Managing Monetary Policy Normalization*

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Abstract

This paper proposes a new framework for monetary policy that introduces a novel transmission mechanism based on a liquidity channel. In our framework, liquidity conditions influence aggregate demand, giving central banks an additional tool of control through balance sheet operations. This mechanism has broad implications. First, balance sheet policies are effective tools for managing aggregate demand even outside the zero lower bound. Second, the size of the central bank balance sheet—and the optimal supply of liquidity—is not solely dictated by private sector reserve demand, but reflects broader fiscal and liquidity management objectives. Finally, in response to a shock that pushes the economy into a liquidity trap, optimal policy calls for an expansion of reserves after hitting the lower bound, with quantitative tightening beginning prior to the interest rate liftoff, and both policies normalizing simultaneously. These findings offer new foundations for understanding the role of central bank balance sheets in macroeconomic stabilization.

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1 Introduction

The global financial crisis of 2008-09 and the recent pandemic shock led major central banks to lower their official policy rates to historically low levels, adopting unconventional monetary policies such as quantitative easing (QE). Quantitative easing consisted in large-scale purchases of government debt and, in some cases, private-sector financial assets to provide monetary accommodation and achieve policy objectives. These operations were financed by issuing bank reserves, resulting in a significant expansion of both central bank assets and liabilities.

As economies recovered and inflationary pressures increased, central banks began normalizing monetary policy by gradually scaling back accommodation. The initial step in this process was tapering, or reducing the pace of asset purchases. This was followed by a combination of interest rate hikes and quantitative tightening (QT), which involved reducing the size of central bank balance sheets. However, the approach to QT has varied across jurisdictions, influenced by institutional differences and strategic decisions regarding the timing, pace, and sequence of policy actions.

As central banks navigate the path of policy normalization, few critical questions arise: How should the balance between interest rate management and balance sheet adjustments be designed? What determines the optimal size of central bank balance sheets and the appropriate supply of liquidity in the economy? Are reserves being adjusted in a way that supports monetary policy effectiveness? Understanding these issues is essential, as central banks recalibrate their policies to meet their objectives.

These questions call for a reassessment of the traditional framework used in monetary policy analysis. In the Neo-Wicksellian model (see Galí, 2008, and Woodford, 2003), central banks have full control over inflation and output through adjustments in the policy rate. Within this framework, reserves play no meaningful role, and the size of central bank balance sheets is irrelevant. Once the policy rate is set, it is a sufficient tool to manage aggregate demand, influence prices, and steer economic activity.

This perspective relies on a crucial assumption: the policy rate directly corresponds to the nominal interest rate that consumers and firms face when making consumption and saving decisions.

To address the policy questions raised above, we develop a new framework for monetary policy analysis that introduces a liquidity channel as a central component of the monetary transmission mechanism. This framework departs from the standard approach by explicitly incorporating the role of the central bank balance sheet as an active policy instruments alongside the interest rate on reserves. While the traditional model is encompassed as a special case, our framework offers a novel perspective on how nominal interest rates interact with balance-sheet policies. In particular, the policy rate no longer directly maps to the nominal interest rate faced by households in their consumption and saving decisions. Instead, household behavior is influenced by liquidity conditions shaped by central bank actions—a mechanism we term the liquidity channel. This distinction has important implications for how inflation and output are managed, and for how central banks design the combination of interest rate and balance sheet tools.

To capture the key features of recent central bank policies, our framework incorporates two essential components: 1) An explicit role for the banking system, as the sole holder of government securities (including central bank reserves) backing deposits; 2) The role of deposits as assets that provide liquidity services to households.

We begin by examining why, in this environment, the policy rate set by the central bank does not necessarily coincide with the interest rate that matters for household consumption and saving decisions. The distinction arises from the structure of financial intermediation: in our framework, only banks hold central bank reserves and treasury bills, which they use to back household deposits. These deposits serve as the primary liquid asset available to households and therefore carry a liquidity premium.

The interest rate on deposits is influenced by the interest rate on reserves set by the central bank. However, because deposits provide liquidity services, their return includes a premium that is not present in other, less liquid assets held by households. It is the return on these illiquid assets—not deposits—that influences intertemporal consumption and saving decisions and thus aggregate demand. As a result, the interest rate relevant for aggregate demand is linked to the policy rate only indirectly, through its effect on the deposit rate and the liquidity premium.

This disconnect gives rise to a novel transmission mechanism – the liquidity channel – through which monetary policy operates. Through the banking sector, equilibrium in the supply of government and private liquidity generates a multiplier effect from government liquidity to private liquidity (i.e., deposits). Crucially, the size of this multiplier depends on the degree of pledgeability of private assets as additional collateral for deposits.

By adjusting the quantity of reserves, central banks can influence the liquidity premium through this multiplier effect on deposits. This, in turn, affects the interest rate on illiquid securities, thereby providing an additional instrument for influencing aggregate demand beyond the conventional policy rate.

An important implication of this framework is that fiscal policy also plays a role in shaping liquidity conditions. Since government bonds serve as collateral backing deposits, the issuance of liquid debt by the fiscal authority affects the liquidity premium. In this way, both monetary and fiscal policy jointly determine inflation and output—marking a departure from conventional frameworks that view monetary policy as the primary driver of macroeconomic outcomes.

The standard Neo-Wicksellian framework is nested within our model as a special case. This occurs when government debt provides no non-pecuniary benefits—either to banks or directly to households—or when liquidity conditions are such that agents derive no marginal benefit from holding liquid assets, i.e liquidity is fully satiated.

Our framework introduces a novel aggregate demand equation that extends the standard New-Keynesian model by explicitly incorporating the role of central bank reserves and the supply of public liquidity. Unlike conventional models, where output depends solely on current and expected future real interest rates, our analysis shows that aggregate demand is also influenced by the supply of liquidity in the economy, which is determined by the combined actions of the central bank and the fiscal authority. An increase in liquidity reduces liquidity premia, making it less costly for households to hold liquid assets, and thereby stimulates output.

This perspective also changes the way interest rate policy affects aggregate demand. While current real interest rates continue to play a central role, the effect of future real rates on current output is reduced. As a result, the model predicts a weaker impact of forward guidance compared to the standard framework, consistent with empirical observation.

We use the model to study the optimal supply of public liquidity and how interest rate and liquidity policies should be managed in an economy facing a temporary liquidity trap.

The first key result is that the optimal supply of liquidity should remain below the satiation level. Maintaining a positive liquidity premium reduces government borrowing costs and, consequently, the reliance on distortionary taxation. Moreover, a higher degree of substitutability between private and public securities as collateral in the banking sector lowers the amount of public liquidity required to support the optimal supply of safe assets, further mitigating the tax burden.

The second main result of our analysis concerns the optimal policy response to shocks that push the economy to the effective lower bound. A key feature of our framework is that it endogenizes demand and supply shocks to safe assets as the primary drivers of movements in the natural rate of interest. In this context, the optimal combination of interest rate and liquidity policy involves a deliberate sequencing of actions. Specifically, once the policy rate reaches the lower bound, the optimal response requires an increase in public liquidity to follow after few quarters the policy rate is at the zero lower bound. This additional liquidity supports aggregate demand through the liquidity channel and helps counteract the constraints on conventional monetary policy.

Importantly, our framework also implies that the process of removing accommodation should begin with a reduction in liquidity—quantitative tightening—before the policy rate is lifted from the lower bound. In other words, the exit strategy involves starting balance sheet normalization ahead of interest rate liftoff, with both instruments ultimately returning to their steady-state levels.

More broadly, the results can be interpreted through the lens of distinct policy objectives—stabilizing inflation versus stabilizing output. If the goal is to maintain inflation close to target, the optimal policy prescribes a gradual and moderate increase in government liquidity, peaking near the end of the liquidity trap and beginning to decline just before the policy rate liftoff. The policy rate itself remains at the lower bound beyond the duration of the shock to avoid premature tightening. In contrast, if output stabilization is prioritized, the optimal response calls for a more front-loaded increase in liquidity, peaking early in the trap and being fully reabsorbed by the time policy rates normalize. In this case, the interest rate liftoff occurs earlier—coinciding with the disappereance of the shock.

Given that standard welfare functions in monetary models typically place more weight on inflation stabilization, the first configuration tends to dominate in the optimal policy design. However, our framework allows for a flexible analysis of these trade-offs, highlighting the importance of jointly managing interest rate and liquidity tools when conventional policy space is constrained.

1.1 Related literature

This work relates to several strands of the macro-finance literature. First, it connects to the influential literature initiated by Krishnamurthy and Vissing-Jorgensen (2011), which documents the quantitative importance of the convenience yield on U.S. Treasury debt.[1] Vissing-Jorgensen (2023a, 2023b), Lopez-Salido and Vissing-Jorgensen (2023) and Afonso et al. (2023a) have estimated a reserve demand function for the banking sector, illustrating the relationship between central bank reserves and the convenience yield. Furthermore, Vissing-Jorgensen (2023a, 2023b) investigates the optimal supply of liquidity based on these estimates.[2]

Our contribution is to embed a banking sector into a New Keynesian framework, showing how financing frictions in the intermediation activity and the non-pecuniary benefits of deposits generate a multiplier between government liquidity (including reserves) and final private liquidity (deposits), with implications for money market spreads tied to liquidity premia. We also analyze the optimal provision of liquidity using a fully microfounded approach that emphasizes the resource constraints imposed by distortionary taxation. In this respect, our analysis echoes Friedman (1960), who raised the fundamental question of whether liquidity should be supplied by the private or public sector.

Our analysis is also related to recent literature that introduces departures from the standard New Keynesian framework. Benigno and Nisticò (2017) develop a model in which the central bank operates with two instruments: the interest rate on reserves and the quantity of reserves. In their framework, reserves provide liquidity services through a cash-in-advance constraint alongside a privately issued asset. They use this model to study how an exogenous reduction in the liquidity properties of private assets affects inflation and output under different monetary policy regimes. However, their banking sector is stylized and does not provide a general framework that nests the standard Neo-Wicksellian paradigm, as our model does.

More recently, Diba and Loisel (2020, 2021) have also proposed New Keynesian models where the central bank operates with two policy instruments. In their setting, financial intermediaries demand reserves to reduce the costs of supplying loans, which are in turn demanded by firms due to a working-capital constraint. In their model, reserves enter directly into the aggregate supply equation. By contrast, in our framework, reserves are held to collateralize deposits, and the money-market channel is distinct from the loan market. Diba and Loisel (2020) show that equilibrium can be determinate even with interest-rate pegging, as reserves act as an additional policy instrument. Diba and Loisel (2021) focus on the quantitative properties of policy at the zero lower bound, demonstrating that their model is consistent with limited deflation and low inflation volatility.

Piazzesi, Rogers, and Schneider (2021) also emphasize the disconnect between money-market rates and the interest rate relevant for consumption and saving decisions. They develop a banking model in which monetary policy operates through either a corridor or a floor system, with the

[1] See also Krishnamurthy, Nagel, and Vissing-Jorgensen (2018) for a related analysis using euro-area data.
[2] See also Afonso et al. (2023b).

main objective of comparing the pass-through of the policy rate to other money-market rates across regimes. They find that equilibrium can be determinate even without a Taylor rule. Arce et al. (2020) also explore the relationship between central bank balance sheet size and the interbank rate. Bigio and Sannikov (2021) integrate monetary policy into a corridor system through a banking model featuring both liquidity and credit channels. However, in their setup, once the corridor around the policy rate collapses to zero, only the interest rate on reserves remains effective, and the quantity of reserves becomes irrelevant. In contrast, in our model, reserves always constitute an active policy instrument and remain relevant for inflation and output even in a floor system, provided they deliver non-pecuniary benefits.[3]

While the aforementioned contributions offer important insights into the role of reserves, liquidity, and monetary transmission mechanisms, they do not explicitly address how the optimal supply of liquidity should be determined, nor how liquidity, interest-rate policy, and fiscal policy should be jointly set during a liquidity trap episode. Our analysis complements this literature by providing a unified framework in which these elements are treated jointly, with particular attention to the implications for the design and normalization of monetary policy.

Earlier contributions such as Canzoneri et al. (2008) and Canzoneri, Cumby, and Diba (2017) also explore environments in which the policy rate diverges from the rate relevant for intertemporal consumption decisions. Cúrdia and Woodford (2010, 2011) present models with borrowers and savers, where credit spreads arise due to financial intermediation. However, in their setting, the policy rate still governs the consumption/saving choices of savers. Although the central bank's balance sheet serves as an additional policy tool in the presence of financial frictions, it operates through the credit spread channel, not through a liquidity channel as emphasized in our framework.

There is both an older and more recent literature that has studied the optimal supply of liquidity. Calvo (1978) and Woodford (1990) analyze monetary economies in which government liabilities that provide liquidity services—namely money—do not bear interest, and where the government finances its needs solely through distortionary taxation. They show that it is optimal to supply money below the satiation level, in contrast with Friedman's rule, which would emerge under lump-sum taxation.

We generalize their results to a monetary economy with sticky prices, in which liquidity is provided through interest-bearing government liabilities. Importantly, we introduce the novel role of the pledgeability of private assets in the production of private liquidity as a mechanism to reduce reliance on government debt. We establish these results within a non-stochastic optimal policy framework under commitment, in the spirit of a timeless perspective. Sims (2022) derives a similar conclusion in a non-stationary solution to a Ramsey problem in a monetary economy with flexible prices. In his model, liquidity satiation is reached only asymptotically, whereas in our framework, as in Calvo (1978), it occurs at a finite level. Relatedly, Angeletos, Collard, and Dellas (2022) obtain comparable results in a setting with real debt and provide microfoundations for the liquidity services of government liabilities.

[3] See also De Fiore, Hoerova, and Uhlig (2018) for a model featuring money-market frictions.

A key extension relative to these contributions is our analysis of optimal monetary and fiscal policy in a stochastic economy, particularly in response to shocks that drive the economy to the zero lower bound. This allows us to study how interest-rate and liquidity policies should be jointly managed under such conditions.

In this respect, our work is also related to the literature on optimal interest rate policy in liquidity traps, including Eggertsson and Woodford (2003, 2004) and Werning (2011). The main difference is that, in our framework, liquidity becomes an active policy instrument during a liquidity-trap episode. While Eggertsson and Woodford (2004) highlight the role of public debt in smoothing distortionary taxation, our framework shows how public debt, by supplying liquidity, can directly stimulate aggregate demand.

Our analysis is also connected to the literature on the so-called "forward-guidance puzzle," as identified by Del Negro, Giannoni, and Patterson (2013), where standard New Keynesian models tend to overstate the effectiveness of forward guidance in stimulating current demand. Recent attempts to resolve this puzzle, such as Werning (2015) and McKay, Nakamura, and Steinsson (2016), rely on incomplete markets. In contrast, our framework generates a new aggregate demand equation in which forward guidance is inherently less powerful, even under complete markets. A similar attenuation effect is obtained in Diba and Loisel (2020), although through a different transmission channel.

The present work starts with Section 2, providing the main intuition for why our framework departs from the standard Neo-Wicksellian paradigm. Section 3 presents the model and Section 4 characterizes the equilibrium. Section 5 studies the model in a log-linear approximation to discuss its main novelties. Section 6 discusses the optimal supply of liquidity while Section 7 studies how interest-rate and liquidity policies should be managed in a liquidity trap. Section 8 concludes the work.

2 Reserve Effectiveness: the "Liquidity Channel"

In this section, we highlight the key distinction between our framework and the traditional Neo-Wicksellian paradigm. In the latter, the economy is typically described by a standard AS-AD model in which the policy rate directly influences the aggregate demand (AD) equation. To illustrate this, consider the standard Euler equation in a perfect-foresight setting:

$$U_c(C_t) = \beta \frac{(1+i_t)}{\Pi_{t+1}} U_c(C_{t+1}), \tag{1}$$

where $U_c(\cdot)$ denotes the marginal utility of consumption at time t, $\beta \in (0,1)$ is the time preference rate, i_t is the nominal interest rate, and Π_{t+1} is the gross inflation rate between t and $t+1$. A central assumption in the Neo-Wicksellian framework is that the policy rate set by the central bank coincides with the nominal rate influencing the AD block. Raising the policy rate reduces demand, conditional on expected future consumption and inflation, and thus allows the central bank to steer the paths of inflation and output.

Our framework retains the Euler equation (1), but introduces a crucial difference: there is no direct connection between the central bankâ's policy rate and the nominal rate that enters household consumption and saving decisions. Instead, we introduce the concept of a market nominal interest rate, denoted i^B, which is the risk-free rate on private, illiquid securities. Under perfect foresight, the household Euler equation becomes:

$$U_c(C_t) = \beta \frac{(1+i_t^B)}{\Pi_{t+1}} U_c(C_{t+1}). \tag{2}$$

In addition to borrowing or lending through private illiquid securities, households can also hold safe, liquid assets Q_t issued by financial intermediaries. These assets provide liquidity services. Households' portfolio choices determine the spread between the interest rate on safe assets, i^Q, and the market nominal rate i^B:

$$1 + i_t^Q = (1-\mu_t)(1+i_t^B), \tag{3}$$

where $\mu_t \geq 0$ is the liquidity premium, given by:

$$\mu_t = V_q\left(\frac{Q_t}{P_t}\right),$$

where $V_q(\cdot)$ is the marginal utility from holding liquid assets, Q_t is the nominal amount of safe assets, and P_t is the price level. We assume that $V_q(Q_t/P_t) = 0$ whenever $Q_t/P_t \geq \bar{q}$, for some satiation level $\bar{q} > 0$, implying that additional liquidity has no value beyond this threshold.

To understand the transmission mechanism in our framework, we model the banking sector explicitly. Intermediaries issue deposits (safe assets), raise equity, and invest in both government debt (reserves) and private securities, both of which can be pledged as collateral. In equilibrium, the interest rate on deposits, i^Q, becomes a weighted average of the policy rate (i.e., the interest on reserves, i^R) and the market nominal interest rate, i^B:

$$1 + i_t^Q = (1-\rho_{\gamma,t})(1+i_t^B) + \rho_{\gamma,t}(1+i_t^R), \tag{4}$$

where $\rho_{\gamma,t} \in [0,1]$ is a time-varying variable that depends on the share of private assets that can be pledged as collateral.[4]

The equilibrium condition in the banking sector further implies that the quantity of safe assets is a multiple of government debt (or reserves):

$$Q_t = \frac{B_t^g}{\rho_{\gamma,t}}. \tag{5}$$

This multiplier varies over time, depending on the pledgeability of private collateral. As the pledgeability of private assets declines (i.e., $\rho_{\gamma,t}$ rises), the supply of private liquidity falls for a

[4] We will show that $\rho_{\gamma,t}$ increases as the share of pledgeable private debt falls.

given level of government liquidity. This feature aligns with the empirical evidence from the 2007–2008 financial crisis, when balance-sheet constraints in the financial sector reduced the availability of private safe assets.

Combining (3), (4), and (5) yields the following key relationship between the market nominal interest rate and the policy rate:

$$1 + i_t^B = \frac{\rho_{\gamma,t}}{\rho_{\gamma,t} - V_q\left(\frac{1}{\rho_{\gamma,t}}\frac{B_t^g}{P_t}\right)}(1 + i_t^R). \tag{6}$$

This expression provides several insights into the role of reserves (government debt) in monetary policy:

- *Reserves as an Independent Stabilization Tool.*
  Reserves can be used independently of the policy rate to stabilize the economy, even when the zero lower bound is not binding. An increase in reserves ($\uparrow B_t^g$), holding everything else constant, lowers the liquidity premium and thus the market nominal interest rate ($\downarrow i_t^B$), which stimulates aggregate demand. This effect holds as long as the economy has not reached full liquidity satiation ($V_q(Q_t/P_t) > 0$).

- *Amplification of Policy Rate Effects.*
  Adjustments in the policy rate i^R have amplified effects on the market rate i^B due to the liquidity premium.[5]

The liquidity channel described above becomes ineffective when liquidity is abundant and fully satiated, i.e., when $V_q(Q_t/P_t) = 0$. In this case, reserves (or government debt) no longer influence the market nominal interest rate.

A central implication of our framework is that the supply of liquidity is inherently tied to fiscal capacity. Since we assume that central bank reserves and Treasury bills are perfect substitutes in terms of the liquidity services they provide, the determination of inflation and output becomes a joint monetary-fiscal policy problem.

