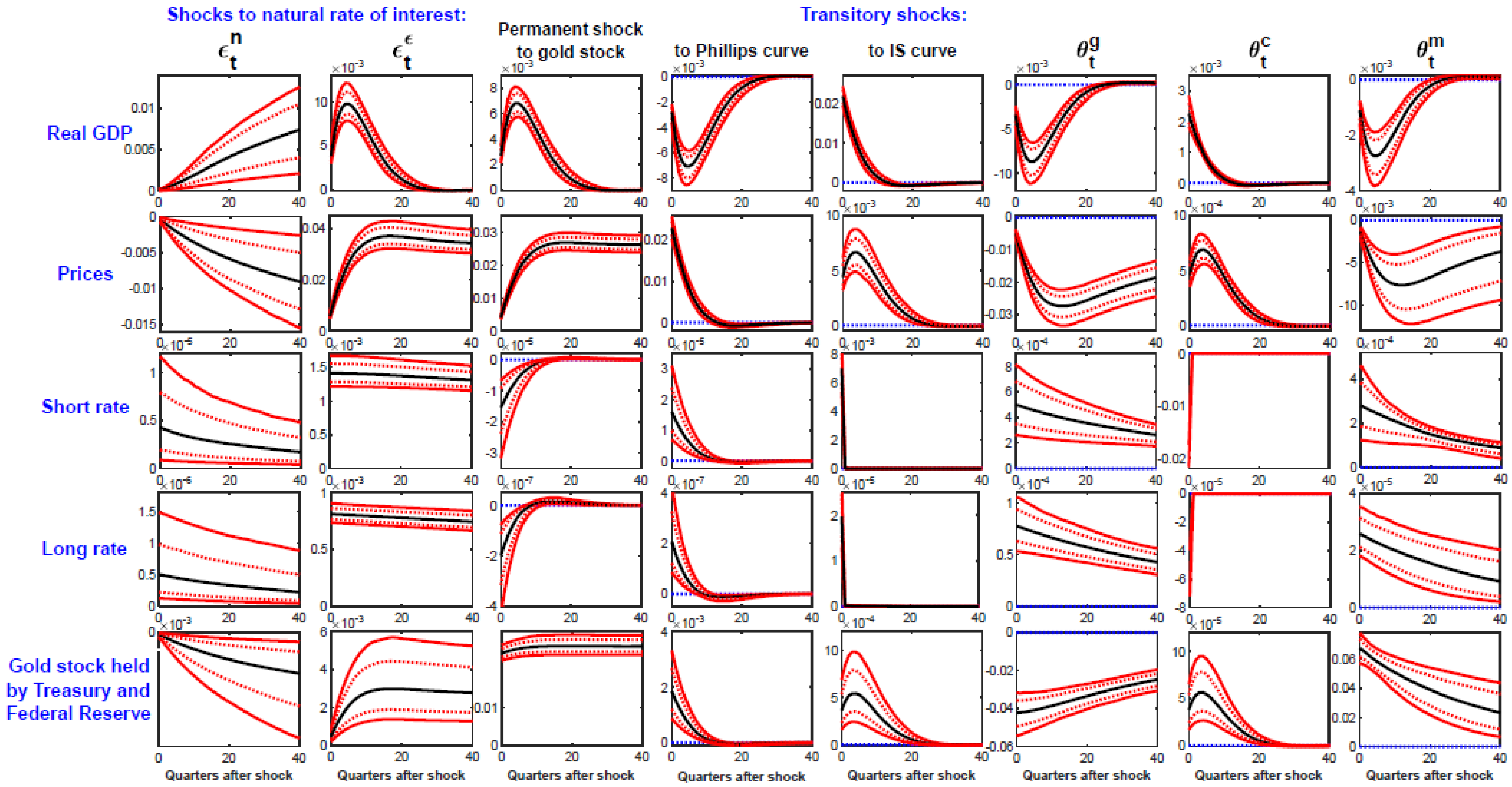


Figure A.2 United States, 1879Q1-1913Q3: Impulse-response functions of the New Keynesian model to the structural shocks (medians of the posterior distributions, and 68 and 90 per cent coverage credible sets)

