

It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve*

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Abstract

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We develop a New Keynesian model with search-and-matching frictions that microfound a nonlinear inverse- L (Inv- L) Phillips curve. The model yields a piecewise-linear relationship whose kink is defined by a time-varying Beveridge threshold in labor-market tightness, $\theta_t = v_t/u_t$. When tightness exceeds this threshold, the Phillips-curve slope and the pass-through of supply shocks rise sharply, providing a mechanism for the post-pandemic inflation surge and its subsequent moderation without a deep recession. Using U.S. quarterly data from 1960 to 2024, we estimate the inverse- L Phillips curve implied by the model and identify the threshold empirically near unity. Above this point, labor-market tightness and supply disturbances exert strong, statistically significant effects on inflation, while below it their impact is muted. The results help explain why policymakers and forecasters underestimated the magnitude of the recent inflation surge. More generally we propose a unified framework to understanding all six U.S. inflation surges since the establishment of the Federal Reserve (five via labor shortage, one via expectations).

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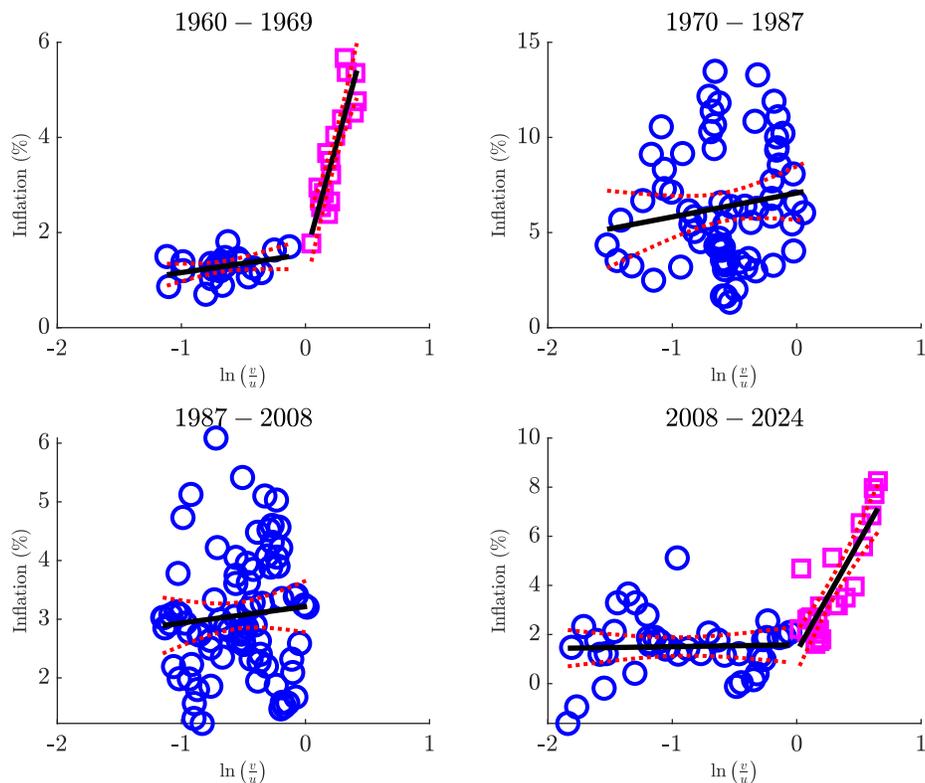


Figure 1: Inflation and Labor Market Tightness in the United States, 1960–2024. The y-axis represents the annualized inflation rate (CPI inflation), while the x-axis represents $\ln(\frac{v}{u})$, the natural logarithm of the vacancy-to-unemployment ratio.

1 Introduction

We extend the canonical New Keynesian (NK) Phillips curve by proposing an inverse- L (Inv- L) New Keynesian Phillips curve.¹ The distinctive shape of the Inv- L curve—a slanted, backward L —captures the sharp rise in U.S. inflation following the COVID-19 pandemic in early 2021 and its subsequent moderation by late 2024. Neither policymakers nor market participants anticipated the magnitude or persistence of this surge (see Figure 8). We argue that the Inv- L NK Phillips curve explains both the forecasting failures and the observed inflation dynamics, and carries important policy implications.

The conventional NK Phillips curve struggles to explain the 2020s inflation episode for three reasons. First, pre-surge estimates indicate a flat, linear relationship, suggesting that economic slack or tightness had little effect on inflation. Second, conventional supply-shock measures show only modest variation during the surge (see Figure 10), predicting negligible effects on core inflation. Third, while conventional wisdom attributes the Great Inflation of the 1970s to unanchored expectations, inflation expectations in the 2020s remained remarkably stable (see Figure 3). Together, these features leave the standard model without an obvious explanation for the post-pandemic surge.

¹See Woodford (2003) and Galí (2016) for textbook treatments.

Our Inv- L NK Phillips curve addresses these challenges. It introduces a nonlinearity that amplifies both demand and supply shocks when labor markets are sufficiently tight – what we term “labor shortages.” This mechanism explains both the onset of the surge and its subsequent moderation.

More broadly, the theoretical framework proposed in this paper provides a unified interpretation of the six major inflation surges that have occurred since the Federal Reserve began operations in 1914. These episodes are documented in Section 2. The unifying theme across five of these episodes is the emergence of *labor shortages*. The exception is the Great Inflation of the 1970’s, which we attribute to unanchored inflation expectations.

Our analysis is motivated by Figure 1, inspired by the seminal work of Phillips (1958). Whereas Phillips related nominal wage growth to unemployment in the United Kingdom, our figure plots annualized quarterly inflation against the U.S. vacancy–unemployment ratio (v/u) from 1960 to 2024, divided into four subperiods. The pattern reveals a distinct inverse- L shape: whenever labor-market conditions exceed a certain threshold close to $v/u = 1$ ($\log(v/u) = 0$), inflation rises sharply with tightness but remains flat otherwise. Purple squares, observed in 1960–1969 and 2008–2024, mark periods of *labor shortage* when v/u crosses this threshold, while blue circles indicate normal conditions below it.

The threshold defining this kink in the Inv- L curve has a clear interpretation. Beveridge (1941) defines $v/u = 1$ as the point of labor market balance—when the number of unemployed workers equals the number of vacant jobs. We refer to the inflection point as the *Beveridge threshold*.

Our first contribution is theoretical. We develop a New Keynesian model that incorporates v/u directly into firms optimal price-setting decisions, drawing on the search and matching literature.² This modification changes the structure of the NK Phillips curve: the relevant measure of economic activity is no longer the output gap or the unemployment rate, but rather labor-market tightness v/u . Moreover, wage-setting in the spirit of Phillips original proposal introduces a nonlinearity that generates the Inv- L shape. The framework produces two main theoretical results. First, the Inv- L NK Phillips curve is flat at low levels of labor market tightness but becomes steep once the Beveridge threshold is crossed. This implies that both supply shocks and labor-market tightness significantly affect inflation in this regime. Second, the threshold itself is endogenous; depending on structural parameters, it can differ from the unitary value and may vary over time due to shocks, such as shifts in matching efficiency and hiring costs.

Our second contribution is empirical. Building on recent work showing that the vacancy–unemployment ratio better captures cyclical conditions than the unemployment rate,³ we estimate the piecewise-linear specification implied by our model and jointly determine the kink point. The results provide formal evidence that the breakpoint lies near $v/u = 1$.⁴

Empirically, crossing the Beveridge threshold markedly amplifies the effects of both v/u and supply shocks on inflation. In our baseline specification, the slope of the Phillips curve with respect to

²See Pissarides (2003) for an overview.

³See Subsection 1.1 for discussion.

⁴Michaillat and Saez (2022) present a model in which this threshold corresponds to the efficient allocation of labor.

v/u increases by roughly a factor of ten once the threshold is exceeded. Below the threshold, supply shocks have a negligible effect on core inflation; above it, their impact becomes large and statistically significant. Recognizing these nonlinearities is crucial for understanding the 2021–2024 episode: ignoring them helps explain why forecasters and policymakers systematically underestimated both the magnitude and persistence of the inflation surge (see Figure 8).

The remainder of the paper proceeds as follows. The next subsection connects our work to existing literature. Section 2 provides a summary of the relationship between labor-market tightness and inflation since the Federal Reserve started operating in 1914. Sections 3 and 4 develop the theoretical model and define the Inv- L NK Phillips curve. Section 5 presents the empirical estimation. Section 6 concludes.

1.1 Related literature

Nonlinear aggregate supply has several antecedents. Blinder’s (2022) “crude Keynesianism” and Friedman’s (1964, 1993) “plucking model” both imply that output is constrained by capacity, while Dupraz, Nakamura, and Steinsson (2019) provide supporting evidence based on downwardly rigid wages. Similar mechanisms arise in secular-stagnation frameworks (Eggertsson, Mehrotra, and Robins 2019) and in models with wage rigidities (Benigno and Ricci 2011; Schmitt-Grohé and Uribe 2016, 2024).

Relative to this literature, our contribution is to embed nonlinearity in a forward-looking New Keynesian model by introducing asymmetric adjustment of new-hire wages and making labor-market tightness $\theta \equiv v/u$ the central driver of inflation dynamics. The framework generates an inverse- L (piecewise-linear) Phillips curve that becomes steeper once the Beveridge threshold is crossed. Incorporating forward looking price setting is critical for accounting for the Great Inflation of the 1970s.

Our work connects to the literature that integrates search and matching into monetary models (e.g., Walsh 2005; Krause and Lubik 2007; Ravenna and Walsh 2008, 2011; Gertler, Sala, and Trigari 2008; Krause, López-Salido, and Lubik 2008; Barnichon 2010; Michaillat 2012). We introduce three tractable innovations: (i) marginal cost equals the *new-hire* (not average) wage, along with time varying hiring costs associated with new hires; (ii) an “employment-agency” device that establishes recruiting wages and resolves wage indeterminacy (Hall 2005); and (iii) asymmetric wage adjustment—new-hire wages may exceed those of incumbents’ in tight markets but do not fall below them in slack markets. Ravenna and Walsh (2008) incorporate v/u into the NKPC through vacancy costs under linearity; our mechanism generates nonlinearity and the Beveridge threshold.

Time-series estimates often reveal a flat slope using traditional slack measures (e.g., Blanchard et al., 2015; McLeay and Tenreyro, 2019; Ball et al., 2022; Barnichon and Shapiro, 2022; Furman and Powell, 2021; Gordon, 1977, 2013). A growing body of evidence shows that labor-market tightness outperforms unemployment and output-gap measures (Barnichon, Oliveira, and Shapiro 2021; Furman and

Powell 2021; Domash and Summers 2022; Ball, Leigh, and Mishra 2022). Several papers emphasize nonlinearity or time variation in slope (Gagnon and Collins, 2019; Benati, 2010; Blanchard et al., 2015; Blanchard, 2016; Matheson and Stavrev, 2013). Our contribution is a parsimonious, theory-guided piecewise specification with a single Beveridge threshold, rather than flexible polynomials (cf. Ball, Leigh, and Mishra 2022). Cross-sectional work reinforces the threshold view: the Phillips curve appears flat when $\log \theta < 0$ (Hazell, Herreño, Nakamura, and Steinsson 2022) but steepens during the post-2020 tightness episode (Cerrato and Gitti 2023; Gitti 2023; Smith, Timmermann, and Wright 2023).⁵

Gagliardone and Gertler (2023) attribute the surge of the 2020s primarily to energy shocks combined with lenient policies. Harding, Lindé, and Trabandt (2023) generate nonlinearities through quasi-kinked demand for goods (Kimball 1995). Comin, Johnson, and Jones (2023) highlight capacity constraints that lead to convex supply curves. Our framework, by contrast, ties the nonlinearity directly to labor shortages: both demand shocks that tighten the market and conventional supply shocks amplify inflation effects once the Beveridge threshold is crossed—consistent with Figure 1.

Guerrieri, Lorenzoni, Straub, and Werning (2022) provide a distinct source of state dependence. In their heterogeneous-sector model, negative sectoral supply shocks can induce aggregate demand shortages when reallocation across sectors is limited and monetary policy is constrained. Their mechanism generates nonlinear aggregate responses of output and inflation to sectoral disturbances, even in the absence of labor-market frictions. By contrast, nonlinearity in our framework originates from the labor market itself: tightness amplifies both demand and supply shocks through asymmetric wage-setting and the endogenous Beveridge threshold. The two approaches are complementary—their highlights how limited reallocation and demand spillovers can turn supply shocks into demand-driven recessions, while ours emphasizes how labor shortages magnify inflationary pressures of both demand and supply shocks.

Bernanke and Blanchard (2024) argue that supply shocks largely accounted for the surge’s onset, while labor-market tightness became increasingly relevant for its persistence. We reconcile this view with our own by noting that average wages and the labor share declined early in the 2020s, while posted and new-hire pay rose markedly (e.g., Crump, Eusepi, Giannoni, and Şahin 2024). Since marginal costs in our model depend on *new-hire* rather than average wages—together with other components of hiring costs for new employees—tightness helps explain inflationary pressures even when average wages lag. Thus, while their framework emphasizes the initial role of supply shocks, our focus on recruiting wages highlights how labor-market pressures persistently contributed to inflation throughout the episode.

Afrouzi, Blanco, Drenik, and Hurst (2025) consider a model in which inflation is treated as an exogenous process. They demonstrate that an unexpected increase in the price level raises on-the-job search, job-to-job flows, and vacancies, thereby shifting the Beveridge curve upward. Our framework endogenizes inflation: once tightness exceeds the Beveridge threshold, the slope of the Phillips curve

⁵For surveys on identification and estimation, see Mavroeidis, Plagborg-Møller, and Stock (2014) and Coibion, Gorodnichenko, and Kamdar (2018). McLeay and Tenreyro (2019) provide a clear overview.

steepens, triggering an inflation surge. The two mechanisms are complementary and mutually reinforcing. Shocks that generate sufficient tightness in our model produce an inflation surge. This surge activates their channel, amplifying labor market tightness. The amplified tightness then triggers additional inflation, further activating their mechanism. This feedback loop suggests an acceleration mechanism. Their granular predictions for quits and vacancy durations complement our aggregate general-equilibrium account of recurrent U.S. inflation surges.

Beaudry, Huo, and Portier (2024, 2025) emphasize that the inflation surge can be explained without a change in slope, with expectations and supply shocks playing dominant roles. In our empirical results, we find that this alternative interpretation is consistent with the data when using one-year consumer inflation expectations from the Michigan Survey of Consumer Expectations. By contrast, our baseline employs the Survey of Professional Forecasters' one-year CPI inflation expectations and demonstrates that our findings are robust across several alternative measures of inflation expectations.⁶

The Beaudry–Huo–Portier interpretation remains a viable and coherent alternative to our theory. One advantage of our narrative is that it explains *why* both markets and policymakers were surprised by the magnitude of the inflation surge in the 2020s, as illustrated in Figure 8—a central question of our paper. The narrative record from this period suggests that the rise and persistence of inflation came as a surprise.⁷ This is captured by the expectation measures we employ. A further strength of our framework is that it provides a unified account of the six major inflation surges in U.S. data since 1914 and clarifies why the Great Inflation of the 1970s fundamentally differs from the other five.

2 U.S. Inflation Surges since the Establishment of the Federal Reserve in 1914

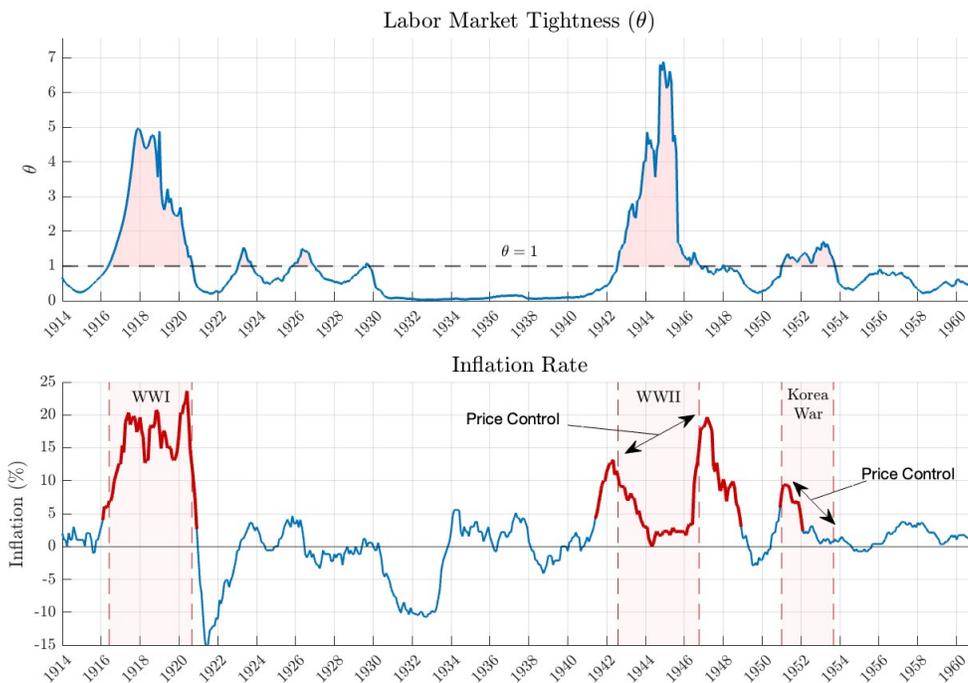
Since the Federal Reserve began operations in 1914, six major *inflation surges* have occurred under its tenure. The lower subplots in each panel of Figure 2 show inflation from 1914 to 2024, measured by the percentage difference in the CPI index relative to the previous year. The line depicting inflation turns from blue to red during inflation surges.