Equation (6) also provides a lens through which to interpret liquidity crises. A decline in the pledgeability of private collateral raises $\rho_{\gamma,t}$, reducing the supply of safe assets Q_t and increasing the market interest rate i_t^B, thereby creating contractionary pressure. In a richer setting with nominal rigidities, we will explore the optimal policy response through adjustments in the policy rate (i^R) and in the supply of government liquidity (B^g), subject to the zero lower bound and the fiscal cost of issuing treasury bonds and central bank reserves.

[5] In equation (6), the term $\frac{\rho_{\gamma,t}}{\rho_{\gamma,t} - V_q(\frac{1}{\rho_{\gamma,t}}\frac{B_t^g}{P_t})}$ exceeds one whenever $V_q(\cdot) > 0$.

3 Model

In this Section, we present the building blocks of the model starting from the banking sector. We then focus on households sector and the government, which encompasses both the treasury and the central bank.

3.1 Banking Sector

At a generic time t, there exists a potentially infinite number of intermediaries that can engage in intermediation without incurring any entry costs. Each intermediary operates for two periods. Intermediaries entering at time t face the following balance sheet constraint:

$$B_t^g + A_t = Q_t + (1 - \delta)N_t, \tag{7}$$

where B_t^g represents holdings of government securities, including central bank reserves and Treasury bills, which are remunerated at the rate i_t^R. A_t denotes holdings of short-term private securities earning the market interest rate i_t^B. Q_t denotes deposits issued by intermediaries, remunerated at i_t^Q, and N_t represents equity raised by intermediaries, which is more costly to issue than debt. The cost is modeled through the parameter δ, with $0 < \delta < 1$.[6]

When intermediaries borrow from the private sector, A_t is negative. In contrast, B_t^g, Q_t and N_t are always non-negative. It follows, by the absence of arbitrage opportunities, that $i_t^B \geq i_t^R$ and $i_t^Q \geq i_t^R$, as otherwise intermediaries could earn infinite profits.

Deposits Q_t, issued by financial intermediaries, function as a "safe asset" for households –that is, a risk-free security providing liquidity services. These deposits are backed by the assets held by the intermediary through the collateral constraint:

$$B_t^g + \gamma_t \max(A_t, 0) \geq \rho Q_t. \tag{8}$$

Here, γ_t is the fraction of private securities A_t that can be pledged as collateral, with $0 \leq \gamma_t \leq 1$, and ρ, with $0 \leq \rho \leq 1$, is the fraction of deposits that must be backed by collateral. Intermediaries' holdings of government debt, B_t^g, reflect the implicit or explicit requirement to use high-quality assets to back the liquid securities they issue.[7]

These assets include Treasury debt and central bank reserves. Importantly, we consider a framework in which the properties of central bank reserves –namely, their ultimate safety and liquidity within the currency system–also extend to Treasury debt. Hence, in what follows, the two are considered equivalent and grouped under B^g.[8]

[6] A more general framework could include intermediaries supplying loans to the private sector to finance physical capital for production, as in Benigno and Benigno (2021). Such a model would capture a credit channel, which is orthogonal to the liquidity channel emphasized here and does not alter the results of the analysis.

[7] This requirement should not be interpreted strictly as a regulatory constraint. Even though reserve requirements have been abolished in the U.S., banks continue to hold government securities including federal funds, Treasury debt, mortgage-backed securities, and other liquid assets.

[8] An important characteristic of central bank liabilities is that they are default-free without the central bank being

Private securities A_t represent risk-free, privately created instruments that can also be used as collateral, albeit to a lesser extent than government debt. Only a fraction γ_t of these can be pledged as collateral, and this fraction can vary over time. For example, in our context, a decline in γ_t may reflect a deterioration in the quality of private assets, such as during the 2007-2008 financial crisis. The interest rate i_t^B on private securities A_t represents the market (nominal) interest rate, as it directly influences households' consumption and saving decisions, as discussed in the next section.

We assume that $0 < \gamma_t < \rho$, otherwise the collateral constraint (8) would never bind, rendering the banking problem trivial.[9]

The parameter ρ determines the extent to which deposits must be backed by assets:

1. When $\rho = 1$, all deposits are fully backed by assets;
2. When $\rho = 1$ and $\gamma_t = 0$, deposits are backed exclusively by government debt, as in a narrow banking system;
3. When $\rho = 0$, there is no collateral requirement.[10]

Intermediaries can also invest in cash, which is dominated in returns by government debt. While the economy is cashless in equilibrium, cash still exists as a store of value. The possibility of converting reserves into cash implies the existence of a zero lower bound on the interest rate on reserves. Consequently, we have the condition:

$$i_t^Q,\ i_t^B \geq i_t^R \geq 0.$$

3.1.1 Banks' Optimization Problem

Intermediaries maximize rents, $\mathcal{R}$, defined as the expected discounted value of profits minus the value of equity:

$$\mathcal{R}_t = E_t \left\{ M_{t+1} \Psi_{t+1} \right\} - N_t, \tag{9}$$

where profits, Ψ_{t+1}, at time $t+1$ are given by:

$$\Psi_{t+1} = (1 + i_t^B) A_t + (1 + i_t^R) B_t^g - (1 + i_t^Q) Q_t. \tag{10}$$

Here, M_{t+1} denotes the household's stochastic discount factor, since consumers are the ultimate owners of financial intermediaries.

Intermediaries are subject to a limited-liability constraint, which requires profits to be non-negative:

$$\Psi_{\min} = (1 + i_t^B) A_t + (1 + i_t^R) B_t^g - (1 + i_t^Q) Q_t \geq 0. \tag{11}$$

subject to a solvency constraint, as discussed in Benigno (2025).

[9] This can be seen by substituting (7) into (8).

[10] In this case, for the central bank to effectively control money-market interest rates through the policy rate, reserves must be supplied in positive quantities.

This constraint is independent of the state realized at time $t+1$.[11]

Intermediaries choose A_t, B_t^g, and Q_t to maximize (9), given (10), subject to the budget constraint (7), the limited-liability constraint (11), and the collateral constraint (8).

It is useful to express the objective function (9) as:

$$\mathcal{R}_t = \left[\frac{1+i_t^R}{1+i_t^B} - 1\right] B_t^g - \left[\frac{1+i_t^Q}{1+i_t^B} - 1\right] Q_t - \delta N_t, \tag{12}$$

where we have substituted the balance sheet constraint (7) into (10) to eliminate A_t, and used the condition $E_t\left\{M_{t+1}(1+i_t^B)\right\} = 1$, which holds in the household optimization problem.

Inspection of (12) reveals the cost associated with issuing equity. As a result, the limited-liability constraint (11) binds. Using (7) to solve for A_t and substituting into (11), we can solve for N_t and substitute it into (12) to obtain:

$$\mathcal{R}_t = \frac{1}{1-\delta}\left\{\left[\frac{1+i_t^R}{1+i_t^B} - 1\right] B_t^g - \left[\frac{1+i_t^Q}{1+i_t^B} - 1\right] Q_t\right\}. \tag{13}$$

The banking equilibrium can be described through three main propositions.

Proposition 1 *When government liquidity is abundant, i.e., $B_t^g + \gamma_t A_t > \rho D_t$, deposit and market interest rates are equalized to the policy rate:*

$$i_t^Q = i_t^B = i_t^R.$$

Proof. Since $i_t^R \leq i_t^B$, for positive government liquidity to be held in equilibrium ($B_t^g > 0$), it must be that $i_t^R = i_t^B$ by (13). Then, applying the zero-rent condition for perfect competition in the market of financial intermediation, we obtain $i_t^Q = i_t^B = i_t^R$. ■

When government liquidity is abundant, the supply of safe assets by intermediaries becomes perfectly elastic at an interest rate equal to the policy rate. As we will see when analyzing the household's problem, at these equalized interest rates the demand for liquidity is high enough to reach satiation.

An additional interesting implication of the above proposition is that the Neo-Wicksellian framework emerges in this case, meaning that our analysis coincides with that of the standard New Keynesian model.

Proposition 2 *When government liquidity is scarce, i.e., $B_t^g + \gamma_t A_t = \rho D_t$ and $0 \leq \gamma_t < \rho$, the interest rate on deposits is given by:*

$$(1+i_t^Q) = \rho_{\gamma,t}(1+i_t^R) + (1-\rho_{\gamma,t})(1+i_t^B), \tag{14}$$

[11] With risky assets, the limited-liability constraint would be state-contingent.

with

$$\rho_{\gamma,t} = \rho(\gamma_t) = 1 - \frac{1-\rho}{1-\gamma_t}.$$

Proof. The result can be proved by solving the limited-liability constraint (11) with equality for A_t and substituting it into the collateral constraint (8). The resulting expression for B_t^g as a function of D_t can then be plugged into (13) obtaining the result. ■

When government liquidity is scarce, the interest rate at which intermediaries are willing to supply safe assets becomes a weighted average of the policy rate and the market interest rate, with the weight given by $\rho_{\gamma,t}$. The supply of such assets is perfectly elastic at this rate.

An interesting implication is that, as the degree of pledgeability of private assets in the collateral constraint increases (i.e., as γ_t rises), the safe interest rate i_t^Q is pulled toward the market rate i_t^B.

The parameter ρ plays a key role in characterizing the equilibrium relationships among money-market interest rates under specific policy regimes:

- **Narrow Banking Regime** ($\rho = 1$)

In a narrow banking system, the rate on safe assets coincides with the policy rate, $i_t^Q = i_t^R$. However, in general, the market interest rate remains higher: $i_t^B > i_t^Q = i_t^R$.

- **No Collateral Requirement** ($\rho = 0$)

When $\rho = 0$, it follows that $i_t^Q = i_t^B$, and $i_t^B = i_t^R$ as long as reserves are positively supplied by the central bank. Therefore, when $\rho = 0$, all interest rates are equalized:

$$i_t^B = i_t^R = i_t^Q,$$

and the Neo-Wicksellian regime is once again nested within the model.

To conclude the characterization of the banking problem in this case, we derive the intermediaries' demand for government liquidity, private assets, and equity.

Proposition 3 *The demand for government liquidity is $B_t^g = \rho_{\gamma,t} Q_t$; the demand for private assets is $A_t = (\rho_{\gamma,t}^{-1} - 1)Q_t$; and the demand for equity is $N_t = 0$.*

Proof. These results follow from combining the zero-rent condition applied on (13) with the balance sheet constraint (7), the collateral constraint (8), and the deposit rate equation (14). ■

The result that the demand for government liquidity is given by $B_t^g = \rho_{\gamma,t} Q_t$ is particularly intriguing when considered alongside the supply of government liquidity, which is determined by the joint actions of monetary and fiscal authorities. It follows that the supply of private safe assets is given by:

$$Q_t = \frac{B_t^g}{\rho_{\gamma,t}},$$

for a given B_t^g, with a time-varying multiplier of $1/\rho_{\gamma,t}$. This implies that government liquidity does not fully determine the supply of private liquidity, since $\rho_{\gamma,t}$ depends on the pledgeability of private assets as collateral.

As this degree of pledgeability increases (i.e., γ_t rises and $\rho_{\gamma,t}$ falls), the supply of safe assets expands for a given supply of government debt. Conversely, a fall in the fraction γ_t reduces the creation of private safe assets – an effect observed during the 2007-2008 financial crisis.

As we will see later, Q_t has direct effects on aggregate demand. Therefore, shocks originating in the banking sector can propagate to the real economy.

Finally, consider the result that the demand for equity is zero. This arises because the assets held by intermediaries are risk-free.[12]

3.2 Households

We consider a representative household that maximizes the following intertemporal utility:

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[\frac{C_t^{1-\sigma^{-1}}}{1-\sigma^{-1}}-\int_0^1\frac{(H_t(j))^{1+\eta}}{1+\eta}dj+\xi_{q,t}V\left(q_t\right)\right]\right\}, \tag{15}$$

where E_{t_0} is expectation operator at time t_0; β, with $0<\beta<1$, is the intertemporal discount factor in preferences; σ, with $\sigma>0$, is the intertemporal elasticity of substitution in consumption, C, which is the Dixit-Stiglitz aggregator of a unit measure of differentiated goods with elasticity of substitution θ.

Households experience disutility from supplying the different varieties of labor $H(j)$, with $j\in[0,1]$. The variety j is used by firm of type j to produce the differentiated good j; η, with $\eta\geq 0$, denotes the inverse of the Frisch elasticity of labor supply.

Finally, households derive utility from the real value of safe assets, q, with $q=Q/P$ and P the price level. The function $V(\cdot)$ is concave and non-decreasing, with a satiation point at a finite level $\bar{q}>0$; $V_q(q_t)=0$ for $q_t\geq\bar{q}$. To ensure a well-defined demand for liquidity, when q_t approaches $\bar{q}$ from below, we assume that $V_{qq}(q_t)$ remains negative in the limit; ξ and ξ_q are preference shocks with ξ_q affecting directly the preference for liquidity.

The household faces the following flow budget constraint:

$$P_tC_t+Q_t+(1+i_{t-1}^B)B_{t-1}+N_t\leq(1+i_{t-1}^Q)Q_{t-1}+B_t+\int_0^1W_t(j)H_t(j)dj+\Psi_t+\Phi_t+T_t. \tag{16}$$

She/He can invest its savings in safe assets Q, which provide liquidity services, and pay an interest rate i^Q. She/He can borrow or lend through private risk-free bonds, B, that pay an interest rate i^B, but do not provide direct liquidity services.[13]

Households finance intermediaries through equity N. On the right-hand side of the budget constraint, households get income from working in each firm, where $W_t(j)$ represents the wage in sector j. They receive profits from intermediaries and firms, denoted by Ψ and Φ respectively. Additionally, T represents exogenous, non-negative government transfers.

[12] If intermediaries supplied risky loans, the demand for equity would be positive in order to absorb potential losses on those loans.

[13] Note that in the household's budget constraint a positive value for B denotes debt.

Household's optimization problem is to maximize utility (15) by choosing stochastic sequences $\{C_t, B_t, Q_t\}_{t=t_0}^{\infty}$ subject to the flow budget constraint (16), an appropriate borrowing limit and initial conditions.

The first order condition with respect to the illiquid bonds, B_t is:

$$E_t \{M_{t+1}\} = \frac{1}{1 + i_t^B}, \tag{17}$$

where M_{t+1}, the nominal stochastic discount factor, is

$$M_{t+1} = \beta \frac{\xi_{t+1} C_{t+1}^{-\sigma^{-1}}}{\xi_t C_t^{-\sigma^{-1}}} \frac{P_t}{P_{t+1}}.$$

The expected value of the nominal stochastic discount factor equals to the price of the illiquid bonds – the inverse of the gross nominal interest rate. The market nominal interest rate, i^B,directly affects the consumption-saving choices.

The first order condition with respect to safe assets, Q_t, implies that

$$1 = \mu_t + (1 + i_t^Q) E_t \{M_{t+1}\}, \tag{18}$$

where μ_t is the liquidity premium, given by

$$\mu_t = \frac{\xi_{q,t} V_q(q_t)}{C_t^{-\sigma^{-1}}},$$

with $V_q(\cdot)$ is the partial derivative of $V(\cdot)$ with respect to Q_t, and $0 \leq \mu_t < 1$.

Combining (17) and (18), we obtain

$$(1 + i_t^Q) = (1 - \mu_t)(1 + i_t^B),$$

indicating that the interest rate on safe assets is lower, or almost equal, than the rate on illiquid bonds. The two rates coincides only when the economy is satiated. The optimal supply of equity, N, is equal to the discounted value of intermediary profits:

$$N_t = E_t \{M_{t+1} \Psi_{t+1}\},$$

consistent with the zero-rent condition applied to (9).

The optimal choice with respect to labor supply requires that the marginal rate of substitution between labor and consumption is equal to the real wage

$$\frac{H_t(j)^\eta}{C_t^{-\sigma^{-1}}} = \frac{W_t(j)}{P_t},$$

for each variety of labor j.

Finally, the intertemporal budget constraint of the consumer holds with equality at all times.

3.3 Firms

Firms are uniformly distributed over the interval $[0, 1]$ and produce goods using labor as their sole input, according to the production $Y_t(j) = H_t(j)$. They face a demand function of the form $Y_t(j) = (P_t(j)/P_t)^{-\theta} Y_t$, in which $P(j)$ is the price of good j and θ is the elasticity of substitution between different varieties of goods, with $\theta > 1$. Prices are sticky following the Calvo model in which a fraction $1 - \alpha$ of firms is allowed to change their prices maximizing the expected present discounted value of its profits. Firms that cannot adjust their prices index them to the target Π. Firms' revenues are taxed at the rate τ_t. We do not detail here the firms' optimization problem and the first-order conditions, since these are standard in the literature. In the next Section, we discuss the resulting Aggregate-Supply equation.

3.4 Government

The government sector includes the treasury and the central bank. Their combined budget constraint is expressed as

$$B_t^g = (1 + i_{t-1}^R) B_{t-1}^g + T_t - \tau_t P_t Y_t, \tag{19}$$

where the short-term debt (B^g) includes treasury bills and central bank reserves and carries the nominal interest rate i^R; T_t with $T_t \geq 0$ denotes exogenous transfers and τ_t represents distortionary taxation on firms' revenues.

4 Equilibrium

We now describe the equilibrium conditions.

Equilibrium in the Goods Market
Goods market equilibrium requires that output equals consumption:

$$Y_t = C_t.$$

Equilibrium in the Market for Private Illiquid Securities
The supply of illiquid securities is perfectly elastic at the rate:

$$\frac{1}{1 + i_t^B} = E_t \left\{ \beta \frac{\xi_{t+1} U_c(Y_{t+1})}{\xi_t U_c(Y_t)} \frac{P_t}{P_{t+1}} \right\}. \tag{20}$$

As shown in the banking equilibrium, the demand for illiquid securities by intermediaries is:

$$A_t = (1 - \rho_{\gamma,t}) B_t^g,$$

and in equilibrium $A_t = B_t$. Therefore, the supply of government liquidity B_t^g determines the equilibrium quantity of private illiquid securities through the factor $1 - \rho_{\gamma,t}$.

In the standard New Keynesian (Neo-Wicksellian) framework, equation (20) captures the mechanism through which monetary policy transmits to output and inflation. However, here i_t^B does not necessarily coincide with the policy rate i_t^R, unless special conditions hold.

Remark 1 *The Neo-Wicksellian framework – where all nominal interest rates are equalized– is nested when: (i) government liquidity is abundant, i.e.,* $B_t^g + \gamma_t A_t > \rho D_t$ *(see Proposition 1); or (ii) government liquidity provides no non-pecuniary benefits, i.e.,* $\rho = 0$*. In both cases,* $i_t^B = i_t^R$*, as discussed in Section 3.1.*

In general, when government liquidity is scarce, the market interest rate – relevant for consumption and saving decisions via (20)– exceeds the policy rate. The relationship between the two depends on the equilibrium in the markets for private and public liquidity.

Equilibrium in the Market for Private Liquid Securities

On the supply side, the banking equilibrium implies that private liquid assets (deposits) are supplied elastically at a rate given by:

$$(1 + i_t^Q) = \rho_{\gamma,t}(1 + i_t^R) + (1 - \rho_{\gamma,t})(1 + i_t^B), \tag{21}$$

with $0 \leq \rho_{\gamma,t} \leq 1$.

On the demand side, households hold safe assets at a premium relative to the market rate. This premium, in equilibrium, reflects the marginal value of liquidity:

$$\frac{1 + i_t^Q}{1 + i_t^B} = \left(1 - \frac{\xi_{q,t} V_q\left(\frac{Q_t}{P_t}\right)}{U_c(Y_t)}\right). \tag{22}$$

Here, $U_c(\cdot)$ denotes the marginal utility of consumption. Equation (22) implicitly defines the demand for private safe assets:

$$\frac{Q_t}{P_t} = Q\left(\underbrace{\xi_{q,t}}_{+}, \underbrace{Y_t}_{+}, \underbrace{i_t^B - i_t^Q}_{-}\right).$$

Demand for private safe assets is proportional to the price level, increases with the liquidity shock $\xi_{q,t}$, and rises with output. Conversely, an increase in the spread between illiquid and liquid assets ($i_t^B - i_t^Q$) raises the opportunity cost of holding safe assets and reduces their demand.

Equilibrium in the Market for Government Securities

The demand for government securities arises from the banking sector to satisfy the collateral constraint:

$$B_t^g = \rho_{\gamma,t} Q_t. \tag{23}$$

The supply of government securities follows from the government's flow budget constraint:

$$B_t^g = (1 + i_{t-1}^R)B_{t-1}^g + T_t - \tau_t Y_t, \tag{24}$$

where monetary and fiscal policy jointly set distortionary taxes τ_t, and the interest rate on reserves i_t^R, whereas lump-sum transfers T_t are exogenous.