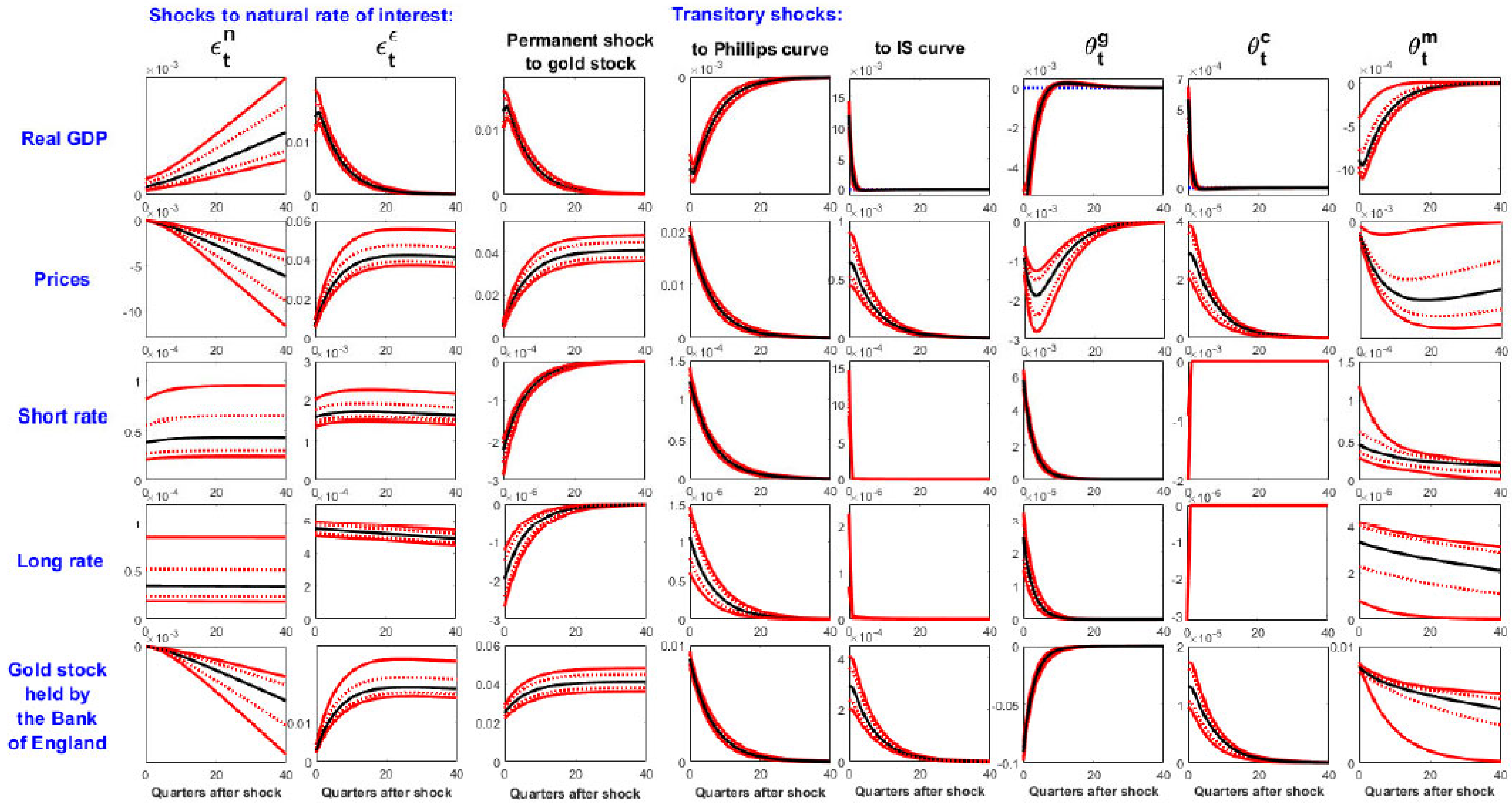


Figure A.3 United Kingdom, 1856Q1-1914Q2: Impulse-response functions of the New Keynesian model to the structural shocks (medians of the posterior distributions, and 68 and 90 per cent coverage credible sets)