Any definition of an inflation surge is somewhat arbitrary. A practical approach defines inflation surges as periods when inflation persistently exceeds 4 percent annually, with an average inflation rate of at least 5 percent. We list the six inflation surges that satisfy this criteria in Table 1.⁹ The surges correspond to WWI, WWII, the Korean War, the Vietnam War, President Johnson's tax cuts in the

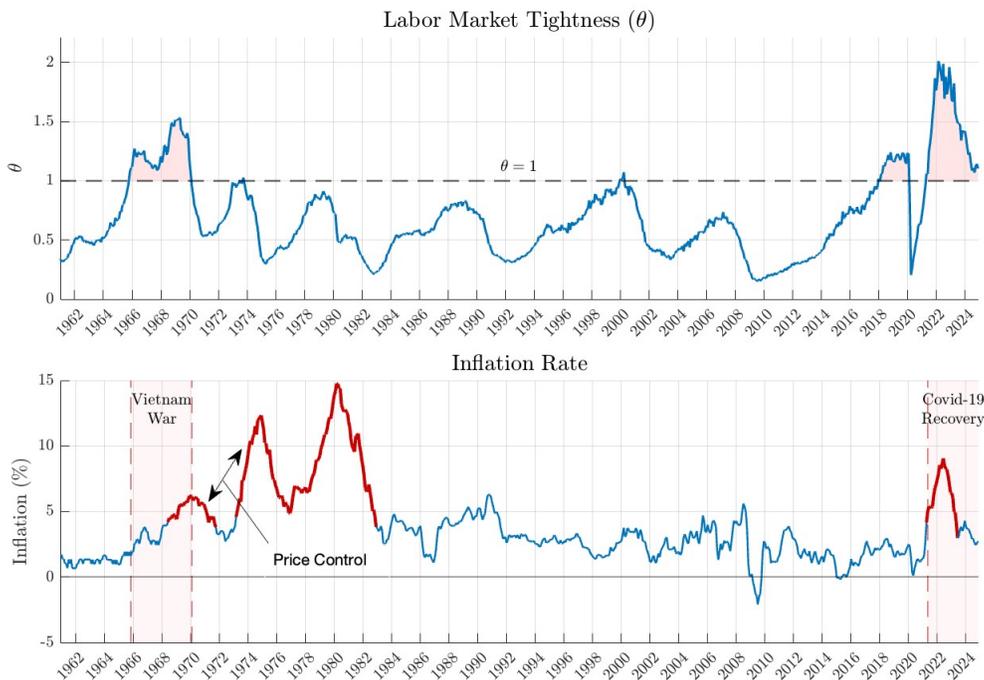
⁶Our alternatives—Cleveland Fed expectations at different horizons, the Livingston Survey, and market-based backcasts (Groen and Middeldorp 2013)—support our estimates. Additionally, see Chodorow-Reich (2024) for a discussion on why these measures are better suited to proxy firms pricing decisions than the Michigan Survey.

⁷This can be established based on market commentary, media analysis, and policymakers' speeches during this time. We have preliminary work in progress documenting this, available upon request.

⁹Peak values use quarterly averages to avoid idiosyncratic monthly spikes.



Panel (a): CPI inflation (annual rates, bottom) and vacancy–unemployment ratio θ (top), 1914Q1–1959Q4. Source: Petrosky-Nadeau and Zhang (2021), Authors’ computation, and BLS.



Panel (b): Vacancy–unemployment ratio θ (bottom) and CPI inflation (annual rates, top), 1960Q1–2024Q4. Source: Petrosky-Nadeau and Zhang (2021), Barnichon (2010), and BLS.

Figure 2: CPI inflation and vacancy–unemployment ratio, 1914–2024.

Table 1: U.S. Inflation Surges Since the Federal Reserve Started Operation in 1914

Episode	Start	End	Duration (months)	Mean π (%)	Peak π (%)	Mean θ	Peak θ
WWI Surge	Aug 1915	Dec 1920	65	12.7	20.2	2.74	5.83
WWII Surge (price controls)	Jan 1941	Dec 1948	96	6.8	17.3	2.03	6.78
Korean War Surge	Dec 1950	Jan 1952	14	7.2	8.7	1.17	1.33
Late 1960s Surge	Jun 1968	Sep 1971	40	5.1	5.9	1.03	1.51
1970s Great Inflation	Mar 1973	Nov 1982	117	8.5	13.5	0.60	0.99
2020s Surge	Apr 2021	May 2023	26	6.4	8.3	1.66	1.91

Notes: Inflation surges are identified as contiguous periods with year-over-year CPI inflation exceeding 4% for at least 12 months and averaging at least 5%. $\theta \equiv v/u$ denotes labor-market tightness (vacancies over unemployment). Mean values are monthly averages over each episode. ⁸ Price controls temporarily contained inflation during WWII, but we treat this inflation surge as one.

late 1960s, the Great Inflation of the 1970s, and the inflation surge of the 2020s in the aftermath of Covid-19.

When inflation is compared to labor market tightness, the upper subplots in each panel of Figure 2 reveal a striking pattern: In five of the six inflation surges, labor market tightness, as measured by θ , exceeds the unitary value.¹⁰ This pattern is summarized in Table 1 which shows that the average θ is above 1 in five surges, often peaking at a substantially higher value. The exception is the 1970s Great Inflation, where θ averaged 0.6, consistent with the idea that the Great Inflation of the 1970's was due to unanchored inflation expectations rather than labor market tightness. Interestingly, the 2020s period of labor market shortage, both in average and peak values, is only eclipsed by the two World Wars.

The six surges vary in duration. The Korean War is associated with the shortest surge, lasting only 14 months. The Great Inflation generated the longest inflation surge, which lasted almost 10 years (9 years and 9 months).

Table 2: Inflation Surges Since 1914: Some Summary Statistics

	Sample size (months)			Mean Inflation (%)		Variance of Inflation	
	N_{all}	$N(\theta \geq 1)$	$N(\theta < 1)$	$\theta \geq 1$	$\theta < 1$	$\theta \geq 1$	$\theta < 1$
Full sample (1914–2024)	1,332	288	1,044	6.2	2.5	31.7	19.2
Excluding 1973–1982	1,212	286	926	6.2	1.7	31.9	14.9

Note: CPI inflation is year-over-year percent; variances are of the monthly inflation rates. Reported data incorporates backcasted theta estimates for 1914–1918.

Observing high inflation alongside a slack labor market during the 1970s surprised many economists. These dynamics were prominently predicted by Phelps (1967) and Friedman (1968) during the inflationary surge of the 1960s when unemployment was at record lows. They warned that once inflation

¹⁰For 1914–1918, we impute θ using a World War II inflation-tightness relationship, as vacancy data begin only in 1919. While imperfect, we prefer using the backcasting, since the inflation data covers the entire period, giving a clear picture of the the WWI inflation surge. See Appendix C for details.

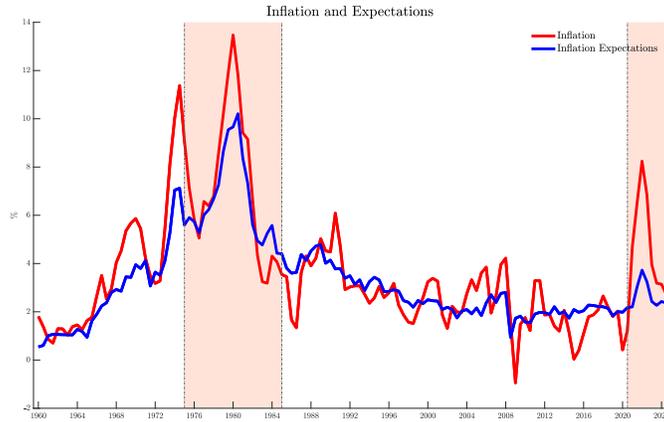


Figure 3: CPI inflation rate (annual, left axis) and 12-month Livingston survey inflation expectations (right axis).

expectations adjusted, expansionary monetary policy would have limited impact. Reacting to a negative supply shock, for example, with monetary expansion would lead to higher inflation with no improvement in unemployment as expectations adjust. The simultaneous increase in inflation and unemployment during the 1970s introduced a new term into the economists' lexicon: stagflation. The 1970s stagflation triggered a significant methodological shift in macroeconomics, with greater emphasis on microfoundations and model-consistent expectations.

Labor market shortage, defined as $\theta > 1$, represents 22 percent of the time using monthly data from 1914-2024 (see Table 2). Average inflation during these periods is about two and a half times greater than during periods of labor market slackness (6.2% vs. 2.4%). Another property of the data is that the variance of inflation is substantially higher during periods of labor market shortage. This becomes especially evident once we omit the period of the Great Inflation. Crossing the Beveridge threshold is associated with substantial increases in both average inflation and its volatility, which will be a prediction of our framework.

These broad empirical patterns, together with Figure 1, provide important motivation for our analysis. They suggest that periods of labor shortages are typically associated with significant inflationary pressures and variability in inflation. At the same time, our framework must account for the Great Inflation, a period when labor markets were not unusually tight, yet inflation surged. The unifying element is the role of inflation expectations: during the 1970s expectations became unanchored, allowing inflation to rise even without labor-market pressure. By contrast, in the recent surge expectations remained relatively well anchored. Figure 3 illustrates this contrast, showing the sharp rise in Livingston survey inflation expectations in the 1970s compared with their stability during the 2020s episode.

3 The Model

In this section we present a minimalistic theoretical framework designed to capture the six U.S. inflation surges since 1914. We extend the canonical New Keynesian model in three dimensions. First, firms' marginal costs depend on the wages of newly hired rather than incumbent workers. Second, we tractably incorporate labor market tightness, θ_t . Third, wage-setting generates a Beveridge threshold, which implies a piecewise log-linear Phillips curve. Crossing the Beveridge threshold accounts for five of the six inflation surges, while forward-looking expectations explain the Great Inflation of the 1970s.

3.1 Households

There is a continuum of households of measure one. Household j 's utility is isoelastic in the consumption good:

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \zeta_T \left(\frac{C_T(j)^{1-\sigma}}{1-\sigma} \right), \quad (1)$$

where E_t is the expectation operator conditional on information at time t , $C_t(j)$ denotes consumption, $\beta \in (0, 1)$ is the discount factor, and ζ_t is a preference shock.

Each household j has a constant number of members participating in the labor force, F , who can be employed or unemployed at a given time:

$$F = N_t(j) + U_t(j), \quad (2)$$

where $N_t(j)$ denotes employed workers and $U_t(j)$ unemployed workers in household j at time t .¹¹

In period $t - 1$, $N_{t-1}(j)$ workers of household j are employed by firms. At the beginning of period t , a fraction s_t of these workers separate from firms and search for a job in the same period. The remaining fraction enters a pool of "existing" workers and, as we will show, in equilibrium, they will be retained by a firm at the existing wage.¹²

We denote by $N_t^{ex}(j)$ the workers of household j that are in the "existing" worker pool at time t :

$$N_t^{ex}(j) = (1 - s_t) N_{t-1}(j), \quad 0 < s_t < 1.$$

Incumbent workers are paid wage W_t^{ex} , which may differ from the wage paid to new hires, denoted W_t^{new} . Total employment for household j can then be written as

$$N_t(j) = N_t^{ex}(j) + N_t^{new}(j), \quad (3)$$

¹¹An earlier version of this work contained in our NBER Working Paper No. 31197 extends the framework to endogenous labor force participation and shows that the main results are unchanged.

¹²The feature of considering the re-employment of "existing" workers rather than treating them as attached to firms enables us to simplify firms' labor demand optimization problem, making our framework highly tractable.

where $N_t^{new}(j)$ denotes new hires. In Section 3.4 we describe how these hires are generated.

The household's budget constraint is

$$B_t(j) + P_t C_t(j) + T_t = (1 + i_{t-1})B_{t-1}(j) + W_t^{ex} N_t^{ex}(j) + W_t^{new} N_t^{new}(j) + Z_t^F + Z_t^E + P_t q_t \bar{O}_t, \quad (4)$$

where $B_t(j)$ is a risk-free nominal bond paying gross nominal interest rate $1 + i_t$, P_t is the consumption price index, T_t are lump-sum taxes, and Z_t^F and Z_t^E are the profits of firms and employment agencies, distributed equally across households. Finally, $P_t q_t \bar{O}_t$ represents revenues from selling the exogenous intermediate input O_t at real price q_t , which can be interpreted as oil.¹³

Household j maximizes (1) subject to (4) by choosing $\{C_t(j), B_t(j)\}$, taking all aggregate variables as given.

The optimal consumption Euler equation is

$$C_t^{-\sigma} = \beta(1 + i_t) E_t \left\{ C_{t+1}^{-\sigma} \Pi_{t+1}^{-1} \right\}, \quad (5)$$

where $\Pi_t \equiv P_t / P_{t-1}$ is gross inflation.

The household's intertemporal budget constraint must hold with equality.¹⁴

3.2 Firms

We consider a standard New Keynesian model with monopolistic competition and Rotemberg price adjustment costs. Firms set optimal prices taking wages as given. Relative to the canonical NK setup, we introduce two elements important for our results, the first novel, the second standard: (i) marginal cost depends on new-hire wages and the hiring fee $(1 + \gamma_t^b)$ rather than on average wages; (ii) an intermediate input (e.g., oil) enters production, affecting marginal cost.

Like households, firms face different wages for existing versus new workers. This constrains firms: they cannot hire at the existing wage beyond the current workforce of "existing" workers. When labor demand exceeds incumbents, the marginal worker is a new hire—this mechanism generates the nonlinearity in the Phillips curve, creating an abrupt kink in marginal costs when the two wage rates differ. Both households and firms take wages as given; their determination is addressed in the next section.

There is a continuum of firms of measure one, each producing a good of variety i . The sources of demand are household and government consumption, C_t and G_t , plus the cost $\gamma_t^c V_t$ of posting vacancies, measured in units of the consumption good, where $\gamma_t^c > 0$ is stochastic. We assume that

¹³The overall supply of the intermediate input is exogenous at \bar{O}_t .

¹⁴This is a necessary condition for optimality. Equivalently, one can impose a transversality condition; see Woodford (2003) for a discussion.

government spending and the vacancy cost $\gamma_t^c V_t$ (see Section 3.4) take the same form as the Dixit–Stiglitz consumption basket. Therefore a representative firm i faces the following demand for its output:

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\epsilon_t} (C_t + G_t + \gamma_t^c V_t), \quad (6)$$

where $p_t(i)$ is the price of variety i . Firms use labor and an intermediate input to produce goods according to

$$y_t(i) = A_t N_t(i)^\alpha O_t(i)^{1-\alpha}, \quad (7)$$

with $0 < \alpha < 1$, where A_t is productivity, $N_t(i) = N_t^{ex}(i) + N_t^{new}(i)$ is the labor employed by firm i , and $O_t(i)$ is the intermediate input (e.g., oil).

The firm's discounted value of current and expected future profits is

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \left\{ p_T(i) y_T(i) - W_T^{ex} N_T^{ex}(i) - (1 + \gamma_T^b) W_T^{new} N_T^{new}(i) - P_T q_T O_T(i) - \frac{\zeta}{2} \left[\left(\frac{p_T(i)}{p_{T-1}(i)} \right) \frac{1}{\Pi} - 1 \right]^2 P_T Y_T \right\}, \quad (8)$$

where $Q_{t,T} \equiv \beta^{T-t} (C_T^{-\sigma} / P_T) / (C_t^{-\sigma} / P_t)$ is the stochastic discount factor households use at time t to value future nominal income at time T . As in Rotemberg (1982), ζ measures the cost of adjusting prices relative to the inflation target Π .¹⁵ The term γ_t^b represents the fee on the wage bill of new workers paid to the employment agency. It can alternatively be interpreted as a per-worker hiring cost paid internally by the firm, measured in units of the consumption good.

The firm cannot hire more existing workers at the incumbent wage than are available in the existing pool:

$$N_t^{ex}(i) \leq N_t^{ex} = \int_0^1 N_t^{ex}(j) dj, \quad (9)$$

where the right-hand side represents the aggregate pool of existing workers. Firms can retain or dismiss these workers freely. To expand employment beyond this pool, they must hire new workers, subject to

$$N_t^{new}(i) \geq 0. \quad (10)$$

Thus total labor employed by firm i is

$$N_t(i) = N_t^{ex}(i) + N_t^{new}(i). \quad (11)$$

The firm chooses all variables indexed by i to maximize (8), subject to (6), (7), (9), (10), and (11). The first-order conditions are shown in Appendix D. We focus on an equilibrium in which constraint (9) is binding while (10) is not, so that

$$N_t(i) > N_t^{ex}.$$

¹⁵Calvo pricing leads to the same AS equation in a first-order approximation. We use Rotemberg costs to simplify the exposition.

As shown in the Appendix, a sufficient condition for firms to always retain their existing workforce before hiring new workers is

$$(1 + \gamma_t^b) W_t^{new} > W_t^{ex}.$$

Since new and existing workers are perfect substitutes, firms prefer to keep existing workers whenever the cost of hiring a new worker exceeds the wage of incumbents. The key implication is that whenever $N_t(i) > N_t^{ex}$, the marginal cost of production depends only on the wage of new workers and the hiring cost.

Because all firms are identical, there is a symmetric equilibrium with $p_t(i) = P_t$ and $y_t(i) = Y_t$. Aggregating the firms' first-order conditions (see Appendix D) yields the Phillips curve:

$$\left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} = \frac{\epsilon_t - 1}{\varsigma} \left[\frac{\mu_t}{A_t} \left(\frac{(1 + \gamma_t^b) w_t^{new}}{\alpha} \right)^\alpha \left(\frac{q_t}{1 - \alpha} \right)^{1 - \alpha} - 1 \right] + \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \right], \quad (12)$$

where $\mu_t \equiv \epsilon_t / (\epsilon_t - 1)$ and w_t^{new} is the real wage paid to new workers. Aggregate output is

$$Y_t = A_t N_t^\alpha O_t^{1 - \alpha}, \quad (13)$$

where the intermediate input is demand determined given the exogenous price by¹⁶

$$O_t = \frac{Y_t}{A_t} \left(\frac{1 - \alpha}{\alpha} (1 + \gamma_t^b) \frac{w_t^{new}}{q_t} \right)^\alpha. \quad (14)$$

Finally, total employment and unemployment sum to the labor force:

$$N_t + U_t = F. \quad (15)$$

3.3 Wage Determination

Phillips's (1958) central insight is that the relationship between nominal wage growth and labor market tightness is highly nonlinear. Workers refuse jobs at wages below "prevailing rates," so despite high unemployment, "wage rates fall only very slowly." Yet the converse is not true: workers readily accept above-prevailing wages. Thus in tight labor markets, "we should expect employers to bid wage rates up quite rapidly." To model wage determination we formalize Phillips idea. This is the sense in which Phillips is "back" as suggested by the title.