Determinants of the Market Nominal Interest Rate

We can use the equilibrium conditions for government and private safe asset markets to characterize the determination of the market nominal interest rate. Combining equations (21), (22), and (23), we obtain:

$$(1 + i_t^B) = \frac{\rho_{\gamma,t}}{\left(\rho_{\gamma,t} - \frac{\xi_{q,t} V_q\left(\frac{1}{\rho_{\gamma,t}} \frac{B_t^g}{P_t}\right)}{U_c(Y_t)}\right)}(1 + i_t^R). \tag{25}$$

This expression shows the proportional relationship between the market nominal interest rate and the policy rate. However, the proportionality factor is shaped by the supply of government liquidity (B_t^g), and by determinants of private liquidity, as the liquidity preference shock ($\xi_{q,t}$) and the pledgeability of private assets, which is a factor influencing ($\rho_{\gamma,t}$). When combined with (20), this yields a novel aggregate demand relationship distinct from the standard New Keynesian model.

Aggregate Supply Block

The aggregate supply equation is implied by the standard first-order conditions by firms and is given by the following set of three equations:

$$\left(\frac{1 - \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1+\theta\eta}{\theta-1}} = \frac{F_t}{K_t}, \tag{26}$$

in which F_t and K_t are given by

$$F_t = \xi_t(1 - \tau_t)U_c(Y_t)Y_t + \alpha\beta E_t\left\{\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1} F_{t+1}\right\}, \tag{27}$$

$$K_t = \xi_t Y_t^{1+\eta} + \alpha\beta E_t\left\{\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta(1+\eta)} K_{t+1}\right\}, \tag{28}$$

Economy's Resource Constraint

Finally, the intertemporal resource constraint of the economy, which mirrors the intertemporal budget constraint of the private sector, implies:

$$\frac{(1 + i_{t-1}^R)B_{t-1}^g}{P_t} = E_t\left\{\sum_{T=t}^{\infty} \beta^{T-t}\frac{\xi_T U_c(Y_T)}{\xi_t U_c(Y_t)}\left[\tau_T Y_T - \frac{T_T}{P_T} + \frac{i_t^B - i_t^R}{1 + i_t^B}\frac{B_t^g}{P_t}\right]\right\}, \tag{29}$$

at each time t and for every possible contingency.

The left-hand side captures the real value of the government's outstanding liabilities to the private sector. This must equal the present discounted value of expected future real primary surpluses (tax revenues net of transfers), plus the (implicit) revenues the government obtains from issuing interest-bearing liabilities (reserves or treasury notes) at rates below the market rate – i.e., seigniorage-like gains.

Equilibrium

Equilibrium is a set of stochastic sequences $\left\{i_t^B, i_t^R, Y_t, P_t, B_t^g, K_t, F_t, \tau_t, \Pi_t\right\}_{t=t_0}^{\infty}$ satisfying equilibrium conditions (20), (24), (25), (26), (27), (28), (29) and $\Pi_t = P_t/P_{t-1}$, for each $t \geq t_0$, with $i_t^B \geq i_t^R \geq 0$, given the stochastic sequence $\{\xi_t, \xi_{q,t}, T_t, \rho_{\gamma,t}\}_{t=t_0}^{\infty}$ and initial conditions $i_{t_0-1}^R$, $B_{t_0-1}^g$. There are two degrees of freedom to specify monetary and fiscal policy, which can set the stochastic sequences for the policy rate and tax rate, $\left\{i_t^R, \tau_t\right\}_{t=t_0}^{\infty}$.

5 A New Framework for Monetary Policy Analysis

In this section, we present the model in its log-linearized form around the steady state, in order to compare it with the benchmark New Keynesian Neo-Wicksellian framework. The details of the log-linear approximation are provided in Appendix A.

Aggregate Demand

The aggregate demand (AD) block builds on the Euler equation, as in the New Keynesian model. The key difference here is that the relevant nominal rate is the market interest rate, i^B, rather than the policy rate, as shown in equation (20). The log-linearized AD equation is:

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma\left(\hat{i}_t^B - E_t(\pi_{t+1} - \pi) - \tilde{r}_t^n\right), \tag{30}$$

where $\tilde{r}_t^n$ is a function of the preference shock ξ, given by $\tilde{r}_t^n = E_t\hat{\xi}_{t+1} - \hat{\xi}_t$. Variables with hats denote log-deviations from steady state.

The market and policy interest rates are connected through equation (25), which implies, in its log-linearized form:

$$\hat{i}_t^B = \underbrace{\hat{i}_t^R}_{\text{Policy rate}} + \frac{\nu}{\rho_\gamma - \nu}\sigma^{-1}\underbrace{\hat{Y}_t}_{\text{Output}} - \frac{\nu}{\rho_\gamma - \nu}\sigma_q^{-1}\underbrace{\hat{b}_t^g}_{\text{Government debt}} + \frac{\nu}{\rho_\gamma - \nu}\underbrace{\left(\hat{\xi}_{q,t} + (\sigma_q^{-1} - 1)\hat{\rho}_{\gamma,t}\right)}_{\text{Shocks}}, \tag{31}$$

where $\nu = V_q/U_c$ is the steady-state ratio of the marginal utility of liquidity to that of consumption. Note that $\sigma_q = -V_q/(V_{qq}q)$ denotes the intertemporal elasticity of substitution in liquidity, and ρ_γ is the steady-state value of $\rho_{\gamma,t}$.[14]

[14]A condition for equilibrium, discussed in the Appendix A, is that $\nu < \rho_\gamma$.

In the standard NK model, $i_t^B = i_t^R$, a condition that arises under full liquidity satiation $\nu = 0$. Equation (31) shows that, under liquidity scarcity ($\nu > 0$), three additional forces shape the market rate.

The first factor is output. Higher output increases the liquidity premium, pushing the market rate upward. This alters the AD channel and reduces the power of forward guidance, as it will be shown shortly.

The second factor is government debt. An increase in the supply of government debt (including central bank reserves), for given output, reduces the liquidity premium, lowering the market rate.

Finally, the third force are shocks in the market of private liquidity. A rise in liquidity demand ($\hat{\xi}_{q,t}$) or a fall in private collateral pledgeability (reflected in $\hat{\rho}_{\gamma,t}$) increases the liquidity premium and market rate.[15]

Substituting equation (31) into (30) yields a modified AD equation. We consider two cases.

First, when $\nu = 0$, liquidity is fully satiated in steady state: $i_t^B = i_t^Q = i_t^R$. The AD equation reduces to the standard NK form:

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma \left(\hat{i}_t^R - E_t(\pi_{t+1} - \pi) - \tilde{r}_t^n \right). \tag{32}$$

Second, when $\nu > 0$:

$$\hat{Y}_t = (1 - \rho_\gamma^{-1}\nu) E_t \hat{Y}_{t+1} - \sigma(1 - \rho_\gamma^{-1}\nu)(\hat{i}_t^R - E_t(\pi_{t+1} - \pi) - \tilde{r}_t^n) + q_y^{-1}\rho_\gamma^{-1}\nu(\hat{b}_t^g - q_\xi \hat{\xi}_{q,t} - q_\rho \hat{\rho}_{\gamma,t}), \tag{33}$$

where $q_\xi \equiv \sigma_q$, $q_y \equiv \sigma_q/\sigma$, and $q_\rho \equiv q_\xi(\sigma_q^{-1} - 1)$. There are three important novel features shown by the AD equation when liquidity is not fully satiated: first, there is a role for liquidity, captured by government debt, $\hat{b}_t^g$, in affecting the aggregate demand equation (liquidity channel) with an increase in government liquidity having an expansionary effect; second, the coefficient $(1 - \rho_\gamma^{-1}\nu)$ in front of the expected level of output is positive and less than the unitary value, which has implications for the effectiveness of forward guidance; third, positive demand shocks to liquidity, given by an increase in $\hat{\xi}_q$, and negative supply shocks to liquidity, given by a rise in $\hat{\rho}_{\gamma,t}$, both lower aggregate demand.

To further clarify the differences from the standard NK model, solve equation (33) forward:

$$\hat{Y}_t = -\nu_\rho \sigma E_t \sum_{T=t}^{\infty} \nu_\rho^{T-t} \left(\hat{i}_T^R - (\pi_{T+1} - \pi) - \tilde{r}_T^n \right) + q_y^{-1}\rho_\gamma^{-1}\nu E_t \sum_{T=t}^{\infty} \nu_\rho^{T-t} \left(\hat{b}_T^g - q_\xi \hat{\xi}_{q,T} - q_\rho \hat{\rho}_{\gamma,T} \right), \tag{34}$$

where $\nu_\rho \equiv 1 - \rho_\gamma^{-1}\nu \in (0, 1)$.

The above equation shows that not only the current real rate has less impact on output, for given intertemporal elasticity of substitution in consumption σ, but also movements in the expected future rates influence current output less and with a decaying weight. This finding shows

[15] There are two channels at play to understand the effect of $\hat{\rho}_{\gamma,t}$ on $\hat{i}_t^B$. A decrease in γ, which raises $\hat{\rho}_{\gamma,t}$, reduces the supply of deposits for a given B_t^g (see equation 23), exerting upward pressure on $\hat{i}_t^B$. Conversely, a rise in $\hat{\rho}_{\gamma,t}$ decreases $\hat{i}_t^B$ through (21). When $\sigma_q < 1$, the first channel dominates. This condition is assumed in our calibrated examples.

that forward guidance has a reduced impact in this framework with respect to the standard New-Keynesian model. A similar argument applies to the effectiveness of the supply of liquidity in influencing current aggregate demand, which is in general a novel channel.

The supply of government liquidity is at the end related to the intertemporal resource constraint of the economy and ultimately to the tax rate. A first-order approximation of (29) implies that

$$\begin{aligned}\hat{b}^g_{t-1} - (\pi_t - \pi) - \sigma^{-1}\hat{Y}_t + \hat{\imath}^R_{t-1} \quad &= \quad [b_y\hat{Y}_t + \varrho\tilde{\tau}_t - \varrho\tilde{T}_t + b_\xi\hat{\xi}_{q,t} + b_q\hat{b}^g_t] \\ &\quad + \beta E_t[\hat{b}^g_t - (\pi_{t+1} - \pi) - \sigma^{-1}\hat{Y}_{t+1} + \hat{\imath}^R_t - \tilde{r}^n_t],\end{aligned}$$

in which b_y, ϱ, b_ξ, b_q are parameters defined in the Appendix A and $\tilde{T}_t \equiv (T_t - T)/Y$.

Aggregate Supply

The aggregate supply (AS) block follows the standard New Keynesian Phillips Curve, but incorporates distortionary taxation:

$$(\pi_t - \pi) = \kappa(\hat{Y}_t + \psi_\tau\tilde{\tau}_t) + \beta E_t(\pi_{t+1} - \pi), \tag{35}$$

where $\kappa > 0$ and $\psi_\tau > 0$ are structural parameters derived in Appendix A. Here, $\pi_t \equiv \ln(P_t/P_{t-1})$ and $\pi \equiv \ln \Pi$ is the steady-state inflation target; $\tilde{\tau}_t \equiv \tau_t - \tau$ denotes the tax gap.

Inflation deviations from target depend positively on output and expected inflation, and are amplified by increases in the distortionary tax rate.

6 The Optimal Supply of Liquidity

Our model features a central role for government debt in the economy: it drives the supply of private liquidity, which provides households with non-pecuniary benefits. However, issuing government debt comes at a cost, as it must be financed through distortionary taxation. In this section, we examine a key policy question: what is the optimal supply of liquidity? Our approach also offers a first-pass analysis of the optimal size of the government's consolidated liabilities, taking into account the liquidity multiplier between public and private liquidity—that is, the liquidity channel.

We carry out this analysis in the deterministic version of the model. To simplify the exposition, we abstract from nominal rigidities and assume that inflation is fixed at its target level, Π. As demonstrated in Appendix D, this assumption is without loss of generality: targeting inflation is optimal even in the more general framework.[16]

The optimal supply of liquidity is determined by the interaction of two opposing forces. On one hand, household utility depends positively on $V(q)$, implying that it is optimal to provide enough liquidity to reach the satiation threshold $\bar{q}$. On the other hand, supplying less liquidity can

[16] We analyze optimal policy under commitment from a timeless perspective, using a recursive formulation of the Ramsey problem augmented with initial constraints that make the solution stationary.

be beneficial, as it reduces the need for distortionary taxation, due to the liquidity premium that arises when liquidity is scarce.

The balance between these forces implies that the optimal level of liquidity is strictly below the satiation threshold $\bar{q}$.

We also explore how the optimal provision of liquidity is affected by the degree to which private assets can be pledged as collateral. When private assets are more easily pledgeable, the optimal supply of liquidity increases. Conversely, the optimal level of government debt—as well as the associated tax rate—declines.

6.1 Optimal Liquidity Problem

In the deterministic steady state the optimal policy problem requires the maximization of households utility subject to resource and technological constraint. Specifically, we can rewrite households' utility as:

$$U_{t_0} = \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{Y_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \frac{Y_t^{1+\eta}}{1+\eta} + V(q_t) \right] \right\}, \tag{36}$$

in which we have set $\xi_t = \xi_{q,t} = 1$ and noted that $C_t = Y_t$ and $H_t(j) = Y_t$, for each j. We specify the utility from safe assets as:

$$\begin{array}{ll} V(q) = \ln\left(\frac{q}{\bar{q}}\right) - \frac{q}{\bar{q}} & \text{for} \quad q < \bar{q} \\ V(q) = -1 & \text{for} \quad q \geq \bar{q}. \end{array}$$

This function is non-decreasing in q with a satiation point at $\bar{q}$, where $V_q(q) \longrightarrow 0$ and $V_{qq}(q)$ remains finite as q approaches $\bar{q}$.

In the flexible-price allocation, the level of output is a function of the tax rate through the functional form:

$$Y_t = Y(\tau) \equiv \left[\frac{(1-\tau)}{\mu_\theta} \right]^{\frac{1}{\eta+\sigma^{-1}}}. \tag{37}$$

An increase in the tax on revenues lower output for a given level of monopolistic distortions, $\mu_\theta \equiv \theta/(\theta-1)$: this effect represents the costs of using distortionary taxation to finance public expenditure (i.e. transfers, T_t, in our case).

The optimization problem is also subject to the following intertemporal resource constraint

$$Z_{t_0} = \sum_{T=t_0}^{\infty} \beta^{T-t_0} \left[Y_T^{-\sigma^{-1}} \left(\tau_T Y_T - \frac{T_T}{P_T} \right) + \frac{V_q(q_t)\, b_t^g}{\rho_\gamma} \right], \tag{38}$$

which is derived from equation (29), having defined $b_t^g \equiv B_t^g / P_t$ and

$$Z_{t_0} \equiv Y_{t_0}^{-\sigma^{-1}} \frac{(1+i_{t_0-1}^R) b_{t_0-1}^g}{\Pi}.$$

In going from (29) to (38), we have used equations (21) and (22) and assumed a constant ρ_γ. [17]

[17] In obtaining this constraint, we have assumed that $\rho_\gamma^{-1} b_t^g = q_t$ since only when $V_q = 0$, $\rho_\gamma^{-1} b_t^g > q_t$.

The resource constraint links the outstanding liabilities of the government sector at time t_0 (the LHS of (38)) with the present discounted value of its inlays (the RHS of (38)). There are two components: the primary surplus $\left(\tau_t Y_t - \frac{T_t}{P_t}\right)$, and the (implicit) revenues generated from issuing liabilities that are priced at discount relative to the market nominal rate. The term $V_q\left(q_t\right) b_t^g$ is indeed proportional to $i^B - i^Q > 0$ when the economy is not satiated, representing an alternative resource generated by the liquidity premium.

The optimal tax, τ_t, and liquidity policy, q_t, maximizes (36) under the constraint (38) considering (37) and given a constraint $Z_{t_0} = \bar{Z}$, which is going to be specified in such a way to be self-consistent with the equilibrium values of the variables involved in Z_{t_0}. This is consistent with the notion of commitment from a timeless perspective.[18]

6.1.1 Results

The first-order condition with respect to τ_t implies that

$$\left(1 - \frac{1-\tau_t}{\mu_\theta}\right) = \lambda[(1+\eta)(1-\tau_t) - (1-\sigma^{-1}) - \sigma^{-1}g], \tag{39}$$

in which λ is the Lagrange multiplier attached to the constraint (38) and $g = T/(PY)$ is the transfer to GDP ratio. The condition shows that the optimal tax rate is constant at $\tau_t = \tau$.

The first-order condition with respect to q_t implies that

$$V_q(q_t) = -\lambda(V_q(q_t) + V_{qq}(q_t)q_t). \tag{40}$$

This condition determines the equilibrium level of liquidity.

Proposition 4 *There are two possible steady-state equilibria: a full-liquidity satiation with $q = \bar{q}$ and $\tau = \overline{\tau}$, and a below-satiation liquidity equilibrium with $q = q^* < \bar{q}$ and $\tau = \tau^*$.*

Proof. Note that (40) has two solutions: one in which $q_t \geq \bar{q}$ and all derivatives of the function $V(\cdot)$ are zero, the second with $q = q^* < \bar{q}$. Details of the proof are in Appendix B. ■

We now establish that the below satiation equilibrium leads to higher welfare. We prove this result analytically for a special case, and then we show that it holds numerically under our preferred calibration.

Proposition 5 *Assume $g = 0$, $\mu_\theta = 1$, $\eta = 0$, $\sigma = 1$. The equilibrium (τ^*, q^*) with $q^* < \bar{q}$ has higher welfare than the equilibrium with full satiation of liquidity $(\overline{\tau}, \bar{q})$. Moreover*

$$\tau^* = \frac{1-\beta}{1+\beta}\rho_\gamma q^* \leq \frac{\overline{\tau}}{2}.$$

[18] The additional constraint $Z_{t_0} = \bar{Z}$ is such to make the optimization problem recursive and the solution stationary, as discussed in Benigno and Woodford (2003).

Proof. In Appendix C. ■

Three results emerge from the special case with log utility ($\sigma = 1$), linear disutility with respect to labor ($\eta = 0$), no markup distortions ($\mu_\theta = 1$) and zero government transfers ($g = 0$).

The optimal provision of liquidity remains below the satiation threshold. The rationale for this lies in balancing two key factors we have identified. It involves striking a balance between meeting liquidity demand (represented by $V(q)$ in the utility function that is maximum when $V_q(q) = 0$) and minimizing taxation to stimulate output.

To optimize this trade-off, the policy sets liquidity such that $V_q(q)$ remains positive in (38), mantaining a liquidity premium. The liquidity premium allows the government to finance its outstanding debt at below-market interest rates reducing the reliance on distortionary taxation. This result is reminiscent of earlier findings by Calvo (1978), Woodford (1990), Sims (2022) and Angeletos et al. (2022). Our results complement and generalize these findings to a setting in which policymakers act under commitment from a timeless perspective, by allowing for both sticky or flexible prices and by using a preference specification where liquidity has a satiation point at a finite level.

The second implications of (5) is that in the optimal equilibrium, the tax rate required to finance the outstanding liability of the government sector is more than halved compared to the tax rate under full liquidity satiation.

The third result shows that the optimal tax rate is a linear function of the parameter ρ_γ which is a decreasing function of the fraction, γ, of private assets that can be pledged as collateral.

When $\gamma = 0$ the parameter ρ_γ is equal to ρ and the solution for optimal tax becomes $\tau^* = \frac{1-\beta}{1+\beta}\rho q^*$.

As γ increases, more private assets serve as collateral, reducing the dependence on public liquidity and ρ_γ goes to zero.[19] Therefore the supply of government debt, given by $\rho_\gamma q^*$, falls when more private assets can be pledged, and the tax rate falls too. When ρ_γ goes to zero, the tax rate τ^* reaches zero as well as the tax rate $\bar{\tau}$ in the full satiation solution. The two solutions coincide and liquidity is fully satiated. Not surprisingly, there are no financing needs, no monopolistic distortions and the first best is achieved.

In the general case in which government transfers g are positive, the previous result no longer holds. When $g > 0$, there is a financing need and since taxes are distortionary it is optimal to rely on the liquidity premium to balance distortions even when $\rho_\gamma \longrightarrow 0$, as shown in Appendix C.

[19] Recall that when $\gamma \geq \rho$, the collateral constraint is not binding and liquidity premia are zero.