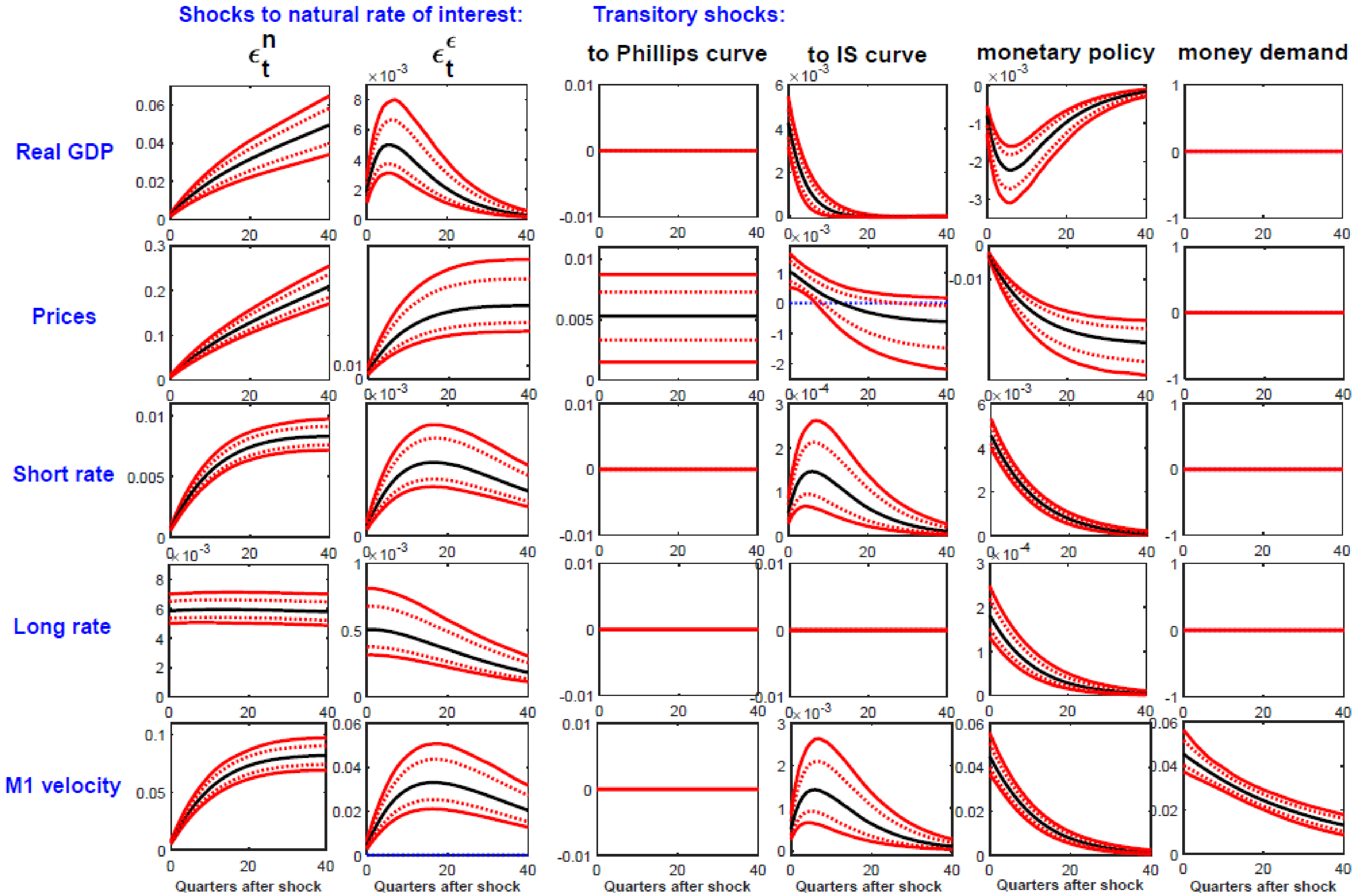


Figure A.4 United Kingdom, 1992Q4-2008Q2: Impulse-response functions of the New Keynesian model to the structural shocks (medians of the posterior distributions, and 68 and 90 per cent coverage credible sets)