We assume that the wage of a new hire at time t is

$$W_t^{new} = \max\{W_t^{ex}, P_t w_t^{flex}\}, \quad (16)$$

¹⁶In equilibrium, demand of the intermediate input can be lower than the exogenous supply \bar{O}_t .

where w_t^{flex} is the flexible real wage (the market-clearing wage absent constraints) and W_t^{ex} is the wage of workers attached to existing firm relationships.

To see how this captures Phillips’s idea, consider a weak labor market. The prevailing wage rate W_t^{ex} exceeds the market-clearing wage $P_t w_t^{flex}$. The max operator shows newly hired workers refuse wages below the prevailing wage at the firm, so if $W_t^{ex} > P_t w_t^{flex}$ then $W_t^{new} = W_t^{ex}$. Yet workers are perfectly happy to work for higher wages than the existing workforce, so when labor markets tighten and $W_t^{ex} < P_t w_t^{flex}$, then $W_t^{new} = P_t w_t^{flex}$.¹⁷ In real terms:

$$w_t^{new} = \max\{w_t^{ex}, w_t^{flex}\}. \quad (17)$$

Search-and-matching models feature wage indeterminacy since employment relationships generate surplus. Researchers commonly divide this surplus through Nash bargaining. Alternatively, monetary models with search frictions treat wages as exogenous, a polar case we examine in Section 3.4.1.¹⁸ One of our theoretical contributions is to depart from these approaches going back to Phillips’s original idea of asymmetric wage in response to labor market tightness and slack.

We now show how Equation (17) naturally gives rise to a more general theory of downward nominal wage rigidity—an assumption that is standard in the literature (see Section 3.4.2). The key idea is to model an anchor toward which wages are continuously pulled by defining the level they would attain under full flexibility. To formalize this concept of flexible wages, we introduce employment agencies that determine hiring and wage posting in equilibrium.

We model employment agencies using a standard matching function to generate hires. This structure determines the flexible wage rate. While the agency can be viewed as either an internal HR department or an external intermediary, we introduce it as a stand-alone entity to decouple hiring and pricing decisions.¹⁹

3.4 Employment agencies

There exists a continuum of employment agencies of measure one, corresponding to the number of firms. Employment agencies identify suitable workers for employment by firms. At a generic time t , the pool of job seekers consists of the unemployed at time $t - 1$, $U_{t-1} = \int_0^1 U_{t-1}(j) dj$ (integrating across households), and the workers who have been separated from firms at the beginning of the period, $s_t N_{t-1} = s_t \int_0^1 N_{t-1}(j) dj$.

The unemployed at the beginning of period t are then

$$\tilde{U}_t = U_{t-1} + s_t N_{t-1}. \quad (18)$$

¹⁷Consistent with the derivation of the AS equation (12), the inequality $(1 + \gamma_t^b)W_t^{new} > W_t^{ex}$ is always satisfied.

¹⁸See, e.g., Blanchard and Galí (2010a) and related literature cited in the Introduction.

¹⁹Unlike Michaillat (2014), we express hiring costs in terms of final goods rather than labor inputs. If hiring were modeled in labor terms, it would directly interact with firms’ pricing decisions. By contrast, in our formulation, the two margins remain separable.

We assume that all unemployed individuals at the beginning of period t are searching for jobs and that hires (H_t) occur through the matching function

$$H_t = m_t (\tilde{U}_t)^\eta (V_t)^{1-\eta}, \quad (19)$$

in which m_t denotes matching efficiency, V_t is total vacancies, and $0 < \eta < 1$. The assumption that workers separated from firms at the beginning of the period also search for a job captures the idea that part of hires reflects job-to-job transitions.²⁰

Since each agency is small, it takes as given the wage rate and the rate of employment matches per vacancy posted. Using the hiring technology (19), the number of matches per vacancy posted under flexible wages is

$$n(\theta_t) = \frac{H_t}{V_t} = m_t (\tilde{U}_t)^\eta (V_t)^{-\eta} = m_t \theta_t^{-\eta}, \quad (20)$$

where $\theta_t \equiv V_t / \tilde{U}_t$.

Consider the problem of a representative agency j . It charges a one-time fee γ_t^b to the firm that is proportional to the salary of a new worker it screens for employment, while incurring a cost γ_t^c for every vacancy it posts. The number of matches it generates is then $n(\theta_t)V_t(j) = m_t \theta_t^{-\eta} V_t(j)$. Revenues and costs are measured in units of the final good. Profits are

$$\max_{V_t(j)} \left(\underbrace{\gamma_t^b w_t^{flex} m_t \theta_t^{-\eta}}_{\text{marginal benefit}} - \underbrace{\gamma_t^c}_{\text{marginal cost}} \right) V_t(j). \quad (21)$$

In an interior solution, as long as the marginal benefit is greater than or equal to the marginal cost, the agency posts vacancies. This raises labor market tightness θ_t and reduces hires per vacancy. Equilibrium is reached when marginal benefits equal marginal costs:

$$\underbrace{\gamma_t^b w_t^{flex} m_t \theta_t^{-\eta}}_{\text{marginal benefit}} = \underbrace{\gamma_t^c}_{\text{marginal cost}}, \quad (22)$$

so that the flexible wage rate is

$$w_t^{flex} = \frac{\gamma_t^c}{\gamma_t^b} \frac{1}{m_t} \theta_t^\eta. \quad (23)$$

Suppose for the time being that the wages of existing and new workers are both determined by the flexible wage:

$$w_t^{ex} = w_t^{new} = w_t^{flex}. \quad (24)$$

This is *the* fundamental assumption we relax next. In the meantime, assuming no constraint on how

²⁰See also Blanchard and Galí (2010a) for a similar modeling strategy.

much wages can decline, this equation closes the model once we add the goods market equilibrium:²¹

$$Y_t = C_t + G_t + \gamma_t^c V_t, \quad (25)$$

and we consider the dynamic of total employment

$$N_t = m_t (\tilde{U}_t)^\eta (V_t)^{1-\eta} + (1 - s_t) N_{t-1} \quad (26)$$

and a specification of the monetary policy rule

$$i_t = \psi(\Pi_t, \xi_t, A_t, G_t, \dots). \quad (27)$$

A full specification of the equilibrium with flexible wages can now be stated formally.²²

3.4.1 Exogenous real wage rigidities

When all wages are fully flexible, we saw that hiring agencies post vacancies until wages fall and tightness increases sufficiently so that the marginal cost equals the marginal benefit of posting a vacancy. We now consider the possibility that the wage of existing workers is greater than the flexible wage rate, i.e.,

$$w_t^{ex} > w_t^{flex}.$$

We follow Phillips's suggestion, formalized in Equation (17), that new workers refuse to work for a wage below the existing wage rate. This implies that the wage of new workers is equal to the existing wage:

$$w_t^{new} = w_t^{ex}. \quad (28)$$

Before moving to the more general analysis in the next section, consider the case in which the existing wage rate is exogenous, as in Blanchard and Galí (2010a) :

$$w_t^{ex} = w(\Omega_t), \quad (29)$$

where the vector Ω_t includes all exogenous shocks.

By fixing the wage rate, the equilibrium can be defined as before (see footnote 22), replacing equation (23) with $w_t^{ex} = w(\Omega_t)$ together with (28). It should be noted that the employment agency problem does not appear explicitly in any of the equations defining the equilibrium. Yet, it is useful to be clear about its role in the background.

²¹We abstract from the resources used by the quadratic cost of price setting. See Eggertsson and Singh (2019) for various justifications.

²²An equilibrium with flexible wages and prices is defined by a collection of stochastic processes for $\{i_t, \Pi_t, Y_t, C_t, V_t, O_t, N_t, U_t, \tilde{U}_t, \theta_t, w_t^{ex}, w_t^{new}, w_t^{flex}\}_{t=0}^\infty$ that satisfy (5), (12), (13), (14), (15), (18), (23), (24), (25), (26), (27) and $\theta_t = V_t / \tilde{U}_t$, given exogenous processes $\{A_t, G_t, \xi_t, s_t, m_t, q_t, \gamma_t^c, \gamma_t^b, \epsilon_t\}_{t=0}^\infty$ and initial conditions N_{t_0-1} and U_{t_0-1} .

Denote the equilibrium labor demanded by firms by \tilde{N}_t^d , and the corresponding equilibrium aggregate vacancies by \tilde{V}_t . These vacancies represent the optimal posting by the employment agencies, assuming that the number of matches per job vacancy is given by

$$n(\theta_t) = \begin{cases} m_t \theta_t^{-\eta} & \text{if } V_t(j) \leq \tilde{V}_t, \\ 0 & \text{if } V_t(j) > \tilde{V}_t. \end{cases} \quad (30)$$

This says that once the hiring agencies have satisfied the labor demanded by firms, \tilde{N}_t^d , posting additional vacancies generates no further hires. Firms have already satisfied their need for labor at the exogenous wage and are not looking to hire additional workers. Previously, this logic did not apply, since more vacancy posting lowered wages and thereby triggered an increase in labor demand. That process continued until the marginal cost and benefit of posting a vacancy were equalized. With wages fixed, this is no longer the case. It becomes optimal for hiring agencies to post vacancies elastically to satisfy firms' demand for new hires at the exogenous wage, i.e. up to \tilde{V}_t , while posting no more vacancies beyond that point, since firms have already hired all the workers they desire.²³

Hiring agencies are not made explicit in typical New Keynesian models that integrate search and matching frictions with exogenous wages. It is thus implicitly assumed that vacancies respond fully elastically to labor demand. The agency problem formulated here provides a simple narrative for this commonly maintained assumption.

While treating real wages as exogenous is often convenient, our focus is on how labor-market tightness influences inflation. We therefore depart from the fixed-wage assumption.

3.4.2 Endogenizing a wage norm: bringing back the Phillips curve

So far we have not attempted to microfound a theory of downward rigid wages, nor do we intend to do so. The empirical evidence in favor of downward nominal wage rigidity is substantial.²⁴ As a theoretical matter, we view our specification as consistent with theories based on fairness norms (where new hires cannot be paid less than incumbents), insider-outsider dynamics, or efficiency wages.

As discussed in Section 3.3, Phillips' key insight was the nonlinearity of the wage Phillips curve: while new workers are reluctant to accept wages below existing levels, they are quite willing to accept higher wages, leading to a rapid acceleration of wages in very tight labor markets. But what can we reasonably assume about the behavior of existing workers? There is considerable evidence that the wages of existing workers are downward rigid, yet some degree of adjustment is observed in the data.

²³In this equilibrium the hiring agency is making a profit on its last vacancy posting.

²⁴There is extensive evidence of downward nominal wage rigidity: early observations go back at least to Malthus (1798)—“it rarely happens that the nominal price of labor universally falls”; Bewley (1999) documents firms' reluctance to cut pay; recent evidence includes U.S. administrative data (Fallick, Lettau, and Wascher 2011), worker-level survey data (Barattieri, Basu, and Gottschalk 2014), and cross-country evidence (Schmitt-Grohé and Uribe 2016). See also Fortin (2015) for Canadian evidence.

In this analysis, we adopt a pragmatic approach. We assume that the wages of existing workers evolve according to a relatively flexible specification. This nests the fully flexible wage at one extreme and the exogenous wage, commonly assumed in the literature just reviewed, at the other:

$$W_t^{ex} = (W_{t-1}^{ex} (\Pi_{t+1}^e)^\delta)^\lambda (P_t w_t^{flex})^{1-\lambda} \phi_t. \quad (31)$$

To understand this specification, consider the special case $\lambda = 1$, $\delta = 0$ and $\phi_t = 1$. Then $W_t^{ex} = W_{t-1}^{ex}$, i.e. existing workers' wages remain constant at their previous-period nominal value, capturing Keynes' idea that wages are nominally downward rigid in the absence of shocks. For $\lambda < 1$, wages of existing workers are anchored by the labor market conditions implied by the flexible wage rate, toward which they are gradually pulled, with perfect wage flexibility arising as $\lambda \rightarrow 0$.

We have already introduced Phillips' idea in equation (16) that new workers will not accept wages lower than those of existing workers but are happy to accept higher ones. This implies that in a slack labor market the wages of new workers are equivalent to the wage rate of existing workers, $W_t^{new} = W_t^{ex}$, but pulled down toward the flexible wage rate at a speed that depends on how close λ is to zero. For high enough λ , the wage rate falls slowly, as Phillips suggested, even if the unemployment rate is high. Conversely, in a tight labor market when $W_t^{flex} > W_t^{ex}$ the real new wage is determined purely by market forces, i.e. w_t^{flex} . How quickly existing wages catch up to the new wages during a period of labor shortage is determined by λ , the lower it is, the more quickly existing wages catch up to new wages.

We introduce an additional feature by including the variable Π_{t+1}^e , relevant if $\delta > 0$. This captures the idea that inflation expectations affect wage-setting behavior.²⁵ Finally, ϕ_t is exogenous, so the wage norm can also depend on external shocks and allowing for flexibility in determining the steady state.

Using equations (17) and (31), we can write the wages of new hires in real terms:

$$w_t^{new} = \begin{cases} w_t^{flex}, & \text{for } \theta_t > \theta_t^*, \\ w_t^{ex} = \left(w_{t-1}^{ex} \frac{(\Pi_{t+1}^e)^\delta}{\Pi_t} \right)^\lambda (w_t^{flex})^{1-\lambda} \phi_t, & \text{for } \theta_t \leq \theta_t^*, \end{cases} \quad (32)$$

In a tight labor market ($\theta_t > \theta_t^*$), the wages of new hires are set at the flexible rate determined by employment agencies' optimizing behavior, which equates the marginal cost and marginal benefit of posting vacancies. Moreover, the flexible wage rate plays a crucial role in determining existing workers' wages, serving as an anchor that gradually pulls them toward the market-clearing level.

By contrast, in a slack labor market ($\theta_t \leq \theta_t^*$), when the wages of new hires are constrained by existing workers' wages, households refuse to accept salaries below the prevailing rate of incumbents. In this case, wages fall only gradually toward the flexible wage rate, despite high unemployment.

²⁵These expectations can, for example, be anchored by the central bank's inflation target. Alternatively, Π_{t+1}^e can capture a "price-wage spiral" dynamic, commonly thought to have played a role in the 1970s, even if we leave this to future research for lack of space.

What remains to finalize the model is the determination of the Beveridge threshold θ^* , which indicates when labor scarcity prevails and the wages of new hires are set flexibly. This threshold is implied by equating $w_t^{flex} = w_t^{ex}$, yielding:

$$\theta_t^* = \left[\frac{\gamma_t^b}{\gamma_t^c} m_t w_{t-1}^{ex} \right]^{\frac{1}{\eta}} \left(\frac{(\Pi_{t+1}^e)^\delta}{\Pi_t} \right)^{\frac{1}{\eta}} (\phi_t)^{\frac{1}{\lambda\eta}}. \quad (33)$$

As the formula suggests, θ_t^* can vary over time and depend on institutional features that differ across regions and countries, captured in reduced form by ϕ_t . We defer discussion of the determinants of θ_t^* to Section 4, where we address the role of other shocks in the Phillips curve. As shown in the Appendix, the steady-state threshold can be derived as a function of structural parameters and need not be equal to one.

We can now formally define equilibrium in the model with nominal wage rigidities.²⁶

4 The Inv-L NK Phillips Curve

Our first step in characterizing the Phillips curve is a log-linear approximation of the New Keynesian Phillips curve given by (12), expressed as:

$$\pi_t - \pi = \frac{1-\epsilon}{\zeta} \alpha \underbrace{\left(d_\gamma \hat{\gamma}_t^b + \hat{w}_t^{new} \right)}_{\text{Marginal Cost of Labor}} + \frac{1-\epsilon}{\zeta} \underbrace{\left(\hat{\mu}_t - \hat{A}_t + (1-\alpha)\hat{q}_t \right)}_{\text{Cost-Push Shocks}} + \beta E_t(\pi_{t+1} - \pi), \quad (34)$$

where a hat denotes the log-deviation of a variable relative to its steady state, $\pi_t \equiv \ln \Pi_t$, $\pi \equiv \ln \Pi$, and the parameter d_γ is defined in Appendix D. The first term highlights that the primary driver of inflation is the marginal cost of hiring a new worker. This cost comprises two components: the wage bill for new workers \hat{w}_t^{new} and the hiring cost $\hat{\gamma}_t^b$. The second term represents what the literature typically identifies as cost-push or supply shocks, while the last term captures inflation expectations, as seen in the standard model.

An important implication of this characterization is that, while marginal costs affect inflation as in the standard New Keynesian model, they are represented differently here. In the canonical framework, marginal costs refer to employee wages, typically measured by aggregates such as the labor share or the Employment Cost Index (ECI). By contrast, our model suggests that the relevant marginal costs are those associated with adding new workers to the workforce. Their wages may or may not correspond to incumbent wages, depending on the state of the labor market. Under normal circumstances, i.e., in the absence of labor shortages, the two move together, consistent with Gertler,

²⁶An equilibrium is defined by a collection of stochastic processes for $\{i_t, \Pi_t, Y_t, C_t, O_t, V_t, N_t, U_t, \tilde{U}_t, \theta_t, w_t^{ex}, w_t^{new}, w_t^{flex}, \theta_t^*\}_{t=0}^\infty$, that satisfy (5), (12), (14), (13), (15), (18), (23), (25), (26), (27), (32), (33) and $\theta_t = \tilde{U}_t/V_t$ given exogenous processes $\{A_t, G_t, \zeta_t, s_t, m_t, q_t, \gamma_t^c, \gamma_t^b, \epsilon_t, \phi_t\}_{t=0}^\infty$ and initial conditions $N_{t_0-1}, U_{t_0-1}, w_{t_0-1}^{ex}$.