Table 1: Comparisons between full-satiation and optimal steady states

(1)	(2)	(3)	(4)	(5)	(6)
$\bar{\tau} - g$	$\tau^* - g$	$\bar{\tau}$	τ^*	$q^*/\bar{q}$	Δc
2%	−7.7%	22%	12.3%	89.9%	0.71%
3%	−7.0%	23%	13.0%	89.6%	0.78%
4%	−6.4%	24%	13.6%	89.4%	0.86%
5%	−5.8%	25%	14.2%	89.1%	0.94%
6%	−5.2%	26%	14.8%	88.8%	1.02%

To illustrate the generality of Proposition 5, we use the calibration specified in the next Section, i.e. $\mu_\theta = 1, 11$, $\eta = 0.47$, $\sigma = 0.5$ and $\rho_\gamma = 0.21$. Moreover, as we are calibrating the model at an annual frequency, we assume $\beta = 0.98$, which is consistent with a two-percent steady-state real interest rate, and $g = 0.2$, representing a government expenditure -to-GDP ratio of 20%. In this example we assume that $\gamma = 0$, therefore $\rho_\gamma = \rho$.[20]

Table 1 presents a comparison of key variables, between the full satiation equilibrium and the optimal steady-state equilibrium with lower level of liquidity.

Our key findings are as follows:

1. Government Surplus versus Deficit

   In the full satiation equilibrium, a surplus is required to back all liquidity, as shown in column 1 of Table 1. In the below satiation equilibrium, the budget balance shifts to a deficit (column 2). This shift occurs because the liquidity premium generates sufficient rents to cover government's liabilities.

2. Tax Rate

   The optimal tax rate τ^* is significantly lower than in the full-satiation equilibrium, nearly halving in some cases. Compare column 3 and 4. This is a direct result of reducing the liquidity supply, which allows taxation to be relaxed.

3. Supply of Liquidity

   The optimal tax rate requires a 10-12% reduction in liquidity supply with respect to full satiation. This is reflected in the $q^*/\bar{q}$ ratio in the fifth column of Table 1.

[20]To calibrate $\bar{q}$, we ensure that the steady state with full satiation of liquidity generates a specific surplus over GDP, consistent with constraint (B.6) in Appendix B. For example, setting $\bar{\tau} - g$ in (B.6) at 0.02, corresponding to a 2% surplus over GDP, implies a tax rate $\bar{\tau}$ at 22%, i.e. $\bar{\tau} = 0.22$. Then, given the other parameter in (B.6) and the imposed $\bar{\tau}$ we derive $\bar{q}$. For the same, $\bar{q}$, we solve (39), (B.7) and (B.8), given in Appendix B, to obtain q^* and τ^*. We then vary $\bar{\tau} - g$ in the range $[0.02, 0.06]$, computing the respective $\bar{q}$, $\bar{Y}$ and $\bar{U}$. For each of the generated $\bar{q}$, we compute τ^*, q^*, and U^*.

4. Welfare Gains

   The last column shows the welfare gains from moving to the optimal equilibrium starting from the solution with full satiation of liquidity, measured as the percentage change in steady-state consumption. These gains are between 0.7% and 1%, depending on the tax rate in the full satiation equilibrium, $\bar{\tau}$.

6.1.2 The Role of Private Assets as Collateral for Private Liquidity

We now examine the role of γ, the fraction of private assets that can be used as collateral to back safe assets issued by the financial intermediaries. Previously, we established that when γ is sufficiently high – specifically, when it exceeds the fraction of safe liabilities to be collateralized ($\gamma \geq \rho$) – the collateral constraint is not binding. In which case, all nominal interest rates are equalized ($i_t^B = i_t^Q = i_t^R$) and supply of safe assets fully satiates liquidity demand. This corresponds to the standard case in the optimal taxation literature, where the optimal steady-state level of government debt is not determined (see among others Benigno and Woodford (2004)). However, when $\gamma < \rho$, the collateral constraint can be binding, leading to a direct relationship between the supply of safe assets and government debt: $Q_t = B_t^g/\rho_\gamma$. As γ converges to ρ, ρ_γ approaches zero. Moreover, ρ_γ is decreasing in γ and reaches ρ when $\gamma = 0$.

To analyze the effects of varying γ, we use the same calibration as before, setting $\bar{\tau} = 0.23$ and the associated $\bar{q}$ when $\gamma = 0$ to obtain $\tau^* = 0.13$ as in the previous Table. Under this conditions we obtain that the debt-to-output ratio is 125.4%. Keeping $\bar{\tau}$ and $\bar{q}$ fixed, we now vary γ up to ρ with the results presented in Table 2.

Table 2: Varying the fraction (γ) of private assets held as collateral

(1)	(2)	(3)	(4)
γ	b^g/Y	τ^*	$q^*/\bar{q}$
0%	125.4%	13.0%	89.6%
5%	100.6%	12.6%	89.8%
10%	73.0%	12.2%	90.0%
15%	42.18%	11.7%	90.2%
21%	0%	11.4%	90.5%

Our key findings are as follows:

1. Declining Government Debt

   As γ increases, less government debt is required to collateralize safe assets, and the debt-to-output ratio declines from 125.4% when $\gamma = 0$ to 0% when $\gamma = 21\%$, as shown in column 2 of Table 2.

2. Tax Rate Reduction

   The optimal tax rate (τ^*) decreases as γ rises but remains positive, staying above 10% even in the limiting case (see column 3).

3. Increased Liquidity Supply

   The supply of liquidity also increases with γ but not to reach satiation, as shown in column 4. Consistently with the discussion of Proposition 5 it is still optimal in the limit not to satiate liquidity because of a positive g.

7 Optimal Monetary Policy Normalization

In this Section, we analyze how interest rate and liquidity policies should be managed when the economy is hit by the stochastic disturbances of the model, namely the preference shock ξ_t and the liquidity shocks $\xi_{q,t}$ and $\hat{\rho}_{\gamma,t}$. What is interesting is that the three shocks have isomorphic effects on an appropriately-defined natural real rate of interest r_t^n, once we "neutralize" their fiscal effects using the transfer policy. Therefore, the analysis of the response to the three shocks can be synthesized in the response to a shock to the natural real rate of interest r_t^n. In this respect, we are interested in a magnitude of such a shock that is enough to bring the policy rate, under the optimal policy problem, to face the zero-lower bound constraint.

The isomorphism between the three shocks is intriguing because it demonstrates that movements in the natural rate may not solely stem from shocks to intertemporal preferences, such as ξ_t, which is the main device used in the literature to drive the model economy to the zero lower bound, as shown in Eggertsson and Woodford (2003). These same movements in the natural rate can originate from disturbances in the market of liquidity, originating from the shortages in the supply of safe assets. This type of shock has been identified as significant in understanding the narrative of the 2007-2008 financial crisis.

In this context, we contrast sub-optimal policy rules with the optimal policy that entails the coordinated choice of both monetary and fiscal policy. The innovative aspect of the framework outlined here, and of the analysis within this section, is that policy-making doesn't solely rely on a single instrument or degree of freedom, such as the interest rate, but also requires the specification of liquidity policy. This is particularly pertinent in offering an interpretation of the balance-sheet dynamics many economies have undergone since the 2007-2008 financial crisis.

Optimal policy is computed using linear-quadratic approximations following the method expounded in Benigno and Woodford (2003). The approximation is taken around the optimal steady state discussed in Section 6 for which the optimal supply of liquidity is below satiation. Details are in Appendix D, where we show that a quadratic approximation of the loss function has the following form

$$L_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2}\lambda_y y_t^2 + \frac{1}{2}\lambda_\pi (\pi_t - \pi)^2 + \frac{1}{2}\lambda_q \hat{q}_t^2 \right\}, \tag{41}$$

for positive parameters λ_y, λ_π and λ_q. The policymaker should care about deviations of an appropriately-defined output gap, $y = \hat{Y}_t - \hat{Y}_t^*$, inflation, π, and real liquidity, $\hat{q}$, from their steady state values. The main difference with respect to standard analyses within the New-Keynesian framework is that there is an additional cost of varying liquidity with respect to the steady state. Since liquidity is a tool that can be used for stabilization purposes, as we have seen, this cost limits its usage, considering also the distortions implied by the required variations in taxes.

The optimal policy problem is subject to three constraints: AS, AD equations and the intertertemporal resource constraint of the government. The aggregate supply (35) is

$$(\pi_t - \pi) = \kappa[y + \psi_\tau(\tilde{\tau}_t - \tilde{\tau}_t^*)] + \beta E_t(\pi_{t+1} - \pi),$$

in which now $\tilde{\tau}_t^*$ represents a combination of the shocks such that when $\tilde{\tau}_t$ achieves that value, output and inflation can be stabilized at their respective targets implied in the loss function.

The AD equation (33) can be written as:

$$y_t = (1 - \rho_\gamma^{-1}\nu)E_t y_{t+1} - \sigma(1 - \rho_\gamma^{-1}\nu)(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) + q_y^{-1}\rho_\gamma^{-1}\nu\hat{b}_t^g, \tag{42}$$

for an appropriately defined natural real rate of interest, r_t^n, given by

$$r_t^n = E_t\hat{\xi}_{t+1} - \hat{\xi}_t + \frac{1}{\sigma}E_t\hat{Y}_{t+1}^* - \frac{1}{\sigma(1 - \rho_\gamma^{-1}\nu)}\hat{Y}_t^* - \frac{\rho_\gamma^{-1}\nu}{(1 - \rho_\gamma^{-1}\upsilon)}\left(\hat{\xi}_{q,t} + (\sigma_q^{-1} - 1)\hat{\rho}_{\gamma,t}\right). \tag{43}$$

The natural real rate of interest r_t^n depends on the four shocks of the model, $\hat{\xi}_t, \hat{\xi}_{q,t}, \hat{\rho}_{\gamma,t}$ and $\hat{T}_t$, since the desired level of output $\hat{Y}_t^*$ is a function of the transfer $\hat{T}_t$, as discussed in Appendix D. What is crucial to observe is that a decline in the natural real rate of interest can arise from either the preference shock, $\hat{\xi}_t$, as conventionally observed in the literature, or from liquidity shocks $\hat{\xi}_{q,t}$ and $\hat{\rho}_{\gamma,t}$. Specifically, positive shocks to $\hat{\xi}_{q,t}$ and $\hat{\rho}_{\gamma,t}$ result in a decrease in r_t^n. As illustrated by the equilibrium in the money market (22), an increase in the demand for liquidity, represented by a positive $\hat{\xi}_{q,t}$, not met by supply or an increase in output, leads to a wider spread in money-market rates. Likewise, a reduction in the fraction of private assets eligible as collateral raises $\hat{\rho}_{\gamma,t}$ and causes a decrease in the natural real rate of interest, provided the decline in safe assets is significant enough to counterbalance the lower interest rate, i.e., when $\sigma_q < 1$. This suggests that disruptions in the liquidity market can capture certain aspects of events like the 2007-2008 financial crisis or the pandemic, during which money-market spreads markedly increased, becoming drivers of the decline in the natural real rate of interest.

An additional constraint of the optimal policy problem is the first-order approximation of (29), which can be written as

$$\hat{b}_{t-1}^g - (\pi_t - \pi) - \sigma^{-1}y_t + (\hat{\imath}_{t-1}^R - r_{t-1}^n) = -f_t + E_t\sum_{T=t}^{\infty}\beta^{T-t}[b_y y_t + b_\tau(\tilde{\tau}_t - \tilde{\tau}_t^*) + b_q\hat{b}_t], \tag{44}$$

for parameters b_y, b_τ, b_q defined in Appendix D; the variable f, as in Eggertsson and Woodford

(2004), captures the "fiscal stress," which measures the extent to which full stabilization of output, inflation and liquidity at their targets implied by the loss function (41), is not compatible with the intertemporal budget constraint of the government. The "fiscal stress" variable includes a combination of all the shocks in the economy. When $f_t = 0$ at all times, which can be obtained by varying appropriately the transfer T to offset the other shocks, it is feasible to reach all three targets in the loss function, provided the movements in the natural real rate of interest, r^n, do not imply violation of the zero-lower bound for the nominal interest rate.[21] By assuming $f_t = 0$ we are then abstracting from the different fiscal consequences of the shocks $\hat{\xi}_t$, $\hat{\xi}_{q,t}$ and $\hat{\rho}_{\gamma,t}$ with the already-mentioned result that they have isomorphic effects on the economy, which can be ultimately captured by movements in the natural real rate of interest. Assuming $f_t = 0$ implies that the optimal policy is simply to achieve all targets in (41) and $\hat{\imath}_t^R = r_t^n$ all times. However, when the natural real rate of interest, r^n, falls substantially, there could be violation of the zero-lower bound for the policy rate, i^R. A trade-off emerges between stabilizing the relevant variables.

We consider, therefore, how policy should be set when the only constraint on the full stabilization of the relevant variables in (41) is given by the existence of the zero-lower bound on the policy rate.[22]

We analyze a shock that brings the natural real rate of interest, r^n, from the steady-state level of 2% to -4% at annual rates for twelve quarters. Given that the steady-state policy rate is set at 4% accounting for a 2% inflation target, the shock to the natural rate of interest could be fully accommodated only if the policy rate could fall at -2%. The zero-lower bound prevents this fall and creates an interesting trade-off among stabilizing the relevant macroeconomic variables.

Preview of the results

In the benchmark calibration, in which the welfare-based loss function gives higher weight to inflation stabilization rather than output gap stabilization, optimal policy in a liquidity trap can be better achieved by a combination of an appropriate interest-rate policy, which implies a longer stay at the zero-lower bound than the duration of the shock, and a tax policy in which taxes are raised at the beginning of the trap and lowered at the end. An active liquidity policy is optimal, but with marginal benefits on the output gap.

These benefits increase when spreads in money markets are calibrated at a higher value to capture features of the 2007-2008 financial crisis. Liquidity should rise early, moved by a lower tax rate, peak in the middle of the liquidity trap and withdrawn before the liftoff of the policy rate, at a fast rate.

[21] In this reasoning, we are considering zero values for the initial conditions $\hat{b}^g_{t_0-1}$, $\hat{\imath}^R_{t_0-1}$,$r^n_{t_0-1}$. We could also allow for different initial conditions requiring, in the case, f_{t_0} to adjust appropriately.

[22] Note that when the optimal supply of liquidity is close to eliminate the distortions in the money market, i.e. $\nu \to 0$, the problem collapses to exactly that analyzed by Eggertsson and Woodford (2004) in the standard New-Keynesian model with absence of lump-sum taxes. Indeed, the AD equation boils down to the standard one in which liquidity does not affect, directly, aggregate demand. The AS equation is already the same as in their framework, as well as the parameters λ_y and λ_π in the loss function (41). With $\nu \to 0$, λ_q goes instead to zero as well as b_q in the constraint (44); b_y and b_τ also approach same values as in Eggertsson and Woodford (2004).

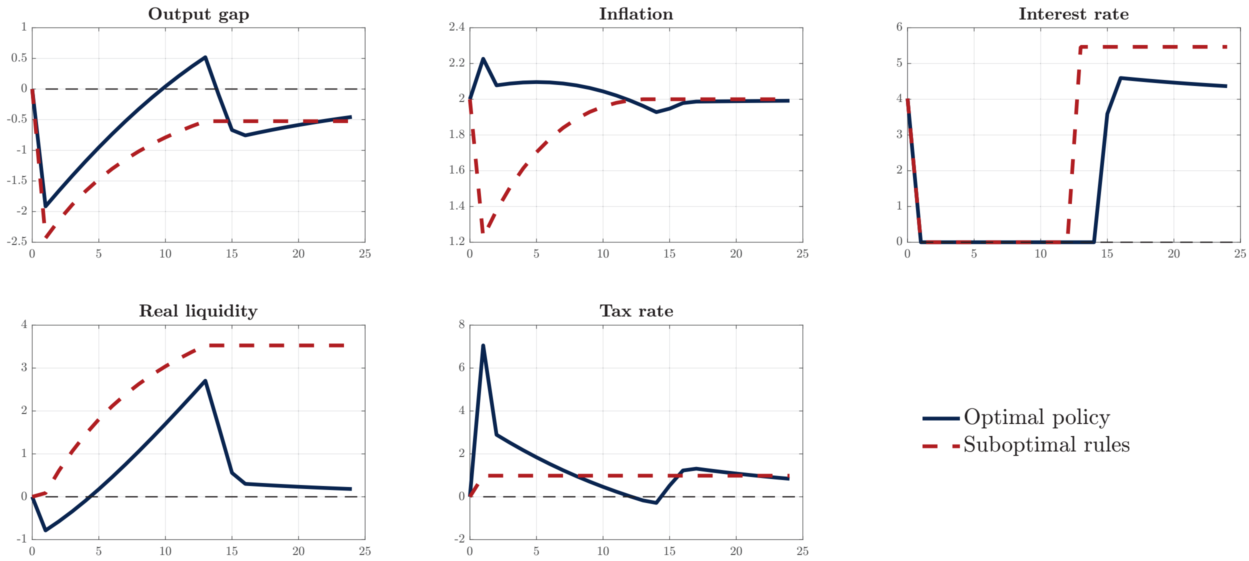

Figure 1: Comparison between optimal policy and sub-optimal rules. Impulse responses follow a negative shock to the natural rate of interest, lowering it to -4% at annual rates for 12 quarters. Output gap is in percent, inflation and interest rates are in percent and at annual rates. Real liquidity is in percent deviations from steady state. The tax rate is in percentage points and shown as deviations from its steady-state value.

When the policymaker's objectives are tilted to stabilize output gap, liquidity policies are more effective and desirable. The policy rate does not need to stay longer at the zero-lower bound than the duration of the shock. Liquidity should peak early in the trap and then withdrawn at a faster pace to be completely absorbed at the same time the policy rate is normalized.

Benchmark calibration

In Figure 1 we compare the optimal policy with sub-optimal policies in which (i) the central bank sets inflation at the target, i.e. $\pi_t = \pi$, whenever it is feasible, otherwise it sets the policy rate to zero and (ii) the fiscal authority keeps the tax gap $\tilde{\tau}_t - \tilde{\tau}_t^*$ at a level that it expects to maintain indefinitely without violating the intertemporal government budget constraint; that is, an expected path of the tax gap such that $E_t(\tilde{\tau}_T - \tilde{\tau}_T^*) = \tilde{\tau}_t - \tilde{\tau}_t^*$ for all $T \geq t$ is consistent with (44).[23]

The Figure shows the costs of the sub-optimal policy with respect to the optimal in terms of contraction in the output gap and inflation below the target. The liftoff of the policy rate from the zero-lower bound occurs exactly at the time in which the shock vanishes. Optimal policy, instead, succeeds to stabilize inflation while keeping moderate variations in the output gap.

There are three important features of the optimal policy that we discuss. First, in line with the literature, optimal policy requires a stay at the zero-lower bound longer with respect to the duration of the shock. In the Figure, the interest rate remains at the zero-lower bound for two additional quarters. What is interesting to note is that the liquidity channel in the AD equation does not imply a shorter stay at the zero-lower bound with respect to the findings of the literature. We are going to elaborate more on this soon. The second result, as well in line with Eggertsson and Woodford (2004), is the use of the tax policy to stabilize the economy. Note that in the

[23] Appendix E provides details on the calibration used.

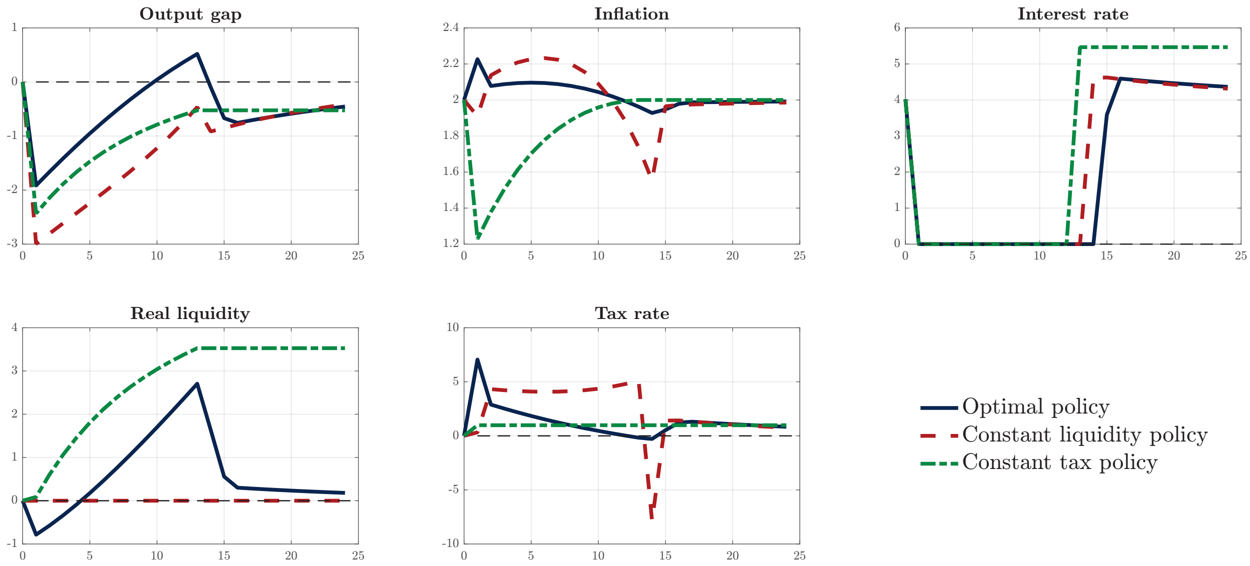

Figure 2: Comparison between optimal policy, optimal policy with constant liquidity, and sub-optimal rules. Impulse responses follow a negative shock to the natural rate of interest, lowering it to -4% at annual rates for 12 quarters. The output gap is expressed in percent; inflation and interest rates are in percent and annualized. Real liquidity is in percent deviations from the steady state. The tax rate is in percentage points and shown as deviations from the steady-state value.

case of disturbances not severe enough for the zero lower bound to bind, the tax gap, $\tilde{\tau} - \tilde{\tau}^*$, would not move at all. Instead, in the case of a larger shock, optimal policy involves raising tax rates during the liquidity trap, while committing to cut them later and more around the time the shock has vanished. As in Eggertsson and Woodford (2004), the tax rate operates through the AS equation and its increase acts to push up inflation at the early stage of the liquidity trap, when the deflationary pressures are stronger, while putting downward pressure when the shock vanishes. The last feature of optimal policy is the path followed by liquidity. As it is shown in the Figure, optimal policy requires a lower increase in liquidity with respect to the sub-optimal policy. The main reason for this counter-intuitive result is in the success of the optimal policy in stabilizing inflation and output. Indeed, the fall in the output gap under the sub-optimal rules produces lower revenues from taxes, which lead to a large accumulation of public liabilities. An interesting feature is the path of liquidity under optimal policy. Whereas the initial fall is due to the rise in the tax rate, liquidity progressively increases to reach its peak just one quarter after the shock vanishes. The withdrawal starts at the same time and in a faster way than the accumulation to reach the initial level of liquidity exactly when the interest rate normalizes.

We now elaborate more on the characteristics of the optimal policy by comparing it with another sub-optimal policy that we label "constant liquidity policy." In this policy framework, fiscal policy adjusts the tax gap to fully stabilize liquidity at its steady-state level, while the monetary authority minimizes the loss function (41) subject to the same constraints as in the general optimal policy problem. However, the monetary authority takes as given the path of fiscal variables, $\tilde{\tau} - \tilde{\tau}^*$, and assumes that the government's intertemporal solvency is guaranteed by fiscal policy. This formulation allows us to characterize how optimal monetary policy would respond to shocks in an

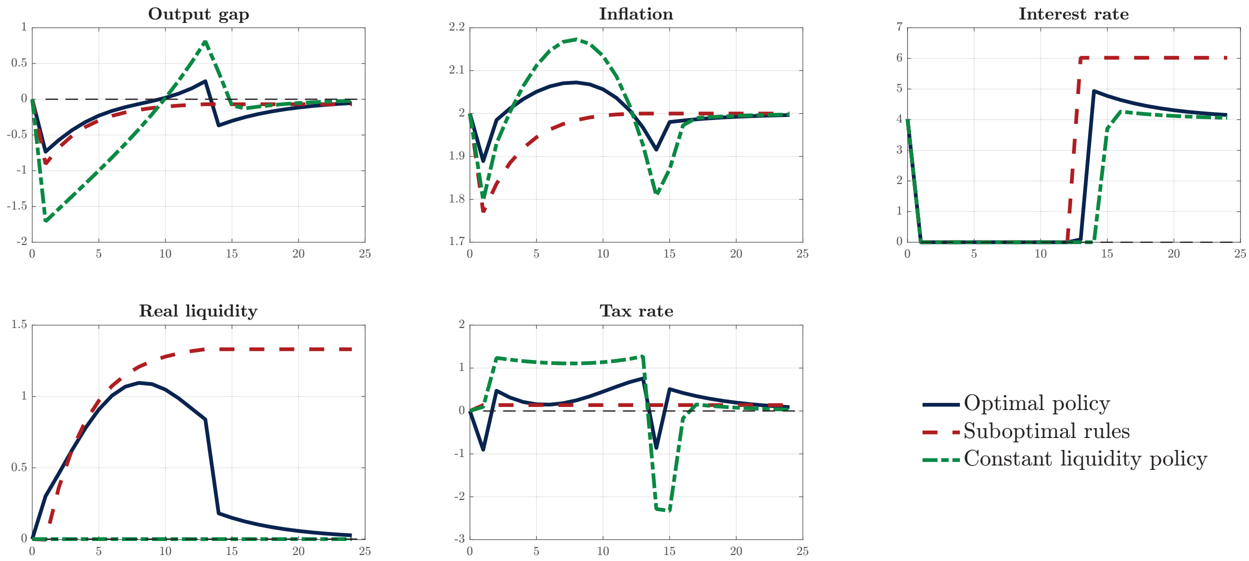

Figure 3: Comparison between optimal policy, optimal policy with constant liquidity, and sub-optimal rules, under high money-market spread. Impulse responses follow a negative shock to the natural rate of interest, lowering it to -4% at annual rates for 12 quarters. Output gap is expressed in percent; inflation and interest rates are in percent and annualized. Real liquidity is shown in percent deviations from the steady state. The tax rate is in percentage points and expressed as deviations from its steady-state value.

environment where liquidity does not play an active role.

Figure 2 adds the "constant liquidity policy" to the two policies of Figure 1. The figure illustrates the costs associated with keeping liquidity constant in terms of output and inflation stabilization. These costs are particularly pronounced for output, given that liquidity has a direct and immediate effect on aggregate demand. Interestingly, the fiscal policy required to maintain constant liquidity—namely, an increase in the tax rate during a liquidity trap—can also help mitigate the disinflationary pressures caused by the shock, due to its effect on the aggregate supply (AS) equation.

This coincidence helps explain why the zero interest rate policy in the "constant liquidity policy" framework is not maintained for a longer horizon to offset the absence of active liquidity management. The tax adjustment partially substitutes for the missing liquidity response, reducing the need for extended monetary accommodation.

Higher Spread in Money Markets

In the previous analysis, the parameter ν, which captures the spread in money markets between liquid and illiquid securities, was calibrated to the average spread between Aaa corporate bonds and Treasury bonds at ten-year maturity in the U.S. economy over the 1971–2005 period. This corresponds to a spread of 1.25% at annual rates. According to the aggregate demand equation (42), a one-percent, once-and-for-all increase in liquidity raises output – everything else equal – by $q_y^{-1}\rho^{-1}\nu$ percentage points. Given $q_y = 0.3143$, $\rho_\gamma = 0.21$, and $\nu = 0.003125$, this implies an output increase of only 0.047 percentage points.

Figure 3 considers instead a spread of 4%, which is more in line with the values observed at the onset of the 2007–2008 financial crisis across several credit market indicators. The figure compares

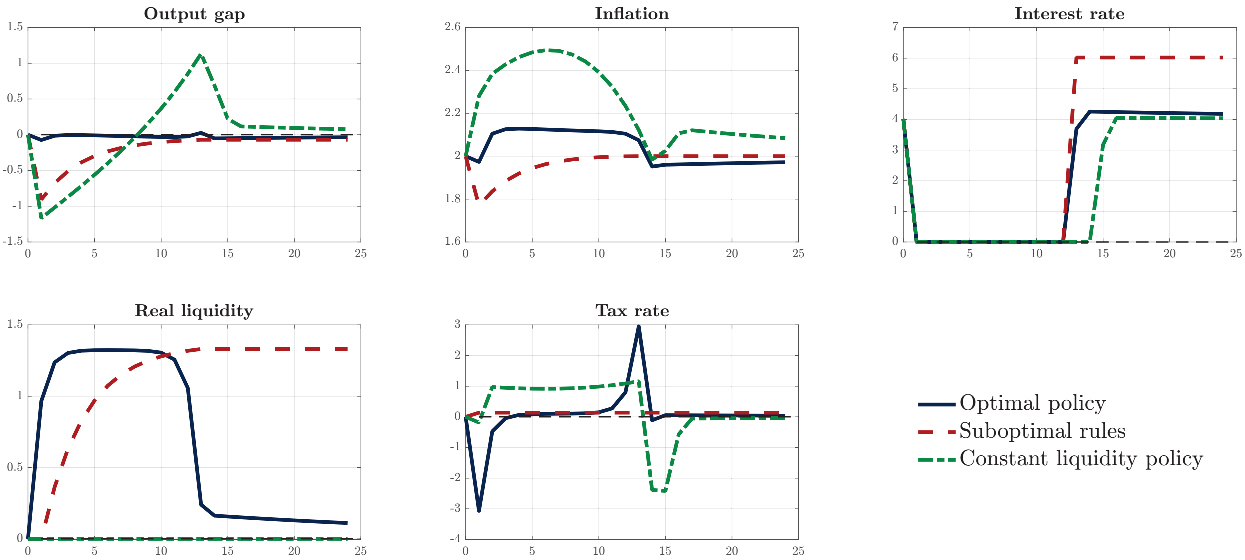

Figure 4: Comparison between optimal policy, optimal policy with constant liquidity, and sub-optimal rules, when λ_y/λ_π is fifty times higher than the benchmark calibration and the money-market spread is high. Impulse responses following a negative shock to the natural rate of interest, which lowers it to -4% at annual rates for 12 quarters. Output gap is in %, inflation and interest rates are in % annualized, real liquidity is in % deviations from steady state, and the tax rate is in percentage points, measured as deviations from steady state.

optimal policy to sub-optimal rules and to the "constant liquidity policy" analyzed in Figure 2. A key difference in the optimal policy, relative to the previous case, lies in the path of liquidity: liquidity now increases immediately at the start of the trap, supported by a reduction in the tax rate – unlike the increase observed in Figure 2. Liquidity then gradually rises, peaking before the shock dissipates, and is withdrawn progressively – though rapidly – after the policy rate lifts off. By the time interest rates return to normal, most of the excess liquidity has already been absorbed.

Figure 3 also shows that the duration of the zero lower bound under the optimal policy is shorter than in the baseline case of Figure 2, though still longer than the duration of the shock. The shorter zero lower bound episode reflects the more effective use of liquidity in stabilizing the economy. In contrast, under the "constant liquidity policy," the interest rate remains at the zero lower bound for one additional quarter to compensate for the absence of an active liquidity response.

Larger Weight on Output-Gap Stabilization

The final experiment is motivated by the observation that liquidity primarily affects aggregate demand, and thus output. The moderate use of liquidity seen in the previous experiments may stem from a high relative weight on inflation stabilization in the policymaker's loss function, which reduces the incentive to actively use liquidity.

In Figure 3, the ratio λ_y/λ_π was set at 0.002. We now consider an extreme case in which this ratio is fifty times higher, while keeping the higher value for ν. Figure 4 displays the resulting impulse responses.

Several features are worth highlighting. Under the revised calibration, optimal policy delivers greater stabilization of the output gap, with a clearer divergence from the "constant liquidity policy." The optimal policy now calls for a rapid and sizable increase in liquidity. Liquidity injection begins early in the trap and is nearly fully withdrawn by the time policy rates normalize. The duration of the zero lower bound episode coincides with the duration of the shock. Unlike earlier cases, there is no need to signal a prolonged period of zero interest rates, since the use of liquidity sufficiently stabilizes output.

In contrast, the "constant liquidity policy" still relies on an extended stay at the zero lower bound to compensate for the absence of liquidity tools, yet it is far less effective in stabilizing the output gap. Finally, note that inflation – being less costly in welfare terms under this calibration– remains above target throughout the liquidity trap episode.

8 Conclusion

We have proposed a new framework for monetary policy analysis that encompasses, as a special case, the Neo-Wicksellian paradigm. The nominal interest rate relevant for consumption/saving decisions can only be controlled by the central bank's simultaneous targeting of the interest rate on reserves and their quantity. The Neo-Wicksellian model is nested when liquidity is fully satiated.

The new framework shows the relevance of the monetary/fiscal policy mix in controlling inflation and output. We have applied it to the study of optimal policy in a liquidity trap, showing the role of tax policy and liquidity in influencing the optimal response of the policy rate to a natural real interest rate shock.

In this version, we have focused on the liquidity channel as the key mechanism through which reserve policies are effective. In subsequent research it would be interesting to study the interplay between the credit channel, as in Benigno and Benigno (2021), and the liquidity channel emphasized in this work.

Finally, the model has been kept as simple as possible for tractability and to compare it with existing analysis in the literature. It requires thorough extension in order to provide realistic quantitative analysis.

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A Log-linear approximation of the equilibrium conditions

Considering first the AD demand side of the model, we have the following first-order approximations of the equilibrium conditions (21), (22) and (20)

$$(1-\nu)\hat{\imath}_t^Q = (\rho_\gamma - \nu)\hat{\imath}_t^R + (1-\rho_\gamma)\hat{\imath}_t^B - \nu\hat{\rho}_{\gamma,t} \tag{A.1}$$

$$\hat{q}_t = q_y\hat{Y}_t - q_i(\hat{\imath}_t^B - \hat{\imath}_t^Q) + q_\xi\hat{\xi}_{q,t} \tag{A.2}$$

$$E_t\hat{Y}_{t+1} = \hat{Y}_t + \sigma(\hat{\imath}_t^B - E_t(\pi_{t+1} - \pi) - \tilde{r}_t^n) \tag{A.3}$$

in which we have defined variables with hat as the log-deviations of the respective variables with respect to the steady state; $\pi_t \equiv \ln P_t/P_{t-1}$, $\tilde{r}_t^n = \hat{\xi}_t - E_t\hat{\xi}_{t+1}$ $\pi \equiv \ln \Pi$, $\sigma \equiv -U_c/(U_{cc}Y)$, $\sigma_q \equiv -V_q/(V_{qq}q)$, $q_y \equiv \sigma_q/\sigma$, $q_i \equiv (1-\nu)\sigma_q/\nu$, $q_\xi \equiv \sigma_q$, with $\nu = V_q/U_c$ in which the derivatives of the function $V(\cdot)$ and $U(\cdot)$ are evaluated at the steady state.

We now turn to the approximation of the AS equation, given by (26) to (28). We obtain

$$(\pi_t - \pi) = \kappa(\hat{Y}_t + \psi_\tau\tilde{\tau}_t) + \beta E_t(\pi_{t+1} - \pi), \tag{A.4}$$

with

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)(\sigma^{-1}+\eta)}{\alpha(1+\theta\eta)}$$

$$\psi_\tau = \frac{1}{(1-\tau)(\eta+\sigma^{-1})}.$$

Finally note that we can derive the intertemporal resource constraint of the economy starting from the flow budget constraint of the government:

$$\frac{B_t^g}{P_t} = \frac{(1+i_{t-1}^R)}{\Pi_t}\frac{B_{t-1}^g}{P_{t-1}} - (\tau_t Y_t - T_t).$$

Defining $b_t^g \equiv B_t^g/P_t$, we can write

$$b_t^g = \frac{(1+i_{t-1}^R)}{\Pi_t}b_{t-1}^g - (\tau_t Y_t - T_t)$$

and therefore

$$\frac{1+i_t^R}{1+i_t^B}b_t^g + \frac{i_t^B - i_t^R}{1+i_t^B}b_t^g = \frac{1+i_{t-1}^R}{\Pi_t}b_{t-1}^g - (\tau_t Y_t - T_t).$$

Since

$$\frac{1}{1+i_t^B} = \beta E_t\left(\frac{U_c(Y_{t+1})\xi_{t+1}}{U_c(Y_t)\xi_t}\frac{1}{\Pi_{t+1}}\right)$$

we can iterate the equation forward using the transversality condition of the households' problem to obtain

$$\frac{(1+i^R_{t-1})}{\Pi_t}U_c(Y_t)\xi_t b^g_{t-1} = E_t\sum_{T=t}^{\infty}\beta^{T-t}U_c(Y_T)\xi_T\left[(\tau_T Y_T - T_T) + \frac{i^B_T - i^R_T}{1+i^B_T}b^g_T\right],$$

which can also be written as (29). Note that

$$\frac{1+i^Q_t}{1+i^B_t} - 1 = \rho_{\gamma,t}\left(\frac{1+i^R_t}{1+i^B_t} - 1\right)$$

and

$$1 - \frac{1+i^Q_t}{1+i^B_t} = \frac{\xi_{q,t}V_q\left(q_t\right)}{U_c\left(Y_t\right)},$$

therefore

$$\left(1 - \frac{1+i^R_t}{1+i^B_t}\right) = \frac{\xi_{q,t}}{\rho_{\gamma,t}}\frac{V_q\left(q_t\right)}{U_c\left(Y_t\right)}.$$

We can then write:

$$\frac{(1+i^R_{t-1})}{\Pi_t}U_c(Y_t)\xi_t b^g_{t-1} = E_t\sum_{T=t}^{\infty}\beta^{T-t}\left[U_c(Y_T)\xi_T(\tau_T Y_T - T_T) + \xi_T\xi_{q,T}V_q(q_T)q_T\right]. \tag{A.5}$$

A first-order approximation of this constraint implies that

$$\begin{aligned}\hat{b}^g_{t-1} - (\pi_t - \pi) - \sigma^{-1}\hat{Y}_t + \hat{\imath}^R_{t-1} \quad &= \quad [b_y\hat{Y}_t + \varrho\tilde{\tau}_t - \varrho\tilde{T}_t + b_\xi\hat{\xi}_{q,t} + b_q(\hat{b}^g_t - \hat{\rho}_{\gamma,t})] \\ &\quad +\beta E_t[\hat{b}^g_t - (\pi_{t+1} - \pi) - \sigma^{-1}\hat{Y}_{t+1} + \hat{\imath}^R_t - \tilde{r}^n_t],\end{aligned}$$

in which $\tilde{T}_t = (T_t - T)/Y$, we have used $\hat{q}_t = \hat{b}^g_t - \hat{\rho}_{\gamma,t}$ as a log-linear approximation of (**??**), and

$$\begin{aligned}\delta &= Y/q \\ \varrho &= \frac{\beta\delta}{\rho(1-\nu/\rho_\gamma)} \\ \omega &= \frac{(\tau-g)\delta}{(\tau-g)\delta+\nu} \\ b_y &= (\varrho\tau - (1-\beta)\omega\sigma^{-1}) \\ b_q &= (1-\beta)(1-\omega)(1-\sigma_q^{-1}) \\ b_\xi &= (1-\beta)(1-\omega).\end{aligned}$$

B Proof of Proposition 4

Note that there are two solutions of (40). At $q_t \geq \bar{q}$, all derivatives of the function $V(\cdot)$ are zero at and above the satiation point of liquidity. Since taxation is distortionary and reduces output, it is optimal to set the lowest possible level of liquidity i.e. $q_t = \bar{q}$. In this equilibrium, the tax rate has to satisfy the resource constraint (38) in which W_{t_0} is evaluated at the candidate equilibrium:

$$\frac{(1-\beta)}{\beta}\rho_\gamma \bar{q} = Y(\bar{\tau})(\bar{\tau} - g). \tag{B.6}$$

Since the revenues derived from taxes, $\tau Y(\tau)$, exhibit a Laffer-curve behavior, there exists a solution provided $\bar{q} \leq q_{\max}$, for an appropriately defined $q_{\max}$; $\bar{\tau}$ is the value of the tax rate in this steady-state equilibrium.