Huckfeldt, and Trigari (2020). During labor shortages, the distinction between the wages of new and existing hires becomes critical, consistent with the evidence presented in Figure 12, which utilizes wage data from the Federal Reserve of Atlanta.²⁷ In our framework, the marginal cost of adding new workers comprises the wages of new hires and hiring costs. These additional hiring costs may also include bonuses – common during the surge of the 2020s – or higher cost for on the job training due to the large number of job-to-job flows during the recovery.²⁸

The nonlinearity of the Phillips curve arises because the wage rate for new hires depends on whether the labor market is tight. Define $\hat{w}_t^{new} \equiv \ln w_t^{new} - \ln \bar{w}$ and $\hat{w}_t^{flex} \equiv \ln w_t^{flex} - \ln \bar{w}^{flex}$. Equation (17) can be rewritten in logarithmic form as

$$\hat{w}_t^{new} = \max(\hat{w}_t^{ex}, -c_w + \hat{w}_t^{flex}), \quad (35)$$

where $c_w = \ln \bar{w} / \bar{w}^{flex}$. As discussed in Appendix D, we consider a steady state with a real wage \bar{w} , where the flexible real wage in this steady state is below it (i.e., $\bar{w}^{flex} < \bar{w}$). The parameter c_w is, therefore, positive. We approximate around $\bar{\theta}$, which lies below the kink point $\bar{\theta}^*$, to characterize “normal times” when the labor market is slack. This is consistent with the empirical evidence presented in Figure 2.

Since the expressions for w_t^{flex} , w_t^{ex} , and θ^* are all linear in logs (see equations (23), (32), (33)), the following expression written in natural logs is exact and involves no approximation error:

$$\hat{w}_t^{new} = \begin{cases} -c_w + \hat{w}_t^{flex} = -c_w + \eta \hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t & \hat{\theta}_t > \hat{\theta}_t^* \\ \hat{w}_t^{ex} = \lambda(\hat{w}_{t-1}^{ex} - (\pi_t - \pi) + \delta E_t(\pi_{t+1} - \pi)) + (1 - \lambda)\hat{w}_t^{flex} & \hat{\theta}_t \leq \hat{\theta}_t^* \end{cases} \quad (36)$$

assuming $\pi_{t+1}^e = E_t \pi_{t+1}$.

Substituting equation (36) into the Phillips curve (34), we obtain:

$$\pi_t - \pi = \begin{cases} -c + \kappa^{tight} \hat{\theta}_t + \kappa_v^{tight} (\hat{v}_t + \hat{\theta}_t^{tight}) + \beta E_t(\pi_{t+1} - \pi) & \hat{\theta}_t > \hat{\theta}_t^* \\ \kappa_w \hat{w}_{t-1} + \kappa \underbrace{\hat{\theta}_t}_{\text{tightness}} + \kappa_v \left(\underbrace{\hat{v}_t}_{\text{cost-push}} + \underbrace{\hat{\theta}_t}_{\text{matching}} \right) + \kappa_\beta E_t(\pi_{t+1} - \pi) & \hat{\theta}_t \leq \hat{\theta}_t^* \end{cases} \quad (37)$$

where the coefficients satisfy $\kappa^{tight} > \kappa > 0$, $\kappa_v^{tight} > \kappa_v > 0$, $\kappa_w > 0$, and $c > 0$. It is unclear whether κ_β is greater than or less than β . The analytical expressions for these coefficients are provided in Appendix D.

We briefly comment on the accuracy of the log-linear approximation. The Phillips curve in equation (34) has the same form as in the standard model, with the only difference being that \hat{w}_t^{new} appears

²⁷ Additionally, see the findings of Crump et al. (2024).

²⁸ Bernanke and Blanchard (2024), for example, argue that labor market tightness did not, at the very beginning of the 2022 inflation surge, play a fundamental role. Their results, however, depend on measuring marginal costs using the ECI, in contrast to our model. More importantly, our framework highlights the interaction between labor tightness and supply shocks. One advantage of our approach is that it explains why supply shocks had such an outsized effect during this period. Moreover, our model is consistent with the lack of disinflation after 2008.

instead of average wages. Moving from equation (34) to the piecewise linear equation (37) only requires the intermediate step of using the max operator and the definition of the new wage above and below the Beveridge curve shown in equation (36). As we have already pointed out, expressing either \hat{w}_t^{new} or \hat{w}_t^{ex} as log deviations involves no approximation error, because both variables are linear in logs. Other aspects of the Phillips curve do involve approximation errors, as in any log-linearization, but there is no reason to believe these are more significant here than in the canonical model. This may seem surprising, given that the Phillips curve is piecewise linear with a non-differentiable kink, and log-linear approximations rely on differentiation. The key insight is that the kink arises from the max operator (35), while the variables inside the max operator are linear in logarithms.

4.1 Three central predictions of the Inv-L NK Phillips Curve and a key result

The theoretical results in equation (37) highlight four major predictions.

First, $\kappa^{tight} > \kappa$: when the labor market is sufficiently tight ($\hat{\theta}_t > \hat{\theta}_t^*$), inflation responds more strongly to labor market tightness than it does under normal circumstances. In this regime, the new-hire wage becomes fully determined by the flexible wage, which is directly proportional to labor market tightness $\hat{\theta}_t$. This direct transmission channel is muted when $\hat{\theta}_t < \hat{\theta}_t^*$, as the new-hire wage is then anchored by the more sluggish incumbent wage.

Second, $\kappa_v^{tight} > \kappa_v$: cost-push and markup shocks have a greater impact on inflation when the labor market is tight.

Third, while real wages enter the Phillips curve as a lagged variable when $\hat{\theta}_t < \hat{\theta}_t^*$, the curve becomes perfectly forward-looking when $\hat{\theta}_t \geq \hat{\theta}_t^*$.

Finally, a key result of our analysis is that we can replace the empirical measure of new wages with the vacancy–unemployment ratio θ . This provides a direct connection to an emerging empirical literature arguing that v/u is a better measure of slack than u_t .²⁹

4.2 The Beveridge threshold and the interpretation of shocks

Using logarithms, we can rewrite Equation (33) in terms of deviations from the steady state and solve for the Beveridge threshold:

$$\hat{\theta}_t^* = \eta^{-1} (\hat{w}_{t-1}^{ex} + \delta E_t \pi_{t+1} - \pi_t + \lambda^{-1} \hat{\phi}_t + \hat{m}_t + \hat{\gamma}_t^b - \hat{\gamma}_t^c). \quad (38)$$

This expression is intuitive. Recall from Equation (36) that the Beveridge threshold is defined by the condition $w_t^{flex} = w_t^{ex}$. As θ_t rises beyond θ_t^* , new wages exceed existing wages, and the slope of the Phillips curve changes.

²⁹Moreover, while data on v/u are readily available over long periods, there is no consensus on the best measure of newly hired workers' wages, and existing series do not yet cover as long a span as v/u . This is a promising area for future research.

The Beveridge threshold decreases when the real wages of existing workers fall below their steady-state value or decline due to current inflation, as nominal wages are sticky. Similarly, a negative exogenous wage shock, $\hat{\phi}_t$, lowers the threshold.

The final three terms in Equation (38)— \hat{m}_t , $\hat{\gamma}_t^b$, and $\hat{\gamma}_t^c$ —influence the flexible wage for a given θ_t , thereby affecting the Beveridge threshold. For example, lower matching efficiency ($\hat{m}_t < 0$) reduces the threshold.

The traditional “cost-push” and supply shocks are denoted by \hat{v}_t and defined as

$$\underbrace{\hat{v}_t}_{\text{Cost-push/supply shocks}} \equiv \underbrace{\hat{\mu}_t}_{\text{Markups}} - \underbrace{\hat{A}_t}_{\text{Productivity}} + (1 - \alpha) \underbrace{\hat{q}_t}_{\text{Oil price}}. \quad (39)$$

Shocks to which the hiring agency is sensitive—specifically, shocks to the matching technology and the marginal cost and benefit of posting vacancies—are summarized by $\hat{\vartheta}_t$:

$$\underbrace{\hat{\vartheta}_t}_{\text{Matching shocks}} \equiv \alpha(d_\gamma - (1 - \lambda)) \underbrace{\hat{\gamma}_t^b}_{\text{Firm hiring cost}} + \alpha(1 - \lambda) \underbrace{\hat{\gamma}_t^c}_{\text{Vacancy cost}} - \alpha(1 - \lambda) \underbrace{\hat{m}_t}_{\text{Matching efficiency}} + \alpha \underbrace{\hat{\phi}_t}_{\text{Wage shock}}, \quad (40)$$

with $d_\gamma \equiv \gamma^b / (1 + \gamma^b)$.

Unlike the cost-push shock \hat{v}_t , which is independent of labor market tightness, matching shocks assume a different form when the labor market is tight. In this case, they are summarized by

$$\hat{\vartheta}_t^{\text{tight}} = \alpha(\hat{\gamma}_t^c - (1 - d_\gamma)\hat{\gamma}_t^b - \hat{m}_t).$$

The reason is that these shocks operate through the determination of flexible wages. In normal times, the flexible wage receives a weight $1 - \lambda$; whereas, during periods of labor tightness, it is directly transmitted to new wages.

We next provide empirical evidence beyond the descriptive results presented in Section 2 by estimating the Phillips curve implied by our model. Estimating Phillips curves is notoriously challenging. We closely follow the literature to test whether our model is consistent with the data. The short answer is yes. While this does not provide conclusive evidence, it serves as a proof of concept.

5 Estimating the Inv-L NK Phillips Curve

Equation (37) delivers an aggregate–supply relation that is piecewise linear in labor–market tightness, with a kink at the Beveridge threshold θ_t^* . Taken literally, it links inflation (π_t) to tightness ($\theta_t \equiv v_t/u_t$), a lagged real–wage term, cost–push and matching shocks, and expected inflation $E_t\pi_{t+1}$. If the model were the true data–generating process and all variables were observed without error, the

parameters could, in principle, be recovered by (piecewise) least-squares estimation, conditional on a known θ_t^* and the exogeneity of the regressors.

Analyses of this type are often subject to endogeneity in various forms, which we address using standard approaches aligned with the literature.³⁰

Guided by Equation (37), we estimate

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) (\ln \theta_t - \ln \theta_t^*) + (\beta_\varrho + \beta_{\varrho_d} D_t) \varrho_t + (\beta_{\pi^e} + \beta_{\pi_d^e} D_t) \pi_t^e + \varepsilon_t, \quad (41)$$

where π_t is the inflation rate (demeaned by 2 percentage points), π_t^e is expected inflation, ϱ_t is our measure of supply shocks, and $D_t = 1$ when $\theta_t \geq \theta^*$ (and $D_t = 0$ otherwise). The coefficients on tightness, supply, and expectations are allowed to shift past the Beveridge threshold θ^* . Subscript d marks the additional effect that applies when $D_t = 1$: for example, the coefficient on $\ln \theta_t$ is β_θ when $D_t = 0$ and $\beta_\theta + \beta_{\theta_d}$ when $D_t = 1$ (similarly for ϱ_t and π_t^e). Note that we proxy the lagged real wage with lagged inflation; in robustness checks we also use alternative measures of lagged real wages.

This specification captures the key nonlinearity implied by the model: crossing the Beveridge threshold changes not only the slope with respect to tightness but also the effect of supply shocks and inflation expectations. Our hypothesis is that the slope steepens and the effect of supply shocks increases once $\theta_t \geq \theta^*$, while the sign of the shift in the expectations coefficient is a priori ambiguous. By contrast, the role of lagged inflation should move from positive (when $\theta_t < \theta^*$) to negligible (when $\theta_t \geq \theta^*$).

We estimate (41) on two samples: (i) a post-financial crisis sample, 2008Q3–2024Q4, for which the regressors are more reliably measured; and (ii) a long sample, 1960Q1–2024Q4, which requires proxies for expectations and vacancies.

In our benchmark regression, π_t denotes core CPI inflation, and θ_t is the vacancy–unemployment ratio, measured using JOLTS data from December 2000 onward and backcast with Barnichon’s (2010) composite vacancy index for earlier years. We construct ϱ_t as the first principal component of three relative price gaps, shown in Figure 10, that are commonly used in the literature: headline CPI relative to core CPI, headline PCE relative to core PCE, and import prices relative to the GDP deflator.³¹ Figure 10 shows that measured supply shocks in the 2020s were modest; nevertheless, conditional on $\theta_t > \theta^*$, the model implies the effect can be substantial.

In the benchmark specification, inflation expectations are proxied by the Survey of Professional Forecasters’ one-year CPI inflation expectations. Figure 11 in the Appendix plots the series.³²

All series are constructed at quarterly frequency from monthly data as described in the Appendix F.

³⁰See Lenoir (1913), Christ (1994), Gordon (1982), Sargent (1975), Mavroeidis et al. (2014), and McLeay and Tenreyro (2019), among others.

³¹Ball et al. (2022) use the first two measures; Blanchard et al. (2022) emphasize the third.

³²The SPF series begins in 1981Q3. We extend it backward using the interpolated Livingston Survey; note that the Livingston 12-month CPI forecast effectively corresponds to a 14-month horizon (Carlson, 1977).

Table 3: Tests for a kink and for the location of the Beveridge threshold ($\theta^* \approx 1$)

	1960–2024	2008–2024
Tests of linearity (no kink) vs. kink alternative		
Global sup test (p-value)	12.0 (0.37)	25.5 (0.53)
Restricted sup [0.75, 1.25] (p)	7.6 (0.15)	22.6 (0.01)
Fixed $\theta = 1$ Wald (p)	6.3 (0.00)	14.1 (0.00)
Tests of location: $H_0 : \theta^* = 1$ vs. $\hat{\theta}^*$		
LR statistic	9.49	0.52
p-value (point)	0.98	0.82
p-value (sup)	1.00	1.00

Notes: Bootstrap p-values based on 999 moving-block resamples.

We estimate the threshold $\hat{\theta}^*$ in equation (41) by profile least squares following Hansen (2000). For each feasible candidate θ^* , we run OLS, record the residual sum of squares $SSR(\theta^*)$, and set $\hat{\theta}^* = \arg \min_{\theta^*} SSR(\theta^*)$.³³ In the short sample (2008–2024), $\hat{\theta}^*$ is close to the unit value (0.9305), whereas in the long sample (1960–2024) it equals 0.5006, although there is another local minimum close to 1 which cannot be statistically rejected as an alternative.

To conduct inference on the existence and location of a structural break, we construct the profile likelihood–ratio curve

$$LR(\theta^*) = \frac{SSR(\theta^*) - SSR(\hat{\theta}^*)}{\hat{\sigma}^2}, \quad \hat{\sigma}^2 = \frac{SSR(\hat{\theta}^*)}{T - k_{\text{eff}}},$$

where T is the sample size and k_{eff} denotes the effective number of regressors. Critical values are obtained via a moving-block residual bootstrap that preserves serial dependence.³⁴

We ask (1) whether a kink exists and (2) whether its location is reasonably approximated by 1; four complementary tests speak to these questions. (Table 3). (i) A *global linearity test* (sup–Wald/sup–LR) of $H_0 : \beta_{\theta_d} = \beta_{\theta_d} = \beta_{\pi_d} = 0$ rejects if the supremum of the restriction test over the full grid exceeds its bootstrap critical value. (ii) A *restricted sup test* repeats (i) on the economically plausible band $\theta^* \in [0.75, 1.25]$, which increases power around $\theta^* = 1$. (iii) A *pointwise LR test* evaluates $H_0 : \theta^* = 1$ against the estimated alternative $\hat{\theta}^*$. We report both the pointwise p-value and the sup p-value based on the maximum LR statistic across all thresholds. (iv) A *fixed-break Wald test* evaluates no kink at $\theta^* = 1$, assessing whether slopes differ above and below unit tightness. The pointwise LR test and fixed-break Wald test address different questions: the LR test compares threshold locations ($\theta^* = 1$ vs. $\hat{\theta}^*$), while the Wald test evaluates slope differences conditional on $\theta^* = 1$.

³³To ensure numerical stability when $D_i(\theta^*)$ is nearly constant, estimation and inference use a rank–revealing QR reduction of the design (orthogonal–triangular decomposition). The likelihood–ratio scaling employs the resulting effective degrees of freedom.

³⁴Specifically, we resample circular blocks of residuals from the fitted model at $\hat{\theta}^*$, recursively reconstruct π_t within the sample, recompute $LR^*(\theta^*)$ over the grid, and take the $(1 - \alpha)$ quantile of $\sup_{\theta^*} LR^*(\theta^*)$. The $(1 - \alpha)$ confidence set for $\hat{\theta}^*$ is the connected region in θ^* where $LR(\theta^*)$ does not exceed the bootstrap sup–critical value.

Table 4: Phillips Curve Estimates (Threshold at $\theta = 1$)

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.3031 *** (0.0972)	0.1020 (0.2745)	0.2030 ** (0.0989)	-0.1788 (0.1869)
$\ln \theta$	0.6988 *** (0.1715)	0.5046 (0.3373)	0.3210 * (0.1807)	0.5027 (0.3602)
$\theta \geq 1$			1.6416 * (0.8700)	4.9959 *** (1.7670)
<i>Supply shock (ρ)</i>	0.0355 * (0.0195)	0.0135 (0.0388)	0.0416 ** (0.0208)	-0.0039 (0.0258)
$\theta \geq 1$			0.1055 (0.0899)	0.2535 ** (0.1058)
<i>Inflation expectations</i>	0.7706 *** (0.1095)	1.6821 ** (0.6779)	0.8959 *** (0.1105)	0.5368 (0.7474)
$\theta \geq 1$			0.7093 ** (0.3123)	0.2601 (1.1818)
<i>Constant</i>	0.4760 *** (0.1383)	0.3511 (0.3658)	0.1612 (0.1407)	0.1646 (0.3350)
R^2 adjusted	0.8184	0.5314	0.8288	0.6515
Observations	260	66	260	66

· ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.