In the second equilibrium, equation (40) implies that $q_t = q^* < \bar{q}$ with q^* given by:

$$V_q(q^*) = -\frac{\lambda}{1+\lambda} V_{qq}(q^*) q^*. \tag{B.7}$$

The solution should also satisfy the resource constraint (38) in which Z_{t_0} is again appropriately evaluated at the equilibrium value. Therefore:

$$(1-\beta)\rho_\gamma q^* \frac{(1+i^{R*})}{\Pi} = Y(\tau^*)(\tau^* - g) + \frac{V_q\,(q^*)\,q^*}{Y(\tau^*)^{-\sigma^{-1}}}.$$

Using the steady-state equilibrium relationships for $1 + i^{R*}$ and $1 + i^{B*}$, we obtain

$$\frac{1+i^{R*}}{1+i^{B*}} = 1 - \frac{1}{\rho_\gamma}\frac{V_q\,(q^*)}{U_c(Y^*)}$$

and

$$1 + i^{B*} = \frac{\Pi}{\beta},$$

which can be used to rewrite the above equation as

$$\frac{(1-\beta)}{\beta}\rho_\gamma q^* = Y(\tau^*)(\tau^* - g) + \frac{1}{\beta}\frac{V_q\,(q^*)\,q^*}{Y(\tau^*)^{-\sigma^{-1}}}. \tag{B.8}$$

The optimal q^* and τ^* solve (39), (B.7) and (B.8) together with the lagrange multiplier λ.

C Proof of Proposition 5

Set the following assumptions on the model parameters: $\mu_\theta = 1$, $\eta = 0$, $\sigma = 1$ and $g = 0$. The set of first-order conditions to obtain the steady-state equilibrium for Y, τ, q is given by

$$Y = 1 - \tau, \tag{C.9}$$

$$\lambda = \frac{\tau}{1-\tau}, \tag{C.10}$$

$$\tau(1-\tau) + (1-\tau)\frac{V_q(q)q}{\beta} = \frac{(1-\beta)}{\beta}\rho_\gamma q, \tag{C.11}$$

$$V_q(q) = -\tau V_{qq}(q) q. \tag{C.12}$$

Note that

$$\begin{aligned} V_q(q) &= \tfrac{1}{q} - \tfrac{1}{q^*} \qquad \text{for} \quad q < \bar{q} \\ &= 0 \qquad\qquad \text{for} \quad q \geq \bar{q} \end{aligned}$$

$$\begin{aligned} V_{qq}(q) &= -\tfrac{1}{q^2} \qquad \text{for} \quad q < \bar{q} \\ &= 0 \qquad\qquad \text{for} \quad q \geq \bar{q} \end{aligned}.$$

The above set of equations has an equilibrium with full satiation of liquidity, when $q \geq \bar{q}$, in which case $V_q(q) = V_{qq}(q) = 0$, see equation (C.12). Therefore the tax rate is determined by equation (C.11)

$$\bar{\tau}(1-\bar{\tau}) = \frac{(1-\beta)}{\beta}\rho_\gamma \bar{q},$$

for which $\bar{\tau}$ solves the quadratic equation

$$(\bar{\tau})^2 - \bar{\tau} + \frac{(1-\beta)}{\beta}\rho_\gamma \bar{q} = 0,$$

which has the lowest root given by

$$\bar{\tau} = \frac{1 - \left(1 - 4\frac{(1-\beta)}{\beta}\rho_\gamma \bar{q}\right)^{\frac{1}{2}}}{2}.$$

Note, however, that the above set of equations has also another solution in which $q^* < \bar{q}$. Equation (C.12) implies that

$$\frac{1-\tau^*}{q^*} = \frac{1}{\bar{q}}$$

and so $q^* = (1-\tau^*)\bar{q}$. Therefore, equation (C.11) implies

$$\tau^*(1-\tau^*) + (1-\tau^*)\frac{V_q(q^*)q^*}{\beta} = \frac{(1-\beta)}{\beta}(1-\tau^*)\rho_\gamma \bar{q}. \tag{C.13}$$

Since

$$V_q q^* = 1 - \frac{q^*}{\bar{q}} = \tau^*$$

we obtain from (C.13) that

$$\tau^* = \frac{1-\beta}{1+\beta}\rho_\gamma \bar{q}.$$

Note, moreover, that we can write (C.13) as

$$\frac{1+\beta}{\beta}\tau^*(1-\tau^*) = \bar{\tau}(1-\bar{\tau})(1-\tau^*)$$

therefore

$$\tau^* = \frac{\beta}{1+\beta}\bar{\tau}(1-\bar{\tau}) < \frac{1}{2}\bar{\tau}$$

Let's compare welfare across the two equilibria. Note that utility is of the form

$$U = \ln Y - Y + \ln \frac{q}{\bar{q}} - \frac{q}{\bar{q}}$$

In the equilibrium with full satiation of liquidity we obtain that

$$\bar{U} = \ln(1 - \bar{\tau}) - (1 - \bar{\tau}) - 1$$

In the other equilibrium we have that

$$U^* = 2[\ln(1 - \tau^*) - (1 - \tau^*)]$$

To prove the result, let's consider the difference

$$U^* - \bar{U} = 2\ln(1 - \tau^*) - \ln(1 - \bar{\tau}) + 2\tau^* - \bar{\tau}$$

To have

$$U^* > \bar{U}$$

it should be that

$$\ln(1 - \tau^*) + \tau^* > \frac{\ln(1 - \bar{\tau}) + \bar{\tau}}{2}.$$

Use the result that $\tau^* < \bar{\tau}/2$, and note that $\ln(1 - \tau) + \tau$ is decreasing with τ, therefore

$$\ln(1 - \tau^*) + \tau^* > \ln\left(1 - \frac{\bar{\tau}}{2}\right) + \frac{\bar{\tau}}{2}.$$

Moreover,

$$\ln\left(1 - \frac{\bar{\tau}}{2}\right) > \frac{\ln(1 - \bar{\tau})}{2}.$$

It follows that

$$\ln(1 - \tau^*) + \tau^* > \ln\left(1 - \frac{\bar{\tau}}{2}\right) + \frac{\bar{\tau}}{2} > \frac{\ln(1 - \bar{\tau}) + \bar{\tau}}{2},$$

concluding the proof.

Consider now the case: $\mu_\theta = 1$, $\eta = 0$, $\sigma = 1$ and $g > 0$. The set of first-order conditions to obtain the steady-state equilibrium for Y, τ, q is given by

$$Y = 1 - \tau, \tag{C.14}$$

$$\lambda = \frac{\tau}{1 - \tau - g}, \tag{C.15}$$

$$(\tau - g)(1 - \tau) + (1 - \tau)\frac{V_q(q)q}{\beta} = \frac{(1 - \beta)}{\beta}\rho_\gamma q, \tag{C.16}$$

$$V_q(q) = -\frac{\tau}{1 - g}V_{qq}(q)q. \tag{C.17}$$

Note that in the optimal solution

$$q^* = \frac{1 - g - \tau^*}{1 - g}\bar{q}$$

and therefore

$$V_q q^* = 1 - \frac{q^*}{\bar{q}} = \frac{\tau^*}{1 - g}.$$

We can then write (C.16) as

$$(\tau^* - g) + \frac{\tau^*}{\beta(1 - g)} = \frac{(1 - \beta)}{\beta} \frac{\rho_\gamma q^*}{1 - \tau^*}.$$

Note that equation (B.8) can be rewritten as

$$\frac{(1 - \beta)}{\beta} \frac{\rho_\gamma q^*}{1 - \tau^*} = (\tau^* - g) + \frac{\tau^*}{\beta(1 - g)}.$$

In the previous expression, the second term on the right hand side represents the resources obtained from liquidity services, which are proportional to the tax rate τ^*. In the limit when ρ_γ goes to zero, then

$$\tau^* \longrightarrow \frac{\beta(1 - g)g}{1 + \beta(1 - g)}$$

while $\bar{\tau} \longrightarrow g$.

D Optimal policy problem

In this Appendix, we consider the optimal policy problem. The objective is the maximization of utility

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\frac{C_t^{1-\sigma^{-1}}}{1 - \sigma^{-1}} -{}_0^1 \frac{(H_t(j))^{1+\eta}}{1 + \eta} dj + \xi_{q,t} V(q_t) \right],$$

in which

$$\begin{aligned} V(q) &= \ln\left(\tfrac{q}{\bar{q}}\right) - \tfrac{q}{\bar{q}} && \text{for} \quad q < \bar{q} \\ &= -1 && \text{for} \quad q \geq \bar{q} \end{aligned}.$$

Note that in equilibrium we can write the above utility as

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \xi_t \left[\frac{Y_t^{1-\sigma^{-1}}}{1 - \sigma^{-1}} + \xi_{q,t} V(q_t) - \frac{Y_t^{1+\eta}}{1 + \eta} \Delta_t, \right]$$

given the definition of Δ_t

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t(j)}{P_t} \right)^{-\theta(1+\eta)} dj,$$

which can be written recursively as

$$\Delta_t = \alpha \Delta_{t-1} \left(\frac{\Pi_t}{\Pi} \right)^{\theta(1+\eta)} + (1 - \alpha) \left(\frac{1 - \alpha \left(\frac{\Pi_t}{\Pi}\right)^{\theta - 1}}{1 - \alpha} \right)^{\frac{\theta(1+\eta)}{\theta - 1}}. \tag{D.18}$$

The optimal policy problem involves choosing stochastic sequences $\{Y_t, q_t, \Delta_t, \tau_t, \Pi_t, F_t, K_t\}_{t=t_0}^{\infty}$ that maximize utility under the constraint (D.18), the AS schedule given by (26)–(28), and the intertemporal resource constraint of the economy (A.5) in which

$$Z_{t_0} = \frac{(1 + i^R_{t_0-1})}{\Pi_{t_0}} U_c(Y_{t_0}) \xi_{t_0} b^g_{t_0-1}.$$

The maximization problem considers as given the stochastic sequences $\{\xi_t, \xi_{q,t}, T_t, \rho_{\gamma,t}\}_{t=t_0}^{\infty}$, initial condition Δ_{t_0-1} and constraints $F_{t_0} = \bar{F}_{t_0}$, $K_{t_0} = \bar{K}_{t_0}$, $Z_{t_0} = \bar{Z}_{t_0}$ that are such to make the optimal policy problem recursive.

We analyze the optimal policy problem through linear-quadratic approximations, in line with Benigno and Woodford (2003). First, we analyze the optimal steady state, then we build a second-order approximation to the policy objective function and study the optimal policy problem from a linear-quadratic perspective.

D.1 The deterministic steady state

Here we compute the steady state of the optimal monetary and fiscal policy problem in a deterministic problem in which the exogenous disturbances $\xi_t, \xi_{q,t}, \rho_{\gamma,t}$ and T_t take constant values $\xi_t = \xi_{q,t} = 1$, $\rho_{\gamma,t} = \rho_\gamma$ and $T_t = T$, for all $t \geq t_0$.

We thus consider the problem of maximizing

$$U_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{Y_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \frac{Y_t^{1+\eta}}{1+\eta} \Delta_t + V(q_t) \right) \tag{D.19}$$

subject to the constraints

$$K_t p \left(\frac{\Pi_t}{\Pi} \right)^{\frac{1+\eta\theta}{\theta-1}} = F_t, \tag{D.20}$$

$$F_t = (1-\tau_t) Y_t^{1-\sigma^{-1}} + \alpha\beta \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\theta-1} F_{t+1}, \tag{D.21}$$

$$K_t = \mu_\theta Y_t^{1+\eta} + \alpha\beta \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\theta(1+\eta)} K_{t+1}, \tag{D.22}$$

$$Z_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\tau_t Y_t^{1-\sigma^{-1}} - T_t Y_t^{-\sigma^{-1}} + V_q(q_t) q_t), \tag{D.23}$$

$$\Delta_t = \alpha \Delta_{t-1} \left(\frac{\Pi_t}{\Pi} \right)^{\theta(1+\eta)} + (1-\alpha) p \left(\frac{\Pi_t}{\Pi} \right)^{-\frac{\theta(1+\eta)}{1-\theta}}, \tag{D.24}$$

given specified initial conditions $\Delta_{t_0-1}, F_{t_0}, K_{t_0}, Z_{t_0}$ where we have defined

$$p\left(\frac{\Pi_t}{\Pi} \right) \equiv \left(\frac{1 - \alpha(\Pi_t/\Pi)^{\theta-1}}{1-\alpha} \right).$$

The maximization of utility, in the optimal policy problem, is subject to the AS equation, given by equations (D.19) – (D.21), to the intertemporal resource constraint, equation (D.22) given the law of motion of Δ_t.

We introduce Lagrange multipliers $\phi_{1,t}$ through $\phi_{5,t}$ corresponding to constraints (D.20) through (D.24) respectively. Note that the lagrange multiplier ϕ_4 is constant. We also introduce multipliers dated t_0 corresponding to the constraints implied by the initial conditions F_{t_0}, K_{t_0}; the latter multipliers are normalized in such a way that the first-order conditions take the same form at date t_0 as at all later dates. The first-order conditions of the maximization problem are then the following. The one with respect to Y_t is

$$\begin{aligned} Y_t^{-\sigma^{-1}} - \Delta_t Y_t^{\eta} - (1-\tau_t)(1-\sigma^{-1})Y_t^{-\sigma^{-1}}\phi_{2,t} - (1+\eta)\mu_\theta Y_t^{\eta}\phi_{3,t} + \tau_t Y_t^{-\sigma^{-1}}\phi_4 \\ -\sigma^{-1}Y_t^{-\sigma^{-1}}\tau_t\phi_4 + \sigma^{-1}Y_t^{-\sigma^{-1}-1}T_t\phi_4 = 0; \end{aligned} \tag{D.25}$$

that with respect to Δ_t is

$$-\frac{Y_t^{1+\eta}}{1+\eta} + \phi_{5,t} - \alpha\beta\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta(1+\eta)}\phi_{5,t+1} = 0; \tag{D.26}$$

that with respect to Π_t is

$$\begin{aligned} \frac{1+\theta\eta}{\theta-1}p\left(\frac{\Pi_t}{\Pi}\right)^{\frac{1+\theta\eta}{\theta-1}-1} p_\pi\left(\frac{\Pi_t}{\Pi}\right)K_t\phi_{1,t} - \alpha(\theta-1)\left(\frac{\Pi_t}{\Pi}\right)^{\theta-2}\frac{F_t}{\Pi}\phi_{2,t-1} \\ -\theta(1+\eta)\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta(1+\eta)-1}\frac{K_t}{\Pi}\phi_{3,t-1}+ \\ -\theta(1+\eta)\alpha\Delta_{t-1}\left(\frac{\Pi_t}{\Pi}\right)^{\theta(1+\eta)-1}\frac{1}{\Pi}\phi_{5,t} - \frac{\theta(1+\eta)}{\theta-1}(1-\alpha)p\left(\frac{\Pi_t}{\Pi}\right)^{\frac{(1+\eta\theta)}{\theta-1}} p_\pi\left(\frac{\Pi_t}{\Pi}\right)\phi_{5t} = 0; \end{aligned} \tag{D.27}$$

that with respect to τ_t is

$$\phi_{2,t} + \phi_4 = 0; \tag{D.28}$$

that with respect to F_t is

$$-\phi_{1,t} + \phi_{2,t} - \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}\phi_{2,t-1} = 0; \tag{D.29}$$

that with respect to K_t is

$$p\left(\frac{\Pi_t}{\Pi}\right)^{\frac{1+\eta\theta}{\theta-1}}\phi_{1t} + \phi_{3t} - \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta(1+\eta)}\phi_{3,t-1} = 0; \tag{D.30}$$

that with respect to q_t is

$$V_q(q_t) = -\phi_4(V_q(q_t) + V_{qq}(q_t)q_t). \tag{D.31}$$

We search for a solution to these first-order conditions in which $\Pi_t = \Pi$, $\Delta_t = \Delta$, $Y_t = Y$, $\tau_t = \tau$, and $q_t = q$ at all times. A steady-state solution of this kind also requires that the Lagrange multipliers take constant values. We furthermore conjecture the existence of a solution in which $\Delta = 1$, $p(\cdot) = 1$, $p_\pi(\cdot) = -(\theta-1)\alpha/[(1-\alpha)\Pi]$, and $K = F$. Using these substitutions, we find that (the steady-state version of) each of the first-order conditions (D.25) – (D.31) is satisfied if the steady-state values satisfy

$$1 - Y_t^{\eta+\sigma^{-1}} = [(1-\sigma^{-1}) + \sigma^{-1}g - (1+\eta)\mu_\theta Y_t^{\eta+\sigma^{-1}}]\phi_2, \tag{D.32}$$

$$(1-\alpha\beta)\phi_5 = \frac{Y^{1+\eta}}{1+\eta},$$

$$\phi_4 = -\phi_2, \tag{D.33}$$

$$\phi_1 = (1-\alpha)\phi_2,$$

$$\phi_3 = -\phi_2,$$

$$V_q(q) = -\phi_4(V_q(q) + V_{qq}(q)q). \tag{D.34}$$

We have defined $g = T/Y$. Similarly, (the steady-state versions of) the constraints (D.20) – (D.24) are satisfied if

$$\frac{(1-\tau)}{\mu_\theta} = Y^{\eta+\sigma^{-1}}, \tag{D.35}$$

$$(\tau Y - gY) + V_q(q)qY^{\sigma^{-1}} = (1-\beta)\rho_\gamma q\frac{(1+i^R)}{\Pi}, \tag{D.36}$$

$$F = K = (1-\alpha\beta)^{-1}\mu_\theta Y^{1+\eta},$$

$$Z = \frac{Y^{-\sigma^{-1}}(1+i^R)\rho_\gamma q}{\Pi}.$$

We can use (D.35) and (D.33) into (D.32) to obtain

$$\phi_4 = \frac{1 - \frac{(1-\tau)}{\mu_\theta}}{(1+\eta)(1-\tau) - (1-\sigma^{-1}) - \sigma^{-1}g} \tag{D.37}$$

which is positive provided $\tau < (\eta + \sigma^{-1}(1-g))/(1+\eta)$. Note that the multiplier ϕ_4 is function of τ and that output is a decreasing function of τ using (D.35).

Note that in the steady state

$$1 + i^B = \frac{\Pi}{\beta}$$

$$\frac{1+i^Q}{1+i^B} = 1 - \frac{V_q(q)}{U_c(Y)}$$

$$1 + i^Q = \rho_\gamma(1+i^R) + (1-\rho_\gamma)(1+i^B).$$

Therefore

$$\frac{(1+i^R)}{\Pi} = \frac{1+i^R}{1+i^B}\frac{(1+i^B)}{\Pi} = \left(1 - \frac{V_q(q)Y^{\sigma^{-1}}}{\rho_\gamma}\right)\frac{1}{\beta},$$

and we can write (D.36) as

$$(\tau - g)Y(\tau) + \frac{V_q(q)q}{\beta Y(\tau)^{-\sigma^{-1}}} = \frac{(1-\beta)}{\beta}\rho_\gamma q$$

which together with

$$V_q(q) = -\phi_4(\tau)(V_q(q) + V_{qq}(q)q).$$

represents a set of two equations to solve for q and τ. We have discussed extensively the solution in Section 6. The remaining equations can then be solved (uniquely) for $K = F$ and for Z.

D.2 A second-order approximation to utility

As a first step to compute optimal policy through linear-quadratic approximations, we take a second-order approximation to the households' utility

$$U_{t_0} = E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\xi_t\left[\frac{Y_t^{1-\sigma^{-1}}-1}{1-\sigma^{-1}} - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t + \xi_{q,t}V(q_t)\right]\right\}. \tag{D.38}$$

Note that

$$\begin{aligned}\xi_t\left[\frac{Y_t^{1-\sigma^{-1}}-1}{1-\sigma^{-1}} + \xi_{q,t}V(q_t) - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t\right] &= U_cY\left[\hat{Y}_t + \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\hat{Y}_t^2\right] + U_cY\hat{Y}_t\hat{\xi}_t - \\ &+V_qq\left[\hat{q}_t + \frac{1}{2}\left(1-\frac{1}{\sigma_q}\right)\hat{q}_t^2\right] + V_qq\hat{q}_t\hat{\xi}_{d,t} - H_lY\hat{Y}_t\hat{\xi}_t \\ &-H_lY\left[\hat{Y}_t + \frac{1}{2}(1+\eta)\hat{Y}_t^2\right] - H(Y)(\Delta_t - 1) + \mathcal{O}(||\xi||^3),\end{aligned}$$

where $\mathcal{O}(||\xi||^3)$ collects terms of order higher than the second and where we have used the following approximation:

$$\left(\frac{Y_t - Y}{Y}\right) = \hat{Y}_t + \frac{1}{2}\hat{Y}_t^2 + \mathcal{O}(||\xi||^3),$$

and similarly for $q_t - q$. Note that we have defined $\xi_{d,t} = \xi_{q,t}\xi_t$ and $\sigma_q \equiv -V_q/V_{qq}q$ in which derivatives are evaluated at the steady state. Note that in the steady $H_l = (1-\Phi)U_c$ where

$$\Phi \equiv 1 - \frac{(1-\tau)}{\mu_\theta} < 1$$

measures the inefficiency of steady-state output Y. We can then write

$$\begin{aligned}\xi_t\left[\frac{Y_t^{1-\rho}-1}{1-\rho} + \xi_{q,t}V(q_t) - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t\right] &= U_cY\left[\Phi\hat{Y}_t + \frac{1}{2}\left(1-\frac{1}{\sigma}\right)\hat{Y}_t^2\right] + \Phi U_cY\hat{Y}_t\hat{\xi}_t - \\ &+V_qq\left[\hat{q}_t + \frac{1}{2}\left(1-\frac{1}{\sigma_q}\right)\hat{q}_t^2\right] + V_qq\hat{q}_t\hat{\xi}_{d,t} \\ &-\frac{1}{2}(1-\Phi)U_cY(1+\eta)\hat{Y}_t^2 - \frac{(1-\Phi)}{1+\eta}U_cY(\Delta_t - 1) \\ &+\mathcal{O}(||\xi||^3),\end{aligned}$$

and in a compact way

$$
\begin{aligned}
U_{t_0} \quad &= \quad U_c Y \cdot E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\Phi \widehat{Y}_t - \frac{1}{2} u_{yy} \widehat{Y}_t^2 + \Phi \widehat{Y}_t \hat{\xi}_t - u_\Delta \hat{\Delta}_t + \\
&\quad +\nu\delta^{-1} \left[\hat{q}_t (1 + \hat{\xi}_{d,t}) + \frac{1}{2} \left(1 - \sigma_q^{-1}\right) \hat{q}_t^2 \right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3),
\end{aligned} \tag{D.39}
$$

in which t.i.p. denotes terms independent of policy and, moreover, we have defined

$$
\begin{aligned}
u_{yy} \quad &\equiv \quad -(1 - \sigma^{-1}) + (1 - \Phi)(1 + \eta), \\
u_\Delta \quad &\equiv \quad \frac{(1 - \Phi)}{1 + \eta},
\end{aligned}
$$

and used the definitions $\nu \equiv V_q / U_c$ and $\delta \equiv Y/q$.

We now take a second-order Taylor expansion of (D.18) around the steady state in which $\Delta_t = 1$ and $\Pi_t = \Pi$ to obtain

$$
\hat{\Delta}_t = \alpha \hat{\Delta}_{t-1} + \frac{\alpha}{1 - \alpha} \theta (1 + \eta)(1 + \eta\theta) \frac{(\pi_t - \pi)^2}{2} + \text{t.i.p.} + \mathcal{O}(||\xi||^3).
$$

Now note that

$$
\hat{\Delta}_t = \alpha^{t-t_0+1} \hat{\Delta}_{t_0-1} + \frac{1}{2} \frac{\alpha\theta}{(1 - \alpha)} (1 + \eta)(1 + \eta\theta) \sum_{s=t_0}^{t} \alpha^{t-s} \, (\pi_s - \pi)^2 + \mathcal{O}(||\xi||^3)
$$

and therefore

$$
\sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_t = \frac{1}{2} \frac{\alpha\theta(1 + \eta)(1 + \eta\theta)}{(1 - \alpha)(1 - \alpha\beta)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t - \pi)^2 + \mathcal{O}(||\xi||^3), \tag{D.40}
$$

neglecting initial condition $\hat{\Delta}_{t_0-1}$.

We substitute (D.40) into (D.39) to obtain

$$
\begin{aligned}
U_{t_0} \quad &= \quad Y U_c \cdot E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\Phi \widehat{Y}_t - \frac{1}{2} u_{yy} \widehat{Y}_t^2 + \Phi \widehat{Y}_t \hat{\xi}_t - \frac{1}{2} u_\pi (\pi_t - \pi)^2 + \\
&\quad +\nu\delta^{-1} \left[\hat{q}_t (1 + \hat{\xi}_{d,t}) + \frac{1}{2} \left(1 - \sigma_q^{-1}\right) \hat{q}_t^2 \right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3),
\end{aligned}
$$

where we have further defined

$$
\kappa \equiv \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha} \frac{(\eta + \sigma^{-1})}{(1 + \eta\theta)}, \qquad u_\pi \equiv \frac{\theta(\eta + \sigma^{-1})(1 - \Phi)}{\kappa}.
$$

We can also write it as

$$
\begin{aligned}
U_{t_0} &= YU_c \cdot E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [a'_x x_t - \frac{1}{2} x'_t A_x x_t - \frac{1}{2} x'_t A_\varepsilon \varepsilon_t - \frac{1}{2} a_\pi (\pi_t - \pi)^2] + \\
&\quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3),
\end{aligned}
$$

where we have used the following definitions

$$
x_t \equiv \begin{bmatrix} \hat{\tau}_t \\ \hat{Y}_t \\ \hat{q}_t \end{bmatrix},
$$

$$
\varepsilon_t \equiv \begin{bmatrix} \hat{\xi}_t \\ \hat{T}_t \\ \hat{\xi}_{d,t} \end{bmatrix}
$$

$$
a'_x \equiv \begin{bmatrix} 0 & \Phi & \nu\delta^{-1} \end{bmatrix}
$$

$$
A_x \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(1-\sigma^{-1}) + (1-\Phi)(1+\eta) & 0 \\ 0 & 0 & -\nu\delta^{-1}\left(1-\sigma_q^{-1}\right) \end{bmatrix}
$$

$$
A_\varepsilon \equiv \begin{bmatrix} 0 & 0 & 0 \\ -\Phi & 0 & 0 \\ 0 & 0 & -\nu\delta^{-1} \end{bmatrix}
$$

$$
a_\pi \equiv \frac{\theta(\eta + \sigma^{-1})(1-\Phi)}{\kappa}.
$$

D.3 A second-order approximation of the AS equation

We follow Benigno and Woodford (2003) to obtain that a second-order approximation of the AS equation is:

$$
\begin{aligned}
V_t &= \frac{1-\alpha}{\alpha} \frac{(1-\alpha\beta)}{(1+\theta\eta)} \Big((\eta + \sigma^{-1})\hat{Y}_t + \omega_\tau \hat{\tau}_t + \frac{1}{2} \frac{\omega_\tau}{(1-\bar{\tau})} \hat{\tau}_t^2 + \frac{1}{2} [(\hat{\xi}_t + (1+\eta)\hat{Y}_t)^2 - \\
&\quad (-\omega_\tau \hat{\tau}_t + \hat{\xi}_t + (1-\sigma^{-1})\hat{Y}_t)^2] \Big) + \frac{\theta(1+\eta)}{2} (\pi_t - \pi)^2 + \beta E_t V_{t+1} + \text{t.i.p.} + \mathcal{O}(||\xi||^3).
\end{aligned}
$$

In a more compact way, we can write

$$
\begin{aligned}
V_t &= \kappa(c'_x x_t + \frac{1}{2} x'_t C_x x_t + x'_t C_\varepsilon \varepsilon_t + \frac{1}{2} c_\pi (\pi_t - \pi)^2) + \beta E_t V_{t+1} \\
&\quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \qquad \text{(D.41)}
\end{aligned}
$$

We have defined

$$\omega_\tau \equiv \tau/(1-\tau)$$

$$\psi \equiv \omega_\tau/(\eta + \sigma^{-1}),$$

$$c_x' \equiv \begin{bmatrix} \psi & 1 & 0 \end{bmatrix},$$

$$C_x \equiv \begin{bmatrix} \psi & (1-\sigma^{-1})\psi & 0 \\ (1-\sigma^{-1})\psi & (2+\eta-\sigma^{-1}) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C_\varepsilon \equiv \begin{bmatrix} \psi & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$c_\pi \equiv \frac{\theta(1+\eta)}{\kappa}.$$

We can also integrate (D.41) forward from time t_0 to obtain

$$\begin{aligned} V_{t_0} \;&=\; E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \kappa (c_x' x_t + \frac{1}{2} x_t' C_x x_t + x_t' C_\varepsilon \varepsilon_t + \frac{1}{2} c_\pi (\pi_t - \pi)^2) \\ &\quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3). \end{aligned} \tag{D.42}$$

Note that in a first-order approximation, (D.41) can be written as simply

$$(\pi_t - \pi) = \kappa[\hat{Y}_t + \psi \hat{\tau}_t] + \beta E_t(\pi_{t+1} - \pi), \tag{D.43}$$

since $V_t = (\pi_t - \pi) + \mathcal{O}(||\xi||^2)$.

D.4 A second-order approximation to the government's intertemporal budget constraint

We now derive a second-order approximation to the intertemporal government budget constraint (A.5), which can be written as

$$Z_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} [\xi_T Y_T^{1-\sigma^{-1}} \tau_T - \xi_T Y_T^{-\sigma^{-1}} T_T + \xi_{d,T} V_q(q_T) q_T], \tag{D.44}$$

and

$$Z_t = \frac{(1+i_{t-1}^R) b_{t-1}^g}{\Pi_t} \xi_t Y_t^{-\sigma^{-1}}. \tag{D.45}$$

First, we take a second-order approximation of the term $\xi_t Y_t^{1-\sigma^{-1}}\tau_t$ obtaining

$$\begin{aligned}
\xi_t Y_t^{1-\sigma^{-1}}\tau_t &= Y^{1-\sigma^{-1}}\tau+(1-\sigma^{-1})Y^{-\sigma^{-1}}\tau\tilde{Y}_t+Y^{1-\sigma^{-1}}\tilde{\tau}_t+Y^{1-\sigma^{-1}}\tau\tilde{\xi}_t+\\
&\quad -\frac{1}{2}\sigma^{-1}(1-\sigma^{-1})Y^{-\sigma^{-1}-1}\tau\tilde{Y}_t^2+(1-\sigma^{-1})Y^{-\sigma^{-1}}\tilde{Y}_t\tilde{\tau}_t+\\
&\quad +(1-\sigma^{-1})Y^{-\sigma^{-1}}\tau\tilde{Y}_t\tilde{\xi}_t+Y^{1-\sigma^{-1}}\tilde{\tau}_t\tilde{\xi}_t+\mathcal{O}(||\xi||^3),\\
&= Y^{1-\sigma^{-1}}\tau+(1-\sigma^{-1})Y^{1-\sigma^{-1}}\tau\hat{Y}_t+Y^{1-\sigma^{-1}}\tau\left(\hat{\tau}_t+\frac{1}{2}\hat{\tau}_t^2\right)+Y^{1-\sigma^{-1}}\tau\hat{\xi}_t\\
&\quad +\frac{1}{2}(1-\sigma^{-1})^2\tau Y^{1-\sigma^{-1}}\hat{Y}_t^2+(1-\sigma^{-1})\tau Y^{1-\sigma^{-1}}\hat{Y}_t\hat{\tau}_t+\\
&\quad +(1-\sigma^{-1})Y^{1-\sigma^{-1}}\tau\hat{Y}_t\hat{\xi}_t+Y^{1-\sigma^{-1}}\tau\hat{\tau}_t\hat{\xi}_t+\text{t.i.p.}+\mathcal{O}(||\xi||^3)\\
&= Y^{1-\sigma^{-1}}\tau+Y^{1-\sigma^{-1}}\tau[(1-\sigma^{-1})\hat{Y}_t+\hat{\tau}_t+\frac{1}{2}\hat{\tau}_t^2+\hat{\xi}_t+\frac{1}{2}(1-\sigma^{-1})^2\hat{Y}_t^2\\
&\quad +(1-\sigma^{-1})\hat{Y}_t\hat{\tau}_t+(1-\sigma^{-1})\hat{Y}_t\hat{\xi}_t+\hat{\tau}_t\hat{\xi}_t]+\text{t.i.p.}+\mathcal{O}(||\xi||^3),
\end{aligned}$$

where a tilde variable denote the deviation of the variable with respect to the steady state. Considering a second-order approximation of the term

$$\begin{aligned}
\xi_t Y_t^{-\sigma^{-1}}T_t &= Y^{-\sigma^{-1}}T-\sigma^{-1}Y^{-\sigma^{-1}-1}T\tilde{Y}_t+Y^{-\sigma^{-1}}\cdot\tilde{T}_t+Y^{-\sigma^{-1}}T\cdot\tilde{\xi}_t+\\
&\quad +\frac{1}{2}\sigma^{-1}(1+\sigma^{-1})Y^{-\sigma^{-1}-2}T\cdot\tilde{Y}_t^2-\sigma^{-1}Y^{-\sigma^{-1}-1}\cdot\tilde{Y}_t\tilde{T}_t+\\
&\quad -\sigma^{-1}Y^{-\sigma^{-1}-1}T\cdot\tilde{Y}_t\tilde{\xi}_t+\mathcal{O}(||\xi||^3),\\
&= Y^{-\sigma^{-1}}T-\sigma^{-1}Y^{-\sigma^{-1}}T\hat{Y}_t+Y^{-\sigma^{-1}}T\cdot\hat{T}_t+Y^{-\sigma^{-1}}T\cdot\tilde{\xi}_t\\
&\quad +\frac{1}{2}\sigma^{-2}TY^{-\sigma^{-1}}\hat{Y}_t^2-\sigma^{-1}Y^{-\sigma^{-1}}T\cdot\hat{Y}_t\hat{T}_t+\\
&\quad -\sigma^{-1}Y^{-\sigma^{-1}}T\cdot\hat{Y}_t\hat{\xi}_t+\text{t.i.p.}+\mathcal{O}(||\xi||^3)\\
&= Y^{1-\sigma^{-1}}g+Y^{1-\sigma^{-1}}g[-\sigma^{-1}\hat{Y}_t+\hat{T}_t+\hat{\xi}_t+\frac{1}{2}\sigma^{-2}\hat{Y}_t^2\\
&\quad -\sigma^{-1}\hat{Y}_t\hat{T}_t-\sigma^{-1}\hat{Y}_t\hat{\xi}_t]+\text{t.i.p.}+\mathcal{O}(||\xi||^3),
\end{aligned}$$

We now take a second-order approximation of the term

$$\begin{aligned}
\xi_{d,t}V_q(q_t)q_t &= V_q q+V_{qq}q\tilde{q}_t+V_q\tilde{q}_t+V_q q\tilde{\xi}_{d,t}+\frac{1}{2}(V_{qqq}q+2V_{qq})\tilde{q}_t^2+(V_q+V_{qq}q)\tilde{q}_t\tilde{\xi}_{d,t}\\
&\quad +\mathcal{O}(||\xi||^3)\\
&= V_q q+(V_{qq}q^2+V_q q)\hat{q}_t+V_q q\hat{\xi}_{d,t}+\frac{1}{2}(V_{qqq}q^3+3V_{qq}q^2+V_q q)\hat{q}_t^2+\\
&\quad +(V_q q+V_{qq}q^2)\hat{q}_t\hat{\xi}_{d,t}+\text{t.i.p.}+\mathcal{O}(||\xi||^3),\\
&= V_q q[1+(1-\sigma_q^{-1})\hat{q}_t+\hat{\xi}_{d,t}+\frac{1}{2}(\tilde{\sigma}_q^{-1}\sigma_q^{-1}-2\sigma_q^{-1}+1)\hat{q}_t^2+\\
&\quad +(1-\sigma_q^{-1})\hat{q}_t\hat{\xi}_{d,t}]+\text{t.i.p.}+\mathcal{O}(||\xi||^3)
\end{aligned}$$

in which we have defined $1+\tilde{\sigma}_q^{-1}=-V_{qqq}q/V_{qq}$.

We can then write

$$
\begin{aligned}
\tilde{Z}_t &= \tau[(1-\sigma^{-1})\hat{Y}_t + \hat{\tau}_t + \frac{1}{2}\hat{\tau}_t^2 + \hat{\xi}_t + \frac{1}{2}(1-\sigma^{-1})^2\hat{Y}_t^2 + (1-\sigma^{-1})\hat{Y}_t\hat{\tau}_t + \\
&+(1-\sigma^{-1})\hat{Y}_t\hat{\xi}_t + \hat{\tau}_t\hat{\xi}_t] - g[-\sigma^{-1}\hat{Y}_t + \hat{T}_t + \hat{\xi}_t + \frac{1}{2}\sigma^{-2}\hat{Y}_t^2 \\
&-\sigma^{-1}\hat{Y}_t\hat{T}r_t - \sigma^{-1}\hat{Y}_t\hat{\xi}_t] \\
&\nu\delta^{-1}[(1-\sigma_q^{-1})\hat{q}_t + \hat{\xi}_{d,t} + \frac{1}{2}(\tilde{\sigma}_q^{-1}\sigma_q^{-1} - 2\sigma_q^{-1} + 1)\hat{q}_t^2 + \\
&+(1-\sigma_q^{-1})\hat{q}_t\hat{\xi}_{d,t}] + \beta E_t\tilde{Z}_{t+1} + \text{t.i.p.} + \mathcal{O}(||\xi||^3)
\end{aligned}
$$

and in a more compact way

$$
\begin{aligned}
\tilde{Z}_t &= [b_x' x_t + b_\varepsilon' \varepsilon_t + \frac{1}{2}x_t' B_x x_t + x_t' B_\varepsilon \varepsilon_t] + \beta E_t \tilde{Z}_{t+1} \\
&+\text{t.i.p.} + \mathcal{O}(||\xi||^3)
\end{aligned}
\tag{D.46}
$$

where $\tilde{Z}_t \equiv (Z_t - \bar{Z})/(U_c Y)$ and

$$
b_x' = \begin{bmatrix} \tau & \tau(1-\sigma^{-1}) + g\sigma^{-1} & \nu\delta^{-1}(1-\sigma_q^{-1}) \end{bmatrix},
$$

$$
b_\varepsilon' = \begin{bmatrix} (\tau - g + \nu\delta^{-1}) & -g & 0 \end{bmatrix}
$$

$$
B_x = \begin{bmatrix} \tau & \tau(1-\sigma^{-1}) & 0 \\ (1-\sigma^{-1}) & \tau(1-\sigma^{-1})^2 - g\sigma^{-2} & 0 \\ 0 & 0 & \nu\delta^{-1}(\tilde{\sigma}_q^{-1}\sigma_q^{-1} - 2\sigma_q^{-1} + 1) \end{bmatrix},
$$

$$
B_\xi = \begin{bmatrix} \tau & 0 & 0 \\ \tau(1-\sigma^{-1}) + \sigma^{-1}g & g\sigma^{-1} & 0 \\ 0 & 0 & \nu\delta^{-1}(1-\sigma_q^{-1}) \end{bmatrix}.
$$

Moreover integrating forward (D.46), we obtain that

$$
\tilde{Z}_{t_0} = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[b_x' x_t + \frac{1}{2}x_t' B_x x_t + x_t' B_\varepsilon \varepsilon_t] + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \tag{D.47}
$$

where we have moved ε_t in t.i.p.

Note that up to first-order terms, we can write

$$
\begin{aligned}
\tilde{Z}_t &= \Big\{[\tau(1-\sigma^{-1}) + g\sigma^{-1}]\hat{Y}_t + \tau\hat{\tau}_t - g\hat{T}_t + (\tau - g + \nu\delta^{-1})\hat{\xi}_t)] \\
&\nu\delta^{-1}\hat{\xi}_{q,t} + \nu\delta^{-1}(1-\sigma_q^{-1})\hat{q}_t\Big\} + \beta E_t\tilde{Z}_{t+1}.
\end{aligned}
$$

in which we have noted that $\hat{\xi}_{d,t} = \hat{\xi}_t + \hat{\xi}_{q,t}$. Note that $\bar{Z} = (1-\beta)^{-1}(U_c Y)(\tau - g + \nu\delta^{-1})$ and

$\hat{Z}_t \equiv (Z_t - \bar{Z})/\bar{Z} = \tilde{Z}_t \cdot (U_c Y/\bar{Z})$. Moreover note that

$$\frac{(1-\beta)}{\beta}\rho_\gamma q\left(1-\frac{\nu}{\rho_\gamma}\right) = (\tau - g)Y + \nu q$$

and therefore

$$\frac{(1-\beta)}{\beta}\rho_\gamma \delta^{-1}\left(1-\frac{\nu}{\rho_\gamma}\right) = (\tau - g) + \nu\delta^{-1}.$$

It also follows that $\bar{Z}/U_c Y = \beta^{-1}\delta^{-1}\rho_\gamma(1-\nu/\rho_\gamma)$. Define $\omega \equiv (\tau - g)/[(1-\beta)\bar{Z}/(U_c Y)]$, therefore $\nu\delta^{-1} = (1-\omega)[(1-\beta)\bar{Z}/(U_c Y)]$. Define also $\varrho \equiv U_c Y/\bar{Z} = \beta\delta\rho_\gamma^{-1}/(1-\nu/\rho_\gamma)$. Therefore, $\hat{Z}_t = \varrho\tilde{Z}_t$.