· Newey-West standard errors.

· (1) and (3): sample 1960 Q1 – 2024 Q4

· (2) and (4): sample 2008 Q3 – 2024 Q4

Four conclusions follow. First, the *global sup test* does not reject linearity in either sample (see Table 3). This test uses critical values adjusted for searching across all candidate thresholds (Hansen, 2000), which guards against false detection of nonlinearity but reduces the power to detect a threshold when one exists (scans many thresholds → low power). Second, focusing the search near unity increases power: the *restricted sup test* strongly rejects linearity in the 2008-2024 sample ($supF = 22.6, p = 0.01$), while the evidence in the full sample is marginal ($supF = 7.6, p = 0.15$). This suggests that the kink is most accurately identified in the recent period, when labor-market tightness fluctuated around unity. Third, the *fixed-break Wald tests* strongly reject linearity at $\theta^* = 1$ in both samples, suggesting that allowing for slope shifts at unit tightness enhances the fit. Fourth, the *pointwise LR tests* cannot reject $\theta^* = 1$ against the estimated alternative $\hat{\theta}^*$ (pointwise $p = 0.82$ in the short sample; $p = 0.98$ in the long sample; $supp = 1.00$ in both samples). This confirms that the data are consistent with a kink at the unitary threshold.³⁵

In Figure 13 in the Appendix, visualizes the location evidence from the LR profile. In both samples, the $LR(\theta^*)$ profile is relatively flat over a broad range, indicating that the exact location of the kink is weakly identified—many thresholds, especially those near unity, fit similarly well. Nonetheless, $\theta^* = 1$ lies at or near the profile’s minimum in both samples, confirming that this threshold fits the data as well as an alternative, with no other value being *statistically significantly* superior.

Overall, the evidence supports a kinked Phillips curve with a break close to $\theta^* = 1$. this evidence is especially compelling in the post-crisis period, when there was significant variation in supply disturbances—a feature lacking during the inflation surge of the late 1960s, the other occasion on which the Beveridge threshold was crossed.

We have previously shown theoretically that the Beveridge threshold may shift over time as function of exogenous shocks. Here, instead, we focus on a single constant threshold. Implicitly, we think of the time variations as random error terms and view estimating a constant threshold as a pragmatic first step. We leave estimation of a time-varying threshold to future work.

We next estimate the parameters of equation (41) in Table 4, fixing $\theta^* = 1$. Columns (1) and (2) report estimates for the full and short samples, respectively, without regime dummies. Columns (3) and (4) re-estimate the equation allowing for regime shifts (i.e., including the dummies). Table 5 in the Appendix reports results when θ^* is jointly estimated. Estimates are virtually unchanged in columns (1) and (2) and qualitatively similar in column (4), consistent with the fact that θ^* is estimated close to 1 in the short sample.

The first main result from Table 4 is that the Phillips curve exhibits statistically significant nonlinearity in θ (at the 1% level). Columns (3) and (4) show this for the full and 2008–2024 samples, respectively. The slope when $\theta > 1$ equals the sum of the coefficients in the second and third rows, whereas for $\theta < 1$ it is given by the second row alone.³⁶ While the point estimate for $\theta < 1$ has the expected sign,

³⁵Sup p-values compare the statistic to a distribution that accounts for searching over many thresholds; this guards against false positives but lowers power. A value of 1.00 therefore reflects non-rejection, not affirmative evidence for $\theta^* = 1$

³⁶For example, in column (4), the slope when $\theta > 1$ equals $0.5027 + 4.9959 = 5.4986$.

the null of a flat Phillips curve cannot be rejected for this region in the full sample. When we constrain the slope to be constant, as in columns (1) and (2), the coefficient is larger and statistically significant, suggesting that the overall significance is driven by periods with $\theta > 1$. The estimated slope increases by roughly a factor of three in the full sample and by a factor of ten in the short sample when moving from a constant-slope specification to the slope in periods with $\theta > 1$.

The second main result is that supply shocks have a larger effect when $\theta > 1$. In the full sample, the point estimate implies roughly twice the impact, although statistical significance is limited. This imprecision is not surprising: the full sample contains only two periods when $\theta > 1$ (the 1960s and the recent episode), and the 1960s featured virtually no supply shocks (see Figure 10 in the Appendix). By contrast, in the 2008–2024 sample we find a large and statistically significant effect of supply shocks when $\theta > 1$, whereas for $\theta < 1$ their effects are both economically and statistically negligible.

Beyond these two main results, Table 4 offers further insights. The coefficient on lagged inflation is statistically significant in the full sample but not in 2008–2024—a period in which $\theta > 1$ for much of the time. This pattern is consistent with the model’s prediction that the role of lagged terms diminishes once the Beveridge threshold is crossed.

A further observation concerns inflation expectations. The coefficient is statistically significant in the full sample both with and without regime dummies, and in the short sample when dummies are excluded. However, it becomes smaller and statistically insignificant over 2008–2024 once dummies are included. In that specification, the two coefficients for the short sample sum to 0.7969, capturing the impact of inflation expectations in tight labor markets, while the corresponding effect during periods of slack labor markets is smaller, at 0.5368.

Finally, the constant term admits an economically intuitive interpretation. It implies the value of θ at which inflation equals 2% in the absence of shocks, denoted by $\bar{\theta}$. We obtain $\bar{\theta} = 0.6052$ for the full sample and $\bar{\theta} = 0.7208$ for the 2008–2024 subsample.³⁷

5.1 Robustness analysis

We assess the robustness of our results to alternative specifications and measurement choices.

Our two central results are broadly robust: (i) the Phillips-curve slope increases markedly when $\theta > 1$, and (ii) under these conditions, supply shocks have a larger impact on inflation even though point estimates vary across specifications. This robustness is encouraging given the well-known sensitivity of Phillips-curve estimates to specification and variable selection.³⁸

³⁷The mean of the supply shock is approximately zero, and all inflation measures are deviations from the 2% target. Because $\ln \theta$ is not mean zero, we recover $\bar{\theta}$ from $\ln \bar{\theta} = -\beta_c/\beta_\theta$, yielding $\bar{\theta} = 0.6052$ and $\bar{\theta} = 0.7208$ for specifications (3) and (4) in Table 4, respectively.

³⁸See, e.g., Mavroeidis, Plagborg-Møller, and Stock (2014).

To address endogeneity, a common approach is to instrument variables with their lags. Our results are confirmed when we instrument lagged inflation and $\ln(v/u)$ with their first lags (Table 6).³⁹ Table 7 shows that the results also hold when the vacancy–unemployment ratio is already lagged in the benchmark regression.

Our findings are robust to traditional measures of supply shocks and to alternative measures of inflation expectations. Tables 8 and 9 confirm the main results when headline-CPI shocks and import-price shocks are entered separately.

Tables 10, 11, and 12 examine alternative expectations: the Cleveland Fed’s 2-year and 5-year inflation expectations, and the 5-year 5-year forward measure of Groen and Middeldorp (2013).⁴⁰ The results are robust in the latter part of the sample where market-based metrics are available. For the full sample that relies on backcasts, point estimates are similar but standard errors are larger and the steep-slope estimate loses statistical significance.

Tables 13 and 14 confirm the results using 1-year expectations from the Cleveland Fed and the 12-month Livingston Survey, which span the entire sample (the latter at a semiannual frequency).

Our results are not robust to the use of the University of Michigan’s 12-month-ahead consumer inflation expectations, as shown in Table 15 and discussed, in slightly different specifications, by Beaudry, Hou, and Portier (2025). In this case, the Phillips curve appears flat, with a very large coefficient on inflation expectations—the sum of the relevant terms exceeds unity, reaching 1.9248 in the short sample with dummies.

The question, then, is why the results using the Michigan series are not robust to the other six measures we employ. More broadly, the issue concerns which expectation metric best represents the expectations that firms base their pricing decisions upon. One interpretation is that the Michigan series reflects more backward-looking behavior than other measures, making it a weaker proxy for the forward-looking expectations relevant for firms’ price setting. In the New Keynesian Phillips curve, firms reset prices based on expected future inflation and economic conditions, and micro evidence (e.g., Nakamura and Steinsson, 2008) shows that prices typically remain unchanged for several quarters. For the Michigan series, we conjecture that realized inflation has a stronger influence on reported expectations than in the other proxies we consider. This feature is problematic, since in the New Keynesian framework it is firms’ expectations of future inflation that drive price adjustments, not the reverse.

Different expectation measures exhibit varying degrees of dependence on observed inflation at the time expectations are formed.⁴¹ In the Appendix, Table 19 reports correlations for 2008Q3–2024Q4.⁴² The 12-month Michigan survey shows an especially sharp correlation of 0.82 during the inflation

³⁹Limitations due to the weak-instrument problem are reviewed in Mavroeidis, Plagborg-Møller, and Stock (2014).

⁴⁰The Groen-Middeldorp series uses market data in recent periods and backcasts earlier decades such as the 1970s.

⁴¹We proxy observed inflation with realized inflation at $t - 1$ for expectations formed at t , ensuring that information is available at the time of formation.

⁴²Focusing on this period avoids extrapolations and backcasts for expectations.

surge period from 2020Q4 to 2024Q4, whereas other—including our baseline at 0.56—are much weaker. Viewing the full sample, the Michigan Survey correlation and SPF series show the strongest correlations (above 0.7), while the 5-year–5-year measure exhibits a negative correlation. The unusually strong response of the Michigan survey, relative to all other measures during the surge, may be connected to its tendency to react more strongly to recent data than other measures, resembling a simple extrapolation. Alternatively, it tends to react more strongly to oil price movements which happen to coincide closely with the surge.

To explore the claim that Michigan’s 12-month measure might be based on simple extrapolation, we present the following suggestive evidence: It is the only series for which observed inflation Granger-causes expected inflation; increases in observed inflation help predict increases in expected inflation.⁴³ Table 20 shows that this Granger causality appears only for the Michigan survey; our benchmark and other alternatives do not exhibit it. This implies that an increase in inflation is more likely to immediately trigger changes in Michigan survey expectations, while other measures assign greater weight to such shocks being transitory.

By contrast, causality from expectations to inflation holds for most measures: increases in expectations Granger-cause observed inflation for all but the Cleveland 5-year and 5-year–5-year forward series. The insensitivity of long-horizon expectations to current inflation is consistent with the view that long-term expectations remained anchored during the recent surge. Importantly, both long-horizon measures deliver results similar to our benchmark (Tables 11 and 12).

The debate over which expectation metric best proxies firms’ expectations remains open. We are particularly interested in the expectation series that influences firms’ expectations, as these determine price setting by firms. Chodorow-Reich (2024, Figure 1) shows a measure of inflation expectation based on the Federal Reserve of Atlanta survey of firms expectations of their own marginal costs; these findings closely match the Survey of Professional Forecasters (SPF), our baseline. By contrast, Coibion and Gorodnichenko (2024) contend that the Michigan Survey is a superior measure, partly because it aligns more closely with the Cleveland Fed’s CEO inflation-expectations data collected since 2019.

Our benchmark choice—the SPF—is guided by empirical and narrative considerations. It yields results consistent with the other six expectation measures and with the widespread perception that the inflation surge surprised most observers. Moreover, the Atlanta Fed’s survey arguably provides a more relevant analog for firms’ price-setting behavior than the Cleveland Fed’s, as it asks respondents to forecast their own unit-cost growth rather than the more abstract concept of overall inflation.

Finally, our results are robust in sign but not statistically significant for the slope when using core PCE (rather than core CPI) as the dependent variable (Table 16), and when measuring labor-market tightness in levels rather than in logs (Table 17). We prefer the benchmark specification that uses $\ln \theta = \ln(v/u)$, as implied by theory.⁴⁴ We also estimate a specification that replaces lagged inflation

⁴³Formally, lagged observed inflation adds statistically significant predictive content for expected inflation. See footnote 41 for the data mapping.

⁴⁴A further practical advantage of using logs is their invariance to whether v or u is in the denominator.

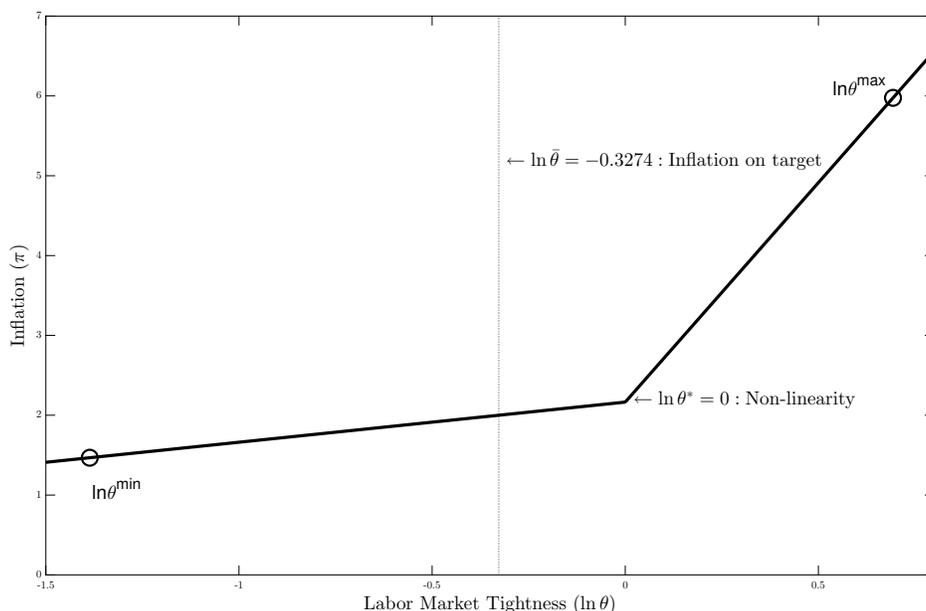


Figure 4: Inv-L NK Phillips Curve as a function of labor market tightness using the estimates of Table 1, column (4).

with lagged real wages, as suggested by the model; Table 18 confirms our main findings under this alternative.

An alternative to our baseline regression – which imposes a single breakpoint – is a time-varying-parameter specification in which the coefficients evolve each period and are estimated via a Kalman filter. Our two central empirical results are robust in this framework. Figure 15 shows that the coefficients on $\ln \theta$ and on the supply-shock index ϱ remain near zero through most of the sample (including the Great Recession) and then rise sharply in 2021–22, precisely when $\theta > 1$. This independent method corroborates the piecewise-linear estimates. Detailed results and discussion are provided in Appendix E.

6 Insights from Our Analysis and Conclusions

Our theoretical analysis links labor-market tightness—measured by the vacancy–unemployment ratio—to inflation through a piecewise linear relation that steepens once the Beveridge threshold is crossed. The empirical evidence strongly supports this specification, identifying a breakpoint in the Phillips curve at (approximately) unity for θ , where vacancies equal unemployment. This conclusion is consistent with visual inspection of the raw data as shown in Figure 1.

The implications are substantial for both measurement and policy.

First, using the estimates in Table 4, we quantify this relationship. Figure 4 plots inflation against the vacancy–unemployment ratio using specification (4) in Table 4, illustrating both the slope of the

Phillips curve and the tightness level consistent with the 2% target. The key result is $\kappa^{\text{tight}} > \kappa$. Once the Beveridge threshold is exceeded, the slope steepens, and inflation rises more sharply as the labor market tightens. The vertical reference line highlights that when tightness surpasses this critical level, inflation exceeds the target, while the curve remains relatively flat near 2% provided $\theta < \theta^*$. Raising θ from 0.7208 (our estimated neutral level consistent with 2% inflation) to $\theta^* \approx 1$ modestly increases inflation from 2% to 2.16%. In the shortage regime, doubling θ from 1 to 2 drives inflation significantly higher, from 2.16% to roughly above 5%.

In our framework, the economy typically operates below the Beveridge threshold, where both new-hire and incumbent wages adjust sluggishly. Once the threshold is breached, however, new-hire wages become highly responsive to tightness, while incumbent wages adjust more slowly, gradually converging toward the flexible wage at a speed governed by λ . The fact that the economy usually remains below the threshold reflects, at least in part, central-bank policy. A plausible interpretation of the 2020s surge is that, under its 2020 framework, the Federal Reserve reduced the role of preemptive tightening in anticipation of full employment (see Eggertsson and Kohn, 2024), reflecting skepticism about the precision with which the natural rate could be determined after the experience of 2015.⁴⁵

Our finding that the slope of the Phillips-curve steepens in tight markets also explains the forecast errors made by policymakers and market participants during the surge. Conventional empirical work, which imposes a single (flat) slope, led forecasters to underestimate the combined impact of post-pandemic demand and supply shocks. In slack conditions, our model is consistent with the view that such shocks would not significantly impact inflation. In tight markets, however – where the slope is steeper and supply shocks are amplified – much larger inflation responses are predicted.

Figure 5 illustrates the predictive gains from threshold effects for the surge in the 2020s. Using only data through 2021Q1—before inflation accelerated—we compare forecasts with and without thresholds. Without thresholds (red line, estimated 1960–2021Q1), the model predicts a peak of only 4.5%, compared to the realized 6.2%, reflecting the widespread forecast errors of 2021. With thresholds included (green line, estimated 1960–2021Q1), the peak forecast rises to 7.0%, close to the realization. Strikingly, estimating the model only on 1960–1969—the last period when $\theta > 1$ —yields a similar peak. A model calibrated more than fifty years earlier matches the 2020s episode precisely because labor-market conditions were similarly tight. Phillips curves without thresholds systematically underpredict inflation in tight markets. That the threshold effect emerges in both the 1960s and the 2020s underscores its robustness. These out-of-sample results validate the empirical relevance of the Beveridge threshold.

Our empirical analysis also clarifies the fitted drivers of the surge. Figures 6 and 7 stack the *within-sample fitted* contributions implied by equation (41) against actual core inflation. Colored bars report each regressor’s fitted component; hatched overlays denote the *incremental* contribution that arises only when $\theta_t > 1$ via the D_t interactions; the gray bar is the residual ε_t . This is an *accounting* decomposition of the fitted value, not a structural attribution of primitive shocks. In particular, it shows how

⁴⁵In 2015, the Fed raised rates, estimating maximum employment at 4.9%. However, unemployment fell further to 3.5% before the pandemic, with little sign of inflationary pressure.

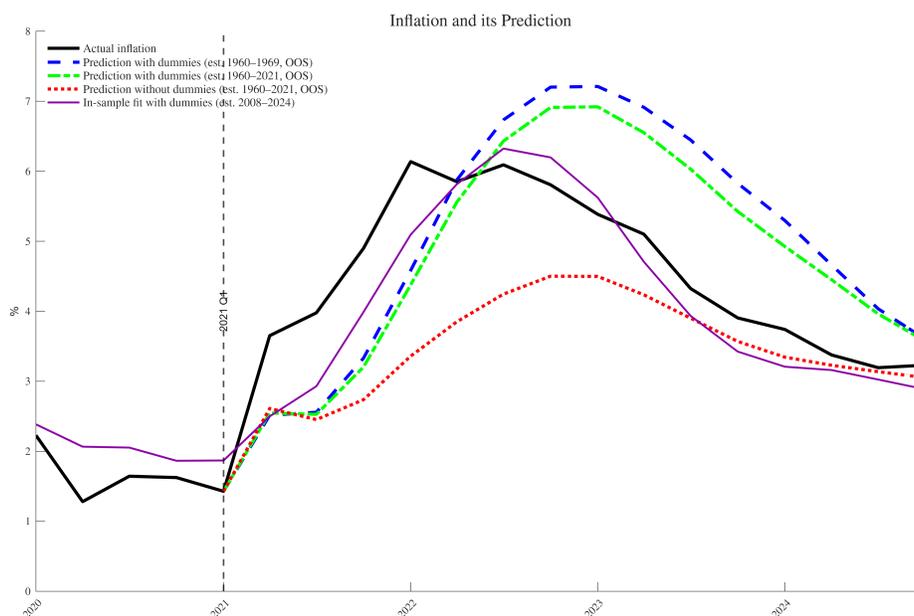


Figure 5: Actual, in-sample fit, and projections of core CPI inflation (YoY, %). Black: actual; green dashed-dot line: dynamic projection from 2021Q1 using a Phillips curve estimated over 1960Q1–2021Q1 with tightness-region dummies; red dotted line: projection from the no-dummy specification estimated on the same sample; thin solid purple line: in-sample fit from the dummy model; dash blue line: projection using coefficients estimated on 1960Q1–1969Q4 with dummies. The vertical dashed line marks the forecast origin (2021Q1); projected series coincide with actual at 2021Q1 and diverge from 2021Q2 onward. All series are year-on-year rates (%).

much of the fitted inflation at date t is accounted for by the observed movement in each regressor, weighted by the estimated coefficients. Because the regressors can be correlated and measured with error, the bars do not form an orthogonal variance decomposition and should not be read as causal shares.

In the post-2008 sample (Figure 6), tightness accounts only about 0.17 percentage points to inflation along the flat branch ($\theta \leq 1$). This is consistent with the flat Phillips curve consensus prior to the surge. The bulk of the surge is accounted for by incremental tightness effect once $\theta > 1$, together with a sizable interaction-augmented supply term early in the episode. Later, the supply component turns negative, but continued labor-market tightness accounts for inflation remaining above the 2% target. Early residuals are large, consistent with bottlenecks and shortages not captured by our pre-2020 supply proxy (cf. Bernanke and Blanchard, 2024).

Figure 7 reports the full-sample decomposition (the analog of Figure 6). In the 1970s episode, the expectations component dominates, beginning when tightness first exceeded unity in the late 1960s; by contrast, in the 2020s tightness dominates and the expectations component remains small. Thus, the full-sample estimates indicate that—unlike the recent surge—the Great Inflation was accounted

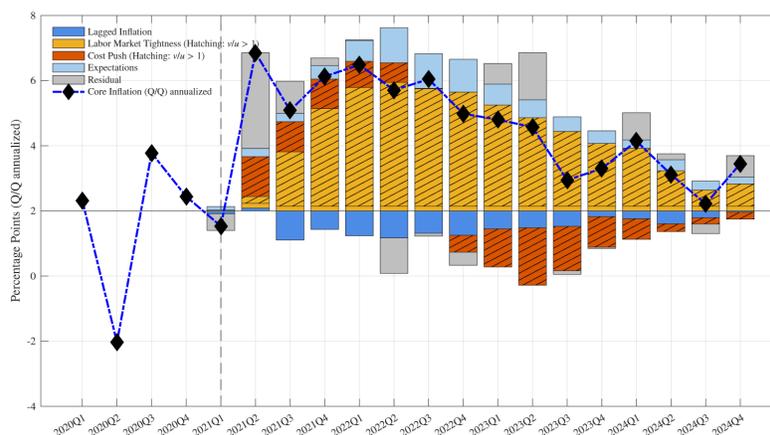


Figure 6: Decomposition of specification (3) in Table 4, 2008Q3–2024Q4, by regressor in equation (41). For $\ln \theta$, hatching denotes the contribution for the portion of θ that exceeds unity; for the supply shock ϱ , hatching denotes the increment when $\theta > 1$. Core inflation and all components are annualized quarter-over-quarter rates.

for primarily by unanchored expectations, consistent with conventional wisdom. More importantly, and in our view underappreciated, this unanchoring appears to have begun during the late-1960s labor shortage—an “original sin” that plausibly set the stage for the expectations-driven inflation of the 1970s. This historical pattern differs markedly from current data, helping to explain why a replay of the 1970s has not materialized, at least so far.

Finally, the framework sheds light on the recent “soft landing.” Between 2022Q2 and 2024Q4, U.S. core CPI inflation fell from 6.2% to 2.5%, while unemployment remained broadly below 4%. Standard (single-slope) Phillips curves typically imply that disinflation of nearly 4 percentage points would require a substantial rise in unemployment. By contrast, the inverse-L Phillips curve accommodates this outcome through two mechanisms. First, when $\theta > \theta^*$ the curve is steep, so disinflation is achieved with relatively small changes in slack. Second, as discussed in Benigno and Eggertsson (2024), the implied Beveridge curve is steeper when $\theta > \theta^*$, so a reduction in tightness is absorbed mainly by vacancies rather than a large increase in unemployment.

The literature on nonlinearities in the Phillips curve and on capacity constraints is still in its early stages. We have emphasized the central role of labor shortages in the United States, reflecting the fact that labor is the single largest input in production and that there was widespread evidence of scarcity during the recent inflation surge. Nonetheless, other capacity constraints likely contributed as well, though these are only partially captured by traditional measures of supply shocks. Future research should provide a more detailed characterization of the production structure and incorporate the role of other scarce inputs. A promising direction is to extend the analysis to sectoral heterogeneity and network linkages. Our objective here has been to paint, in broad strokes, the macroeconomic picture that places capacity constraints—and particularly labor shortages—at the core of inflation surges.

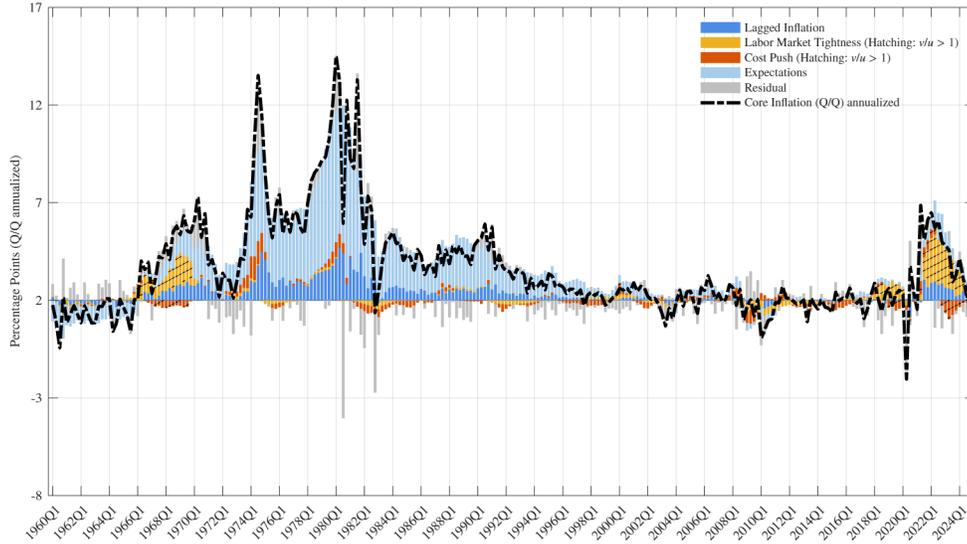


Figure 7: Decomposition of specification (3) in Table 4, 1960Q1–2024Q4, by regressor in equation (41). For $\ln \theta$, hatching denotes the contribution for the portion of θ that exceeds unity; for the supply shock ϱ , hatching denotes the increment when $\theta > 1$. Core inflation and all components are annualized quarter-over-quarter rates.

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A Appendix: Additional Tables

Table 5 presents the OLS estimates of regression (41) with the same variables as Table 4, except that the threshold for the dummy is set at $\theta^* = 0.5006$ for the sample 1960–2024, and at $\theta^* = 0.9305$ for the sample 2008–2024, which correspond to the thresholds in the respective samples that maximize the likelihood of the regressions across different thresholds, as shown in Figure 13.

Table 5: Phillips Curve Estimates with Endogenous Threshold

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.3031*** (0.1002)	0.1020 (0.2626)	0.3455*** (0.0863)	-0.1852 (0.1862)
$\ln \theta$	0.6988*** (0.1769)	0.5046 (0.3394)	0.2050 (0.3264)	0.3081 (0.3228)
$(\ln \theta - \ln \theta^*), \theta \geq \theta^*$			0.5912 (0.4318)	4.9621*** (1.4951)
<i>Supply shock</i> (ρ)	0.0355* (0.0198)	0.0135 (0.0393)	-0.0277 (0.0250)	-0.0113 (0.0229)
$\theta \geq \theta^*$			0.0776** (0.0331)	0.2645** (0.1039)
<i>Inflation expectations</i>	0.7706*** (0.1147)	1.6821** (0.6370)	0.5156*** (0.1366)	0.9080 (0.5486)
$\pi^e, \theta \geq \theta^*$			0.2673* (0.1418)	-0.1282 (0.9067)
<i>Constant</i>	0.4760*** (0.1443)	0.3511 (0.3751)	0.0168 (0.2964)	-0.0584 (0.2945)
R^2 adjusted	0.8184	0.5314	0.8350	0.6546
Observations	260	66	260	66

***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.

Newey–West standard errors (Bartlett, bandwidth 4).

(1) and (3): sample 1960 Q1 – 2024 Q4

(2) and (4): sample 2008 Q3 – 2024 Q4.

Table 6 presents the OLS estimates of regression (41) with the same variables as Table 4, except that the inflation lag and the log of the vacancy-to-unemployment ratio θ are instrumented with the fitted values of their first lags.

Table 6: Phillips Curve Estimates Using an Instrumental-Variables Approach

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.1991 ** (0.0980)	0.1578 (0.1840)	0.0826 (0.0899)	0.0096 (0.1058)
$\ln \theta$	0.7976 *** (0.2021)	-0.0307 (0.3744)	0.3655 * (0.2092)	0.1929 (0.2965)
$\theta \geq 1$			0.8988 (1.1273)	8.1427 *** (2.7372)
<i>Supply shock</i> (ρ)	0.0449 * (0.0229)	0.0057 (0.0370)	0.0479 * (0.0245)	-0.0060 (0.0240)
$\theta \geq 1$			0.1518 (0.0985)	0.4010 *** (0.1117)
<i>Inflation expectations</i>	0.9134 *** (0.0964)	2.3161 *** (0.7553)	1.0346 *** (0.0894)	1.4734 * (0.8211)
$\theta \geq 1$			1.2449 *** (0.2867)	-2.0714 * (1.1474)
<i>Constant</i>	0.5122 *** (0.1746)	-0.0922 (0.3913)	0.1929 (0.1704)	0.0928 (0.3364)
R^2 adjusted	0.8036	0.5078	0.8239	0.5879
Observations	260	66	260	66

• ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.

• Newey–West standard errors.

• (1) and (3): sample 1960 Q1 – 2024 Q4

• (2) and (4): sample 2008 Q3 – 2024 Q4

Table 7 presents the OLS estimates of regression (41) with the same variables as Table 4, except that the log of the vacancy-to-unemployment ratio θ is replaced with its first lag.

Table 7: Phillips Curve Estimates Using Lagged θ

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.2892 *** (0.0972)	0.1155 (0.2829)	0.1912 * (0.0974)	-0.0723 (0.2508)
$\ln \theta$	0.6266 *** (0.1741)	-0.0345 (0.3270)	0.2776 (0.1934)	0.1280 (0.2944)
$\theta \geq 1$			0.8690 (1.0262)	7.7345 *** (2.4568)
<i>Supply shock (ρ)</i>	0.0367 * (0.0191)	0.0037 (0.0371)	0.0432 ** (0.0208)	-0.0105 (0.0256)
$\theta \geq 1$			0.1205 (0.0962)	0.4145 *** (0.1359)
<i>Inflation expectations</i>	0.7803 *** (0.1085)	2.3777 ** (1.0966)	0.8992 *** (0.1080)	1.7336 * (0.8811)
$\theta \geq 1$			1.0224 *** (0.2969)	-2.1130 * (1.0777)
<i>Constant</i>	0.4520 *** (0.1466)	-0.0639 (0.3868)	0.1585 (0.1545)	0.0016 (0.3687)
R^2 adjusted	0.8141	0.5057	0.8283	0.5869
Observations	260	66	260	66

• ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.

• Newey–West standard errors.

• (1) and (3): sample 1960 Q1 – 2024 Q4

• (2) and (4): sample 2008 Q3 – 2024 Q4

Table 8 presents the OLS estimates of regression (41) with the same variables as Table 4, except that we proxy the supply shock with the four-quarter average of the CPI headline shock.

Table 8: Phillips Curve Estimates Using Headline-CPI Shock

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.3204 *** (0.1068)	0.1077 (0.2645)	0.2426 ** (0.1211)	-0.1200 (0.1972)
$\ln \theta$	0.7014 *** (0.1733)	0.5798 (0.3830)	0.4181 ** (0.1871)	0.6245 (0.4201)
$\theta \geq 1$			0.6972 (1.0157)	3.1559 (2.5580)
<i>Supply shock (ρ)</i>	0.1041 (0.0785)	0.1732 (0.2037)	0.0872 (0.0824)	0.0549 (0.1398)
$\theta \geq 1$			0.1795 (0.3036)	0.6877 * (0.3755)
<i>Inflation expectations</i>	0.7780 *** (0.1134)	1.3970 * (0.7420)	0.8811 *** (0.1259)	0.1094 (0.9052)
$\theta \geq 1$			0.6487 * (0.3348)	0.4029 (1.3974)
<i>Constant</i>	0.4351 *** (0.1365)	0.4305 (0.3767)	0.1876 (0.1463)	0.3157 (0.4124)
R^2 adjusted	0.8136	0.5412	0.8191	0.6165
Observations	260	66	260	66

- ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.
- Newey–West standard errors.
- (1) and (3): sample 1960 Q1 – 2024 Q4
- (2) and (4): sample 2008 Q3 – 2024 Q4

Table 9 presents the OLS estimates of regression (41) with the same variables as Table 4, except that we proxy the supply shock with the four-quarter average import-price shock.

Table 9: Phillips Curve Estimates Using Import-Price (Relative to GDP Deflator) as Supply Shock

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.3024 *** (0.0970)	0.1016 (0.2748)	0.2014 ** (0.0983)	-0.1772 (0.1849)
$\ln \theta$	0.7004 *** (0.1716)	0.5026 (0.3354)	0.3273 * (0.1828)	0.5495 (0.3758)
$\theta \geq 1$			1.7586 * (0.9003)	5.0976 *** (1.7505)
<i>Supply shock (ρ)</i>	0.0362 * (0.0198)	0.0131 (0.0395)	0.0428 ** (0.0211)	-0.0020 (0.0272)
$\theta \geq 1$			0.1043 (0.0896)	0.2573 ** (0.1041)
<i>Inflation expectations</i>	0.7710 *** (0.1094)	1.6896 ** (0.6753)	0.8961 *** (0.1097)	0.4435 (0.7893)
$\theta \geq 1$			0.7160 ** (0.3183)	0.3868 (1.2431)
<i>Constant</i>	0.4934 *** (0.1399)	0.3540 (0.3783)	0.1879 (0.1445)	0.2170 (0.3582)
R^2 adjusted	0.8185	0.5312	0.8289	0.6509
Observations	260	66	260	66

- ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.
- Newey–West standard errors.
- (1) and (3): sample 1960 Q1 – 2024 Q4
- (2) and (4): sample 2008 Q3 – 2024 Q4

Table 10 presents the OLS estimates of regression (41) with the same variables as Table 4, except that 1-year CPI inflation expectations from the U.S. Survey of Professional Forecasters are replaced by 2-year Cleveland Fed inflation expectations until 1982Q2, patched backward using the interpolated 12-month Livingston Survey inflation expectations until 1960Q1.