We can then write:

$$\begin{aligned}
\hat{Z}_t &= \varrho\tilde{\tau}_t - \varrho\tilde{T}_t + (\varrho\tau - (1-\beta)\omega\sigma^{-1})\hat{Y}_t + \\
&\quad +(1-\beta)[\hat{\xi}_t + (1-\omega)\hat{\xi}_{q,t} + (1-\omega)(1-\sigma_q^{-1})\hat{q}_t] + \beta E_t \hat{Z}_{t+1},
\end{aligned}$$

in which we have used the definition $\tilde{\tau}_t = \tau_t - \tau$ and from now onwards $\tilde{T}_t = (T_t - T)/Y$. Moreover

$$\hat{Z}_t \equiv \hat{b}^g_{t-1} - (\pi_t - \pi) - \sigma^{-1}\hat{Y}_t + \hat{\xi}_t + \hat{\imath}^R_{t-1}$$

We can write

$$\begin{aligned}
\hat{b}^g_{t-1} - (\pi_t - \pi) - \sigma^{-1}\hat{Y}_t + \hat{\imath}^R_{t-1} &= [b_y\hat{Y}_t + \varrho\tilde{\tau}_t - \varrho\tilde{T}_t + b_\xi\hat{\xi}_{q,t} + b_q\hat{q}_t] \\
&\quad +\beta E_t[\hat{b}^g_t - (\pi_{t+1} - \pi) - \sigma^{-1}\hat{Y}_{t+1} + \hat{\imath}^R_t - \tilde{r}^n_t],
\end{aligned}$$

in which we have defined $\tilde{r}^n_t = \hat{\xi}_t - E_t\hat{\xi}_{t+1}$ and moreover

$$\begin{aligned}
b_y &\equiv (\varrho\tau - (1-\beta)\omega\sigma^{-1}), \\
b_q &\equiv (1-\beta)(1-\omega)(1-\sigma_q^{-1}), \\
b_\xi &\equiv (1-\beta)(1-\omega).
\end{aligned}$$

D.5 A quadratic approximation to the policy objective function

Using the above derivations, we can now obtain a quadratic approximation to the policy objective function. To this end, we combine equation (D.42) and (D.47) in a way to eliminate the linear terms in (D.39). Indeed, we find ϑ_1, ϑ_2 such that

$$\vartheta_1 b'_x + \vartheta_2 c'_x = a'_x \equiv [0 \ \Phi \ \nu\delta^{-1}].$$

The solution is given by

$$\begin{aligned}
\vartheta_1 &= -\frac{\Phi}{\Gamma}, \\
\vartheta_2 &= \frac{\Phi(1-\tau)(\sigma^{-1}+\eta)}{\Gamma},
\end{aligned}$$

where

$$\Gamma = (1-\tau)(1+\eta) - (1-\sigma^{-1}(1-g)).$$

Note that the lagrange multiplier ϕ_4, given in (D.37), is such that $\phi_4 = -\vartheta_1$ and, therefore, given the first-order condition (D.34) it also follows that

$$\vartheta_1 \nu \delta^{-1}(1-\sigma_q^{-1}) = \nu \delta^{-1}.$$

We can, therefore, write

$$\begin{aligned} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Phi \hat{Y}_t &= E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\vartheta_1 b'_x + \vartheta_2 c'_x] x_t = \\ & -E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2} x'_t D_x x_t + x'_t D_\varepsilon \varepsilon_t + \frac{1}{2} d_\pi (\pi_t - \pi)^2] \\ & + \vartheta_1 \tilde{Z}_{t_0} + \vartheta_2 \kappa^{-1} V_{t_0} + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \end{aligned}$$

where

$$D_x \equiv \vartheta_1 B_x + \vartheta_2 C_x, \quad \text{etc.}$$

Hence

$$\begin{aligned} U_{t_0} &= \Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ a'_x x_t - \frac{1}{2} x'_t A_x x_t - x'_t A_\varepsilon \varepsilon_t - \frac{1}{2} a_\pi (\pi_t - \pi)^2 \right\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \\ &= -\Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2} x'_t \Delta_x x_t + x'_t \Delta_\varepsilon \varepsilon_t + \frac{1}{2} \lambda_\pi (\pi_t - \pi)^2 \right\} + \\ & \quad + X_{t_0} + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \\ &= -\Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2} \lambda_y \hat{Y}_t^2 - \lambda_g \hat{T}_t \hat{Y}_t + \lambda_q \hat{q}_t^2 + \frac{1}{2} \lambda_\pi (\pi_t - \pi)^2 \right\} + X_{t_0} + \\ & \quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{D.48}$$

In particular, we obtain that $\Omega = U_c Y$ and that

$$\lambda_y \equiv (1-\Phi)(\sigma^{-1}+\eta) + \Phi(\sigma^{-1}+\eta)\frac{(1-\tau)(1+\eta)}{\Gamma} + \frac{\Phi}{\Gamma}\sigma^{-1} g;$$

$$\lambda_q = \frac{\Phi}{\Gamma} \nu \delta^{-1} \sigma_q^{-1} (\sigma_q^{-1} - \tilde{\sigma}_q^{-1})$$

$$\lambda_g = \frac{\Phi}{\Gamma} g \sigma^{-1}$$

moreover we have defined

$$\lambda_\pi = \frac{\Phi\theta(1-\tau)(\sigma^{-1}+\eta)(1+\eta)}{\Gamma\kappa} + \frac{(1-\Phi)\theta(\sigma^{-1}+\eta)}{\kappa}.$$

Finally,

$$X_{t_0} \equiv U_c Y \cdot [\vartheta_1 \tilde{Z}_{t_0} + \vartheta_2 \kappa^{-1} V_{t_0}]$$

is a transitory component.

Therefore the loss function is given by

$$L_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2}\lambda_y y_t^2 + \frac{1}{2}\lambda_\pi (\pi_t - \pi)^2 + \frac{1}{2}\lambda_q q_t^2 \right\}.$$

in which the output gap is defined by $y_t = \hat{Y}_t - \hat{Y}_t^*$ with $\hat{Y}_t^* \equiv \lambda_y^{-1} \lambda_g \tilde{T}_t / g$.

D.6 A linear-quadratic approximation of the optimal policy problem

Before solving the optimal policy problem in the LQ approximation, we discuss the model equilibrium conditions in a log-linear approximation. The AS equation is given by

$$(\pi_t - \pi) = \kappa[\hat{Y}_t + \psi \hat{\tau}_t] + \beta E_t(\pi_{t+1} - \pi)$$

which can be rewritten also as

$$(\pi_t - \pi) = \kappa[y_t + \psi_\tau(\tilde{\tau}_t - \tilde{\tau}_t^*)] + \beta E_t(\pi_{t+1} - \pi),$$

in which we have defined $\psi_\tau = \psi/\tau$ and $\tilde{\tau}_t^* = -\psi_\tau^{-1} \hat{Y}_t^*$.

The AD block is given by

$$E_t \hat{Y}_{t+1} = \hat{Y}_t + \sigma(\hat{\imath}_t^B - E_t(\pi_{t+1} - \pi) - \tilde{r}_t^n) \tag{D.49}$$

$$\hat{q}_t = q_y \hat{Y}_t - q_i(\hat{\imath}_t^B - \hat{\imath}_t^D) + q_\xi \hat{\xi}_{q,t} \tag{D.50}$$

$$(1-\nu)\hat{\imath}_t^D = (\rho_\gamma - \nu)\hat{\imath}_t^R + (1-\rho_\gamma)\hat{\imath}_t^B - \nu \hat{\rho}_{\gamma,t}. \tag{D.51}$$

in which we have defined a variable with a hat as the log-deviations of the variable with respect to the steady state; $\pi_t \equiv \ln P_{t+1}/P_t$, $\tilde{r}_t^n = \hat{\xi}_t - E_t \hat{\xi}_{t+1}$ $\pi \equiv \ln \Pi$, $\sigma \equiv -U_c/(U_{cc}Y)$, $\sigma_q \equiv -V_q/(V_{qq}q)$, $q_y \equiv \sigma^{-1}/\sigma_q^{-1}$, $q_i = (1-\nu)/(\nu\sigma_q^{-1})$, $q_\xi = \sigma_q$.

Combining (D.49)–(D.51), we obtain the AD equation

$$y_t = (1 - \rho_\gamma^{-1}\nu) E_t y_{t+1} - \sigma(1 - \rho_\gamma^{-1}\nu)(\hat{\imath}_t^R - E_t(\pi_{t+1} - \pi) - r_t^n) + q_y^{-1} \rho_\gamma^{-1} \nu \hat{b}_t^g.$$

in which

$$r_t^n = \tilde{r}_t^n + \frac{1}{\sigma} E_t \hat{Y}_{t+1}^* - \frac{1}{\sigma(1-\rho_\gamma^{-1}\nu)} \hat{Y}_t^* - \frac{\rho_\gamma^{-1}\nu}{(1-\rho_\gamma^{-1}\nu)} (\hat{\xi}_{q,t} + (\sigma_q^{-1} - 1)\hat{\rho}_{\gamma,t}).$$

The intertemporal budget constraint of the government is given by

$$\hat{b}^g_{t-1} - (\pi_t - \pi) - \sigma^{-1}\hat{Y}_t + \hat{\imath}^R_{t-1} = E_t \sum_{T=t}^{\infty} \beta^{T-t}[b_y \hat{Y}_T + \varrho\tilde{\tau}_T - \varrho\tilde{T}_T + b_q(\hat{b}^g_t - \hat{\rho}_{\gamma,t}) + b_\xi \hat{\xi}_{q,t} - \beta\tilde{r}^n_T]$$

which can be written as

$$\hat{b}^g_{t-1} - (\pi_t - \pi) - \sigma^{-1}y_t + (\hat{\imath}^R_{t-1} - r^n_{t-1}) = -f_t + E_t \sum_{T=t}^{\infty} \beta^{T-t}[b_y y_t + \varrho(\tilde{\tau}_T - \tilde{\tau}^*_T) + b_q \hat{b}^g_T]$$

having define the fiscal-stress variable f_t

$$f_t = -E_t \sum_{T=t}^{\infty} \beta^{T-t}[b_y \hat{Y}^*_T + \varrho\tilde{\tau}^*_T - \varrho\tilde{T}_T - \beta\tilde{r}^n_T + b_\xi \hat{\xi}_{q,T} - b_q \hat{\rho}_{\gamma,T}] - r^n_{t-1}.$$

In the evaluation of the optimal policy when considering that the economy is hit by a shock to the natural real rate, we assume a zero fiscal stress at all times, meaning that the transfer policy adjusts so to keep $f_t = 0$ at all times.

The optimal policy problem in a linear-quadratic approximation minimizes the quadratic loss function

$$L_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2}\lambda_y y_t^2 + \frac{1}{2}\lambda_\pi(\pi_t - \pi)^2 + \frac{1}{2}\lambda_q q_t^2 \right\}$$

under the log-linear approximation of the equilibrium conditions:

$$(\pi_t - \pi) = \kappa[y_t + \psi_\tau(\tilde{\tau}_t - \tilde{\tau}^*_t)] + \beta E_t(\pi_{t+1} - \pi).$$

$$y_t = (1 - \rho_\gamma^{-1}\nu)E_t y_{t+1} - \sigma(1 - \rho_\gamma^{-1}\nu)(\hat{\imath}^R_t - E_t(\pi_{t+1} - \pi) - r^n_t) + q_y^{-1}\rho_\gamma^{-1}\nu\hat{b}^g_t,$$

$$\begin{aligned} \hat{b}^g_{t-1} - (\pi_t - \pi) - \sigma^{-1}y_t + \hat{\imath}^R_{t-1} - r^n_{t-1} \quad = \quad & b_y y_t + \varrho(\tilde{\tau}_t - \tilde{\tau}^*_t) + b_q \hat{b}^g_t + \\ & + \beta E_t[\hat{b}^g_t - (\pi_{t+1} - \pi) - \sigma^{-1}y_{t+1} + \hat{\imath}^R_t - r^n_t]. \end{aligned}$$

First-order conditions with respect to $\hat{Y}_t, \pi_t, \hat{\tau}_t, \hat{\imath}^R_t$ and $\hat{b}^g_t$ are given respectively by

$$\lambda_y y_t - \kappa\phi_{1,t} + \phi_{2,t} - \beta^{-1}(1 - \rho_\gamma^{-1}\nu)\phi_{2,t-1} - \sigma^{-1}(\phi_{3,t} - \phi_{3,t-1}) - b_y\phi_{3,t} = 0$$

$$\lambda_\pi(\pi_t - \pi) + \phi_{1,t} - \phi_{1,t-1} - \sigma(1 - \rho_\gamma^{-1}\nu)\beta^{-1}\phi_{2,t-1} - (\phi_{3,t} - \phi_{3,t-1}) = 0$$

$$-\kappa\psi_\tau\phi_{1,t} - \varrho\phi_{3,t} = 0$$

$$\sigma(1 - \rho_\gamma^{-1}\nu)\phi_{2,t} + \beta(E_t\phi_{3,t+1} - \phi_{3,t}) - \phi_{4,t} = 0$$

$$\lambda_q \hat{q}_t - q_y^{-1}\rho_\gamma^{-1}\nu\phi_{2,t} - b_q\phi_{3,t} + \beta(E_t\phi_{3,t+1} - \phi_{3,t}) = 0$$

in which $\phi_{4,t}$ is the lagrange multiplier associated to the zero-lower bound constraint

$$(\hat{\imath}_t^R + \ln(1 + i^R)) \geq 0$$

with $\phi_{4,t} \geq 0$.

E Calibration

We calibrate the model parameters as in the following table:

Table 1: Calibration of parameters

$\beta = 0.995$	$\kappa = 0.02$
$\sigma = 0.5$	$g = 0.2$
$\eta = 0.47$	$\nu = 0.003125$
$\theta = 10$	$\Pi = 1 + 0.02/4$
$\rho_\gamma = 0.21$	

The intertemporal elasticity of substitution in consumption σ is set to 0.5; the inverse of the Frisch elasticity of labor supply is set to $\eta = 0.47$; the elasticity of substitution among the varieties of goods in the consumption basket is set to $\theta = 10$; the slope of the AS equation is set to $\kappa = 0.02$. All the above calibration is taken from Eggertsson and Woodford (2003). The gross inflation rate Π is set to be consistent with an inflation target of 2% at annual rates. The rate of time preference is set to $\beta = 0.995$ so that the steady-state real interest rate is at 2% at annual rates. The parameter ρ_γ is calibrated at 0.21, which is the average of the ratio between liquid assets and deposit of FDIC-Insured Commercial Banks and Savings Institutions in the U.S during the period 1984 Q1 to 2005 Q4. Data are taken from FRED Database. Liquid assets include U.S. Treasury Securities (the series QBPBSTASSCUSTRSC), Federal Funds (the difference between those sold QBPBSTASFEDREVREPO and those purchased QBPBSTLKFEDREPO), mortgage-backed securities (QBPBSTASSCMRTSEC) and Cash and Due from Depositary Institution (QBPBSTASCSHDP). Deposits are the series QBPBSTLKDP. The parameter g is set equal to 0.2, indicating a 20% of public spending over GDP.

The spread between risk-free illiquid and liquid securities, ν, is calibrated as the average of the Moody's Seasoned Aaa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity (series TB3MS from the FRED database), for the period 1991 M1 to 2005 M12. Since this average is equal to 125 basis points at annualized rates, then $\nu = 0.003125$.

Note that

$$
\begin{aligned}
V_q(q) &= \tfrac{1}{q} - \tfrac{1}{q^*} \qquad &\text{for} \quad q < \bar{q} \\
&= 0 \qquad &\text{for} \quad q \geq \bar{q}
\end{aligned}
$$

$$
\begin{aligned}
V_{qq}(q) &= -\tfrac{1}{q^2} \qquad &\text{for} \quad q < \bar{q} \\
&= 0 \qquad &\text{for} \quad q \geq \bar{q}
\end{aligned}.
$$

Therefore,

$$\nu = \frac{V_q(q)}{U_c(Y)} = \left(\frac{1}{q} - \frac{1}{\bar{q}}\right) Y^{\sigma^{-1}}.$$

The following set of equations is solved to obtain $q, Y, \tau, \bar{\tau}, \bar{q}, \bar{Y}, \lambda$

$$\nu = \left(\frac{1}{q} - \frac{1}{\bar{q}}\right) Y^{\sigma^{-1}}$$

$$\frac{(1-\beta)}{\beta}\rho_\gamma \bar{q} = \bar{Y}(\bar{\tau} - g)$$

$$\bar{Y} = \left(\frac{(1-\bar{\tau})}{\mu_\theta}\right)^{\frac{1}{\eta+\sigma^{-1}}}$$

$$Y = \left[\frac{(1-\tau)}{\mu_\theta}\right]^{\frac{1}{\eta+\sigma^{-1}}}$$

$$\lambda = \frac{1 - \frac{(1-\tau)}{\mu_\theta}}{(1+\eta)(1-\tau) - (1-\sigma^{-1}) - \sigma^{-1}g}$$

$$(\tau - g)Y + \frac{\nu}{\beta} = \frac{(1-\beta)}{\beta}\rho_\gamma q$$

$$\frac{\lambda}{1+\lambda} = 1 - \frac{q}{\bar{q}},$$

given the other parameters. The parameter δ is determined by $\delta = Y/q$. The elasticity of substitution σ_q is equal to

$$\sigma_q = -\frac{V_q}{V_{qq}q} = \left(1 - \frac{q}{\bar{q}}\right).$$

Moreover

$$1 + \tilde{\sigma}_q^{-1} = -V_{qqq}q/V_{qq} = 2,$$

Therefore $\tilde{\sigma}_q = 1$.

The following Table cointains the value of the parameters derived through the above procedure:

Table 2: Derived parameters

$$\begin{array}{cc} q = 36.68 & \bar{q} = 43.47 \\ \tau = 0.2415 & Y = 0.8568 \\ \sigma_q = 0.1562 & \tilde{\sigma}_q = 1 \\ \delta = 0.0234 & \end{array},$$

The other parameters in the optimal policy problem can then be derived given their definitions:

$$\begin{aligned}
q_y &= \sigma_q/\sigma \\
q_\xi &= \sigma_q \\
\varrho &= \frac{\beta\delta}{\rho_\gamma(1-\nu/\rho_\gamma)} \\
\Phi &= 1 - \frac{(1-\tau)}{\mu_\theta} \\
\psi_\tau &= \frac{1}{(1-\tau)}\frac{1}{\sigma^{-1}+\eta} \\
\omega &= \frac{(\tau-g)\delta}{(\tau-g)\delta+\nu} \\
\Gamma &= (1-\tau)(1+\eta) - (1-\sigma^{-1}(1-g)) \\
\lambda_y &\equiv (1-\Phi)(\sigma^{-1}+\eta) + \Phi(\sigma^{-1}+\eta)\frac{(1-\tau)(1+\eta)}{\Gamma} + \frac{\Phi}{\Gamma}\sigma^{-1}g \\
\lambda_q &= \frac{\Phi}{\Gamma}\nu\phi^{-1}\sigma_q^{-1}(\sigma_q^{-1}-\tilde{\sigma}_q^{-1}) \\
\lambda_\pi &= \frac{\Phi\theta(1-\tau)(\sigma^{-1}+\eta)(1+\eta)}{\Gamma\kappa} + \frac{(1-\Phi)\theta(\sigma^{-1}+\eta)}{\kappa} \\
b_y &= (\varrho\tau - (1-\beta)\omega\sigma^{-1}) \\
b_q &= (1-\beta)(1-\omega)(1-\sigma_q^{-1}) \\
b_\xi &= (1-\beta)(1-\omega).
\end{aligned}$$

When instead ν is calibrated at 0.05 then the parameters change to

Table 3: Derived parameters when $\nu = 0.01$

$q = 10.47$	$\bar{q} = 12.12$
$\tau = 0.2011$	$Y = 0.8750$
$\sigma_q = 0.1367$	$\tilde{\sigma}_q = 1$
$\delta = 0.0836$	