Table 10: Phillips Curve Estimates Using 2-Year Cleveland Fed Inflation Expectations

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.3696 *** (0.0947)	0.2730 (0.2445)	0.2544 *** (0.0930)	-0.1216 (0.1955)
$\ln \theta$	0.6722 *** (0.1758)	0.7235 * (0.3642)	0.2299 (0.1967)	0.3350 (0.2873)
$\theta \geq 1$			3.7262 *** (0.9425)	5.9788 *** (1.0072)
<i>Supply shock</i> (ρ)	0.0378 ** (0.0192)	0.0187 (0.0395)	0.0448 ** (0.0205)	-0.0249 (0.0229)
$\theta \geq 1$			0.1048 (0.0987)	0.2845 ** (0.1112)
<i>Inflation expectations</i>	0.6612 *** (0.1064)	0.7608 (0.6038)	0.8102 *** (0.1014)	1.0825 * (0.5782)
$\theta \geq 1$			0.0442 (0.3006)	-1.3779 ** (0.6310)
<i>Constant</i>	0.5522 *** (0.1513)	0.9027 ** (0.3892)	0.1915 (0.1634)	0.4214 (0.3392)
R^2 adjusted	0.8134	0.5063	0.8257	0.6792
Observations	260	66	260	66

- ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.
- Newey–West standard errors.
- (1) and (3): sample 1960 Q1 – 2024 Q4
- (2) and (4): sample 2008 Q3 – 2024 Q4

Table 11 presents the OLS estimates of regression (41) with the same variables as Table 4, except that inflation expectations are proxied by the Cleveland Fed’s 5-year inflation expectations until 1982Q2, patched with SPF 1-year GDP-deflator inflation expectations until 1970Q2 and the interpolated 12-month Livingston Survey inflation expectations until 1960Q1.

Table 11: Phillips Curve Estimates Using Cleveland Fed 5-Year Inflation Expectations

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.3957 *** (0.1002)	0.3237 (0.2390)	0.2790 *** (0.1028)	−0.1294 (0.2015)
$\ln \theta$	0.7413 *** (0.1829)	0.8577 ** (0.3957)	0.3282 (0.2028)	0.4501 (0.2883)
$\theta \geq 1$			3.7354 *** (0.9512)	6.0188 *** (1.0307)
<i>Supply shock (ρ)</i>	0.0482 ** (0.0190)	0.0305 (0.0383)	0.0574 *** (0.0205)	−0.0084 (0.0217)
$\theta \geq 1$			0.1025 (0.0962)	0.2581 ** (0.1064)
<i>Inflation expectations</i>	0.6443 *** (0.1202)	0.3463 (0.6506)	0.8022 *** (0.1218)	0.6586 (0.6107)
$\theta \geq 1$			−0.0007 (0.2739)	−1.2571 (0.7538)
<i>Constant</i>	0.5865 *** (0.1541)	0.8651 ** (0.3855)	0.2393 (0.1688)	0.3238 (0.3432)
R^2 adjusted	0.8055	0.4826	0.8171	0.6612
Observations	260	66	260	66

- ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.
- Newey–West standard errors.
- (1) and (3): sample 1960 Q1 – 2024 Q4
- (2) and (4): sample 2008 Q3 – 2024 Q4

Table 12 presents the OLS estimates of regression (41) with the same variables as Table 4, except that inflation expectations are proxied by the 5-year–5-year forward inflation expectations (backcast by Groen and Middelcorp, 2013, until 1971Q4), patched with the interpolated 12-month Livingston Survey inflation expectations until 1960Q1.

Table 12: Phillips Curve Estimates Using 5-Year–5-Year Forward Inflation Expectations

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.5818 *** (0.0839)	0.1401 (0.2411)	0.5633 *** (0.0902)	−0.1910 (0.2148)
$\ln \theta$	0.8490 *** (0.2008)	1.6901 *** (0.4985)	0.7243 *** (0.2136)	1.1782 * (0.6319)
$\theta \geq 1$			0.5756 (0.6069)	4.6325 *** (1.0531)
<i>Supply shock</i> (ρ)	0.0686 *** (0.0212)	0.0071 (0.0293)	0.0716 *** (0.0227)	−0.0149 (0.0242)
$\theta \geq 1$			0.0770 (0.0724)	0.2544 *** (0.0941)
<i>Inflation expectations</i>	0.3350 *** (0.0827)	1.4705 *** (0.4452)	0.3504 *** (0.0862)	0.9280 (0.6567)
$\theta \geq 1$			0.5241 ** (0.2182)	0.5745 (1.2585)
<i>Constant</i>	0.5256 *** (0.1243)	0.8216 ** (0.3632)	0.4217 *** (0.1382)	0.3916 (0.3311)
R^2 adjusted	0.7814	0.5637	0.7822	0.6834
Observations	260	66	260	66

- ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.
- Newey–West standard errors.
- (1) and (3): sample 1960 Q1 – 2024 Q4
- (2) and (4): sample 2008 Q3 – 2024 Q4

Table 13 presents the OLS estimates of regression (41) with the same variables as Table 4, except that inflation expectations are proxied by the Cleveland Fed’s 1-year inflation expectations until 1982Q2, patched with the interpolated 12-month Livingston Survey inflation expectations until 1960Q1.

Table 13: Phillips Curve Estimates Using Cleveland Fed 1-Year Inflation Expectations

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.3433 *** (0.0964)	0.2072 (0.2330)	0.2532 *** (0.0965)	-0.1249 (0.1777)
$\ln \theta$	0.6188 *** (0.1701)	0.6097 ** (0.3040)	0.2246 (0.1898)	0.3024 (0.2901)
$\theta \geq 1$			3.1556 *** (1.0540)	6.3169 *** (1.0933)
<i>Supply shock</i> (ρ)	0.0330 (0.0203)	-0.0010 (0.0376)	0.0388 * (0.0215)	-0.0375 (0.0250)
$\theta \geq 1$			0.0796 (0.0993)	0.3032 ** (0.1144)
<i>Inflation expectations</i>	0.7030 *** (0.1074)	0.9096 ** (0.4387)	0.8182 *** (0.1058)	0.9790 ** (0.4703)
$\theta \geq 1$			0.0222 (0.3327)	-1.1695 ** (0.5154)
<i>Constant</i>	0.5169 *** (0.1448)	0.8112 ** (0.3534)	0.2024 (0.1571)	0.3360 (0.2981)
R^2 adjusted	0.8175	0.5490	0.8262	0.6977
Observations	260	66	260	66

• ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.

• Newey–West standard errors.

• (1) and (3): sample 1960 Q1 – 2024 Q4

• (2) and (4): sample 2008 Q3 – 2024 Q4

Table 14 presents the OLS estimates of regression (41) with the same variables as Table 4, except that inflation expectations are proxied by the 12-month Livingston Survey inflation expectations.

Table 14: Phillips Curve Estimates Using 12-Month Livingston Survey Inflation Expectations

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.2982 *** (0.0911)	0.0946 (0.2059)	0.2032 ** (0.0912)	-0.1568 (0.1772)
$\ln \theta$	0.6761 *** (0.1647)	0.4695 (0.3052)	0.2942 * (0.1778)	0.5193 (0.3566)
$\theta \geq 1$			1.6274 * (0.9325)	5.5134 * (2.8068)
<i>Supply shock (ρ)</i>	0.0344 * (0.0193)	0.0167 (0.0321)	0.0419 ** (0.0202)	0.0017 (0.0247)
$\theta \geq 1$			0.0386 (0.0936)	0.2311 (0.1813)
<i>Inflation expectations</i>	0.7797 *** (0.1012)	1.6724 *** (0.4144)	0.8952 *** (0.1013)	0.3434 (0.5010)
$\theta \geq 1$			0.6341 ** (0.2468)	0.0409 (1.2967)
<i>Constant</i>	0.4975 *** (0.1337)	0.4528 (0.2920)	0.1930 (0.1408)	0.2147 (0.2668)
R^2 adjusted	0.8233	0.5644	0.8327	0.6474
Observations	260	66	260	66

- ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.
- Newey–West standard errors.
- (1) and (3): sample 1960 Q1 – 2024 Q4
- (2) and (4): sample 2008 Q3 – 2024 Q4

Table 15 presents the OLS estimates of regression (41) with the same variables as Table 4, except that inflation expectations are proxied by the 12-month household expectations inflation from the Michigan Survey (median until 1978Q1; mean back to 1960Q1).

Table 15: Phillips Curve Estimates Using 12-Month Michigan Inflation Expectations

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.4656 *** (0.0887)	-0.1430 (0.1697)	0.4667 *** (0.0900)	-0.2538 * (0.1379)
$\ln \theta$	0.3475 *** (0.1162)	1.1007 *** (0.2845)	0.4355 *** (0.1525)	0.6837 *** (0.2558)
$\theta \geq 1$			-0.1310 (0.8092)	-1.2926 (2.4879)
<i>Supply shock (q)</i>	-0.0148 (0.0195)	-0.0375 (0.0248)	-0.0213 (0.0211)	-0.0329 (0.0210)
$\theta \geq 1$			0.1273 ** (0.0593)	0.0588 (0.0700)
<i>Inflation expectations</i>	0.6610 *** (0.1337)	1.4115 *** (0.2726)	0.6675 *** (0.1382)	0.7823 *** (0.2009)
$\theta \geq 1$			-0.0041 (0.1548)	1.1425 ** (0.5232)
<i>Constant</i>	-0.1546 (0.1268)	-0.6398 *** (0.2112)	-0.0997 (0.1385)	-0.5324 * (0.2697)
R^2 adjusted	0.7997	0.6895	0.8001	0.7430
Observations	253	66	253	66

• ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.

• Newey–West standard errors.

• (1) and (3): sample 1960 Q1 – 2024 Q4

• (2) and (4): sample 2008 Q3 – 2024 Q4

Table 16 presents the OLS estimates of regression (41) with the same variables as Table 4, except that PCE core inflation replaces CPI core inflation as the dependent variable.

Table 16: Phillips Curve Estimates Using PCE Core

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.5185 *** (0.0653)	0.3897 (0.2365)	0.4023 *** (0.0739)	0.1216 (0.2114)
$\ln \theta$	0.2910 *** (0.1058)	0.2312 (0.2777)	0.0006 (0.1333)	0.2300 (0.3098)
$\theta \geq 1$			1.5721 ** (0.7694)	3.2802 (1.9847)
<i>Supply shock</i> (ρ)	0.0347 *** (0.0132)	0.0275 (0.0291)	0.0411 *** (0.0145)	0.0257 (0.0214)
$\theta \geq 1$			0.0779 (0.0676)	0.1969 ** (0.0784)
<i>Inflation expectations</i>	0.4341 *** (0.0660)	0.9594 (0.6264)	0.5539 *** (0.0748)	0.3611 (0.7184)
$\theta \geq 1$			0.3889 (0.2867)	0.1772 (1.1208)
<i>Constant</i>	0.1384 (0.0911)	0.0273 (0.3305)	-0.1219 (0.1138)	-0.1604 (0.3462)
R^2 adjusted	0.8664	0.5857	0.8736	0.6566
Observations	260	66	260	66

- ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.
- Newey–West standard errors.
- (1) and (3): sample 1960 Q1 – 2024 Q4
- (2) and (4): sample 2008 Q3 – 2024 Q4

Table 17 presents the OLS estimates of regression (41) with the same variables as Table 4, except that the level of θ is used rather than its log.

Table 17: Phillips Curve Estimates Using θ in Levels

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Inflation lag</i>	0.2681 *** (0.0970)	0.0386 (0.2599)	0.2109 ** (0.0991)	-0.1619 (0.1952)
θ	1.1531 *** (0.2367)	1.3471 * (0.6910)	0.6037 * (0.3315)	1.1341 (0.7401)
$\theta \geq 1$			0.5989 (0.7599)	2.7894 (1.6721)
<i>Supply shock (ρ)</i>	0.0381 * (0.0198)	0.0195 (0.0382)	0.0416 ** (0.0208)	-0.0027 (0.0258)
$\theta \geq 1$			0.0978 (0.0914)	0.2362 ** (0.1109)
<i>Inflation expectations</i>	0.8187 *** (0.1091)	1.1394 (0.7234)	0.8867 *** (0.1111)	0.5587 (0.7682)
$\theta \geq 1$			0.7741 ** (0.3074)	0.1342 (1.2685)
<i>Constant</i>	-0.6709 *** (0.1538)	-0.8125 ** (0.3551)	-0.3837 * (0.1986)	-0.8038 * (0.4112)
R^2 adjusted	0.8232	0.5595	0.8278	0.6449
Observations	260	66	260	66

- ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.
- Newey–West standard errors.
- (1) and (3): sample 1960 Q1 – 2024 Q4
- (2) and (4): sample 2008 Q3 – 2024 Q4

Table 18 presents the OLS estimates of regression (41) with the same variables as Table 4, except that the lag of inflation is replaced by the lag of the detrended real wage.

Table 18: Phillips Curve Estimates Using Lagged Real Wages (Instead of Lagged Inflation)

	(1)	(2)	(3)	(4)
	1960-2024	2008-2024	1960-2024	2008-2024
<i>Lagged real wage</i>	6.9269 (4.7426)	-7.1417 (5.7925)	8.5456 ** (4.2473)	-10.6450 (8.2331)
$\ln \theta$	0.8153 *** (0.2190)	0.6620 ** (0.3174)	0.1595 (0.2329)	0.7296 * (0.4275)
$\theta \geq 1$			3.0059 *** (0.9867)	3.8252 * (1.9456)
<i>Supply shock (ρ)</i>	0.0423 * (0.0249)	0.0085 (0.0402)	0.0521 ** (0.0250)	-0.0125 (0.0248)
$\theta \geq 1$			0.0920 (0.0954)	0.2775 *** (0.0987)
<i>Inflation expectations</i>	1.0961 *** (0.0490)	1.7720 *** (0.3516)	1.1051 *** (0.0484)	0.4377 (0.9747)
$\theta \geq 1$			0.6934 ** (0.3286)	-0.0514 (1.4607)
<i>Constant</i>	14.4640 (9.5115)	-13.8490 (11.6882)	17.1818 ** (8.4606)	-20.9096 (16.2811)
R^2 adjusted	0.8006	0.5345	0.8252	0.6519
Observations	260	66	260	66

• ***, **, * denote statistical significance at the 1, 5, and 10 percent level, respectively.

• Newey–West standard errors.

• (1) and (3): sample 1960 Q1 – 2024 Q4

• (2) and (4): sample 2008 Q3 – 2024 Q4

Table 19: Correlation between CPI core inflation and different measures of inflation expectations.

Inflation expectations	2008Q3 - 2024Q4	2020Q4 - 2024Q4
2-year Cleveland	0.5998	0.1874
1 year SPF	0.7289	0.5588
5-year Cleveland	0.4332	0.0439
5y-5y Forward	-0.0595	0.2941
12-month Michigan Survey	0.7006	0.8226
1-year Cleveland	0.6805	0.4026
12-month Livingston Survey	0.7480	0.6516

Table 20: Granger causality test: Core CPI inflation causing inflation expectations.

Variable	h	p-Value
2008Q3 - 2024Q4		
2-year Cleveland	0	0.4815
1 year SPF	0	0.9859
5-year Cleveland	0	0.3582
5y-5y Forward	0	0.4175
12-month Michigan Survey	1	0.0202
1-year Cleveland	0	0.6241
12-month Livingston Survey	0	0.1881

Table 21: Reverse Granger causality test: Inflation expectations causing CPI core inflation.

Variable	h	p-Value
2008Q3 - 2024Q4		
2-year Cleveland (baseline)	1	0.0267
1 year SPF	1	0.0000
5-year Cleveland	0	0.1077
5y-5y Forward	0	0.4446
12-month Michigan Survey	1	0.0059
1-year Cleveland	1	0.0033
12-month Livingston Survey	1	0.0014

B Appendix: Additional Figures

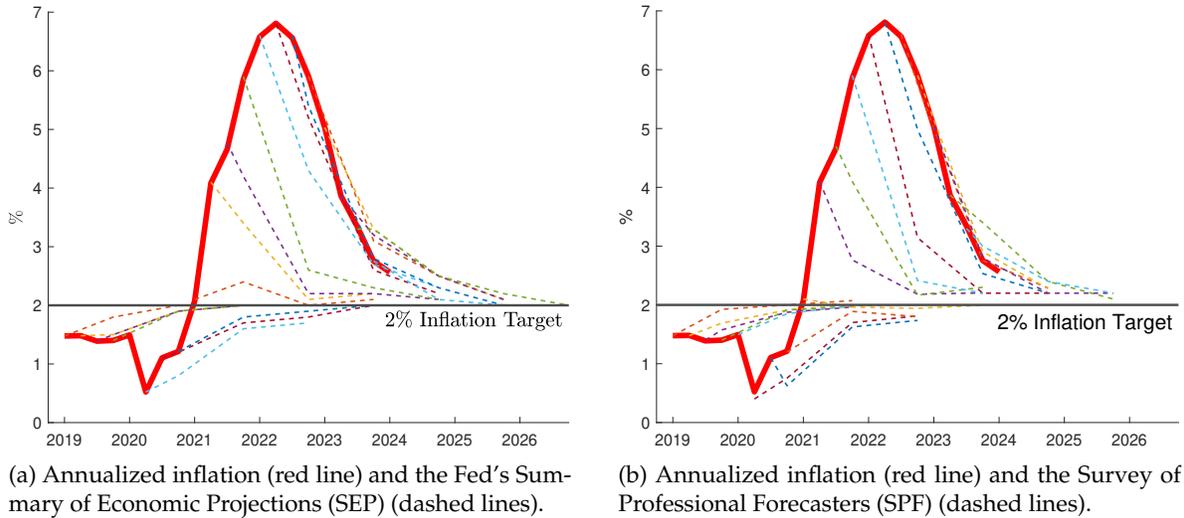


Figure 8: Inflation forecasts during the inflation surge

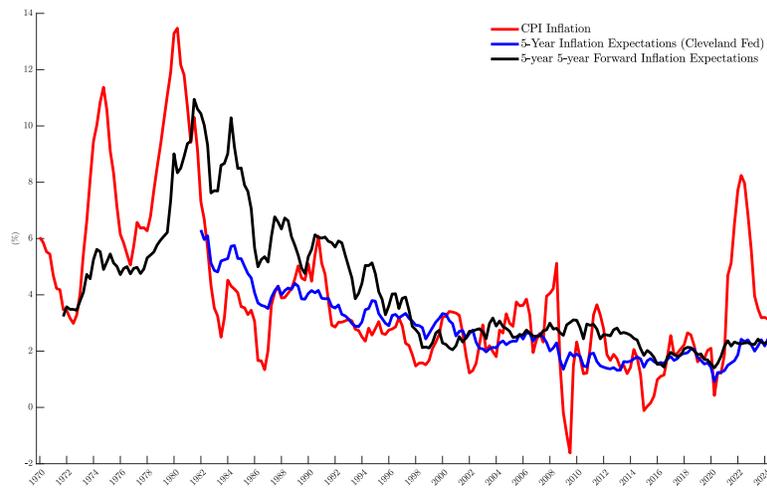


Figure 9: This figure contrasts CPI inflation at annual rates with the five-year expected inflation rate compiled by the Cleveland Fed and five-year five-year forward inflation expectations, which are market-based from 1997 and back-casted by Groen and Middledorp (2013) to 1970.

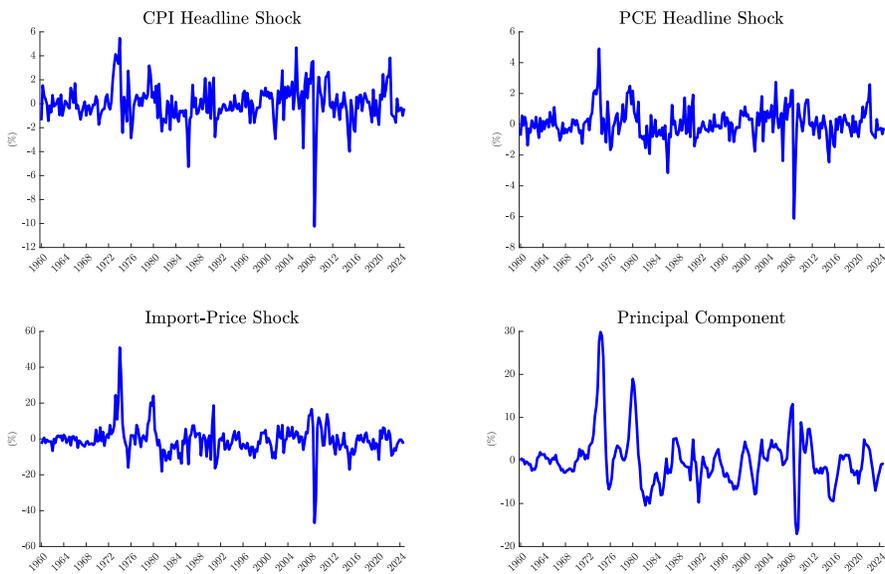


Figure 10: Measures of supply shock and their principal component (four-quarter average)

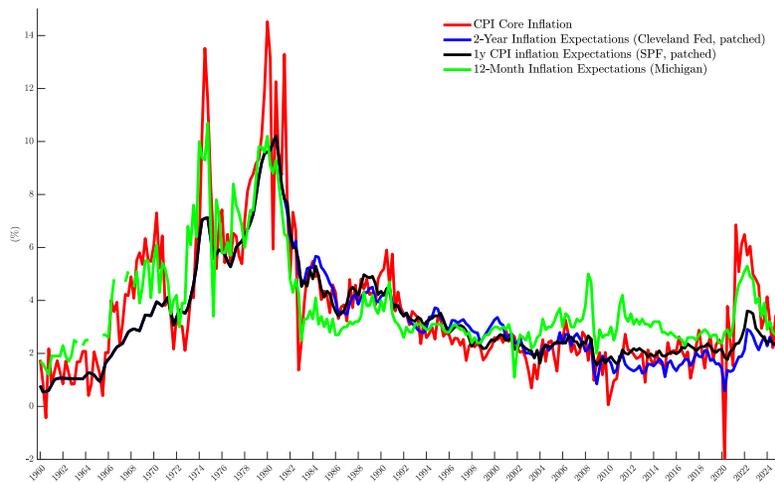


Figure 11: CPI core inflation (annualized quarterly rates). 2-year inflation expectations of the Cleveland Fed patched, before 1982 Q1, with 12-month Livingston survey inflation expectations. 1-year CPI inflation expectations of SPF patched, before 1981 Q3, with 12-month Livingston survey inflation expectations. 12-month households inflation expectations of the Michigan survey.

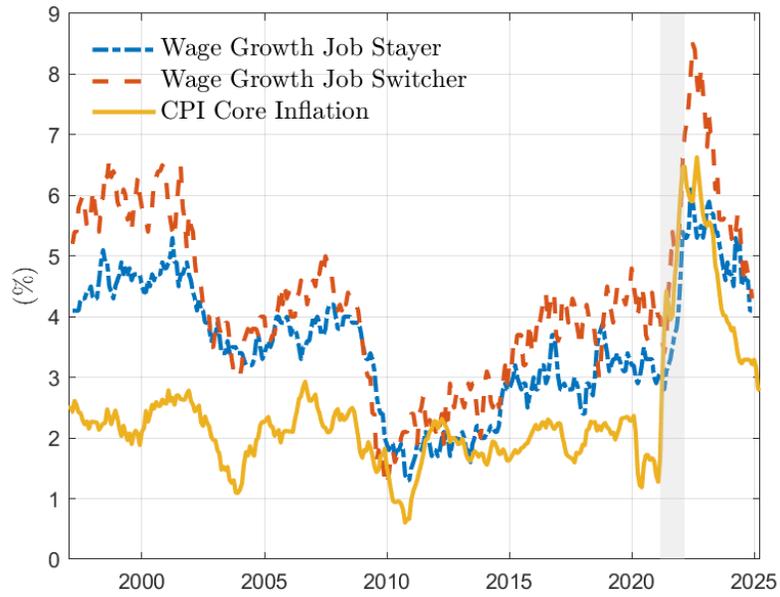


Figure 12: Wage growth (%), overall, and decomposition between job switchers and job stayers, from Wage Growth Tracker of the Federal Reserve Bank of Atlanta.

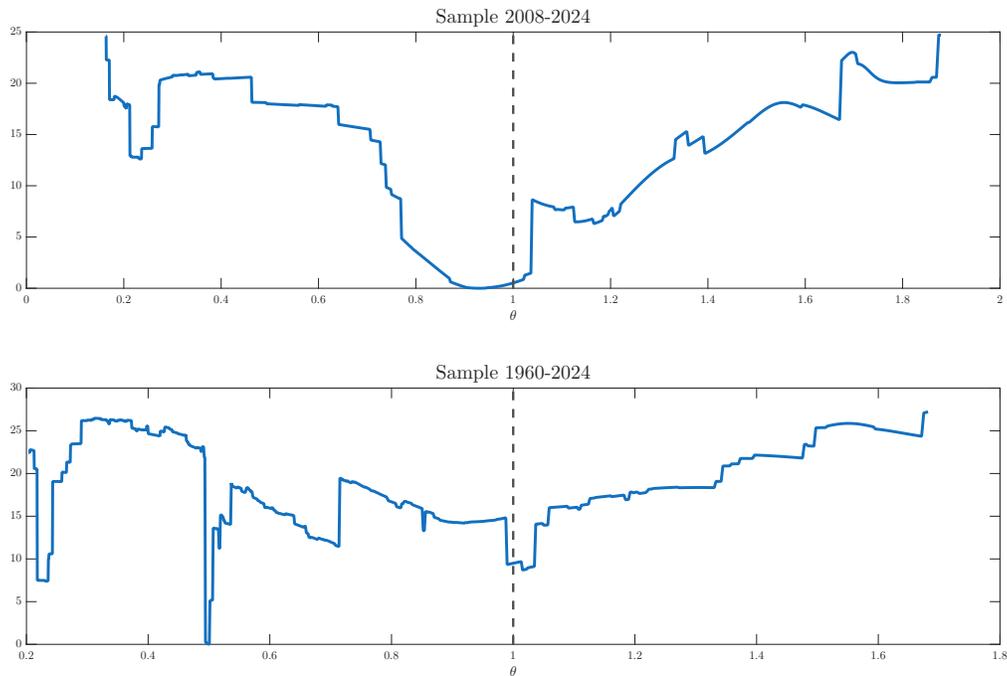


Figure 13: Profile likelihood-ratio statistic $LR(\theta^*)$ for the samples 2008–2024 (top panel) and 1960–2024 (bottom panel). The vertical dashed line marks $\theta = 1$ in levels.

C Appendix: Backcasting θ from 1919 to 1914

A.1 Motivation

Labor market tightness ($\theta_t = V_t/U_t$) necessitates data on vacancies and unemployment. While unemployment and inflation data extend back to 1914, vacancy data became available only from January 1919 onward. We therefore backcast θ_t for the 60-month period from January 1914 to December 1918 using a predictive regression estimated on data from post-1919. The only result dependent on this back-casting is the reported mean and peak during the WWI inflation surge for θ in Table 1, which have no implication for the other numbers reported in that table or any subsequent analyzes. Accordingly, we use a simple and straightforward backcasting method based on Romer (1991).

A.2 Methodology

Following Romer (1989), who estimated prewar GNP using relationships from later periods, we specify:

$$\log(\theta_t) = \beta_0 + \beta_1 \log(U_t^{(6m)} + 1) + \beta_2 \pi_t^{(6m)} + \varepsilon_t \quad (\text{C.1})$$

where $U_t^{(6m)}$ is the 6-month trailing moving average of the unemployment rate, $\pi_t^{(6m)}$ is the 6-month trailing moving average of year-over-year inflation, and the $+1$ shift in $\log(U + 1)$ prevents undefined values at low unemployment and stabilizes elasticities.

A.3 Estimation Results

We estimate the model via OLS using all months from January 1919 to May 2018, during which actual θ_t is observed ($N = 1,193$ observations). Table 22 reports the results.

Table 22: Backcasting Regression Results

Variable	Coefficient	Std. Error	<i>t</i> -statistic	<i>p</i> -value
Constant	3.018	0.052	57.89	<0.001***
$\log(U_t^{(6m)} + 1)$	-1.964	0.025	-77.75	<0.001***
$\pi_t^{(6m)}$	0.021	0.002	9.68	<0.001***
Observations		1,193		
R^2		0.869		
Adjusted R^2		0.869		
Residual Std. Error		0.327 (df = 1,190)		
<i>F</i> -statistic		3,947.91 on 2 and 1,190 df ($p < 0.001$)		

Note: *** $p < 0.001$. Estimation period: January 1919 to May 2018.

The high explanatory power ($R^2 = 0.869$) indicates that smoothed unemployment and inflation are strong predictors of labor market tightness. The negative coefficient on unemployment ($\beta_1 = -1.964$) confirms that labor markets tighten significantly when unemployment decreases, whereas the positive inflation coefficient ($\beta_2 = 0.021$) captures the relationship between tight labor markets and price pressures.

A.4 Backcasting Procedure

We generate predicted values from January 1914 to December 1918 (60 months) using:

$$\hat{\theta}_t = \exp\left(3.018 - 1.964 \cdot \log(U_t^{(6m)} + 1) + 0.021 \cdot \pi_t^{(6m)}\right) \quad (\text{C.2})$$

To ensure continuity at the transition point, we calculate an anchor ratio using the first six months of available data (January–June 1919): $\gamma = \text{mean}(\theta_t / \hat{\theta}_t) = 0.835$. All pre-1919 predictions are scaled by this ratio: $\tilde{\theta}_t = 0.835 \times \hat{\theta}_t$.

The final series combines backcast estimates with actual measurements:

$$\theta_t^{(\text{final})} = \begin{cases} \tilde{\theta}_t & \text{for } t \in \text{Jan 1914} - \text{Dec 1918} \\ \theta_t^{(\text{actual})} & \text{for } t \geq \text{Jan 1919} \end{cases} \quad (\text{C.3})$$

D Appendix: The Model

D.1 Derivation of the AS equation (12)

The firms' discounted value of current and expected future profits are:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \left\{ p_T(i) y_T(i) - W_T^{ex} N_T^{ex}(i) - (1 + \gamma_T^b) W_T^{new} N_T^{new}(i) - P_T q_T O_T(i) - \frac{\zeta}{2} \left(\frac{p_T(i)}{p_{T-1}(i)} \frac{1}{\Pi} - 1 \right)^2 P_T Y_T \right\}$$

where $Q_{t,T} \equiv \beta^{T-t} (C_T^{-\sigma} / P_T) / (C_t^{-\sigma} / P_t)$ is the stochastic discount factor the household uses at time t . Note that the maximization problem is subject to the following constraints:

$$y_t(i) = A_t (N_t^{ex}(i) + N_t^{new}(i))^\alpha O_t(i)^{1-\alpha}, \quad (D.4)$$

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\epsilon_t} Y_t, \quad (D.5)$$

$$0 \leq N_t^{ex}(i) \leq N_t^{ex} = \int_0^1 N_t^{ex}(j) dj, \quad (D.6)$$

$$N_t^{new}(i) \geq 0. \quad (D.7)$$

We can use (D.4) to solve for $N_t^{new}(i)$ to obtain

$$N_t^{new}(i) = \left(\frac{y_t(i)}{A_t} \right)^{\frac{1}{\alpha}} O_t(i)^{\frac{\alpha-1}{\alpha}} - N_t^{ex}(i),$$

which substituted into the objective function yields to:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} \left\{ p_T(i) y_T(i) - (W_T^{ex} - (1 + \gamma_T^b) W_T^{new}) N_T^{ex}(i) - (1 + \gamma_T^b) W_T^{new} \left(\frac{y_T(i)}{A_T} \right)^{\frac{1}{\alpha}} O_T(i)^{\frac{\alpha-1}{\alpha}} + \right. \\ \left. - P_T q_T O_T(i) - \frac{\zeta}{2} \left(\frac{p_T(i)}{p_{T-1}(i)} \frac{1}{\Pi} - 1 \right)^2 P_T Y_T \right\}.$$

Note that whenever $W_t^{ex} < (1 + \gamma_t^b) W_t^{new}$, it follows, using (D.6), that the optimal choice for $N_t^{ex}(i)$ is $N_t^{ex}(i) = N_t^{ex}$. Given this result, first-order conditions with respect to $p_t(i)$ and $O_t(i)$ imply

$$0 = (1 - \epsilon_t) y_t(i) + \epsilon_t (1 + \gamma_t^b) W_t^{new} \frac{1}{\alpha} \left(\frac{y_t(i)}{A_t} \right)^{\frac{1}{\alpha}-1} \frac{y_t(i)}{A_t p_t(i)} O_t(i)^{\frac{\alpha-1}{\alpha}} + \\ - \zeta \left(\frac{p_t(i)}{p_{t-1}(i)} \frac{1}{\Pi} - 1 \right) \frac{1}{p_{t-1}(i) \Pi} P_t Y_t + \zeta E_t \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i) \Pi} - 1 \right) \frac{p_{t+1}(i)}{(p_t(i))^2 \Pi} P_{t+1} Y_{t+1} \right\}$$

and

$$\frac{1-\alpha}{\alpha}(1+\gamma_t^b)W_t^{new}\left(\frac{y_t(i)}{A_t}\right)^{\frac{1}{\alpha}}O_t(i)^{-\frac{1}{\alpha}}=P_tq_t.$$

We can combine the second first-order condition into the first to substitute for $O_t(i)$ and obtain

$$0 = (1-\epsilon_t)y_t(i) + \epsilon_t \left(\frac{(1+\gamma_t^b)W_t^{new}}{\alpha} \right)^\alpha \left(\frac{P_tq_t}{1-\alpha} \right)^{1-\alpha} \frac{y_t(i)}{A_t p_t(i)} + \\ -\varsigma \left(\frac{p_t(i)}{p_{t-1}(i)\Pi} - 1 \right) \frac{1}{p_{t-1}(i)\Pi} P_t Y_t + \varsigma E_t \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i)\Pi} - 1 \right) \frac{p_{t+1}(i)}{(p_t(i))^2 \Pi} P_{t+1} Y_{t+1} \right\}.$$

All firms are going to set the same price therefore $p_t(i) = P_t$ and $y_t(i) = Y$. We can then obtain

$$0 = (1-\epsilon_t) + \frac{\epsilon_t}{A_t} \left(\frac{1+\gamma_t^b}{\alpha} \frac{W_t^{new}}{P_t} \right)^\alpha \left(\frac{q_t}{1-\alpha} \right)^{1-\alpha} - \varsigma \left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} + \\ + \varsigma E_t \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \right\},$$

from which we can obtain equation (12), here restated as

$$\left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} = \frac{\epsilon_t - 1}{\varsigma} \left(\frac{\mu_t}{A_t} \left(\frac{1+\gamma_t^b}{\alpha} \frac{W_t^{new}}{P_t} \right)^\alpha \left(\frac{q_t}{1-\alpha} \right)^{1-\alpha} - 1 \right) + \\ + \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \right\}, \quad (\text{D.8})$$

in which we have defined $\mu_t \equiv \epsilon_t / (\epsilon_t - 1)$.

Note that whenever $W_t^{ex} = (1+\gamma_t^b)W_t^{new}$, the above derivation applies too implying the same AS equation.

D.2 Inv-L Phillips curve characterization

In this Section we derive the Inv-L Phillips curve characterization through a log-linear approximation of equation (D.8) considering that

$$w_t^{new} = \max(w_t^{ex}, w_t^{flex}), \quad (\text{D.9})$$

where

$$w_t^{flex} = \frac{\gamma_t^c}{\gamma_t^b} \frac{1}{m_t} \theta_t^\eta, \quad (\text{D.10})$$

and

$$w_t^{ex} = \left(w_{t-1}^{ex} \frac{(\Pi_{t+1}^e)^\delta}{\Pi_t} \right)^\lambda (w_t^{flex})^{1-\lambda} \phi_t. \quad (\text{D.11})$$

The approximation is going to deliver a piece-wise log-linear function as it is shown in equation (37) in the main text. First, we characterize the steady state of the general equilibrium model, then we take a first-order log-linear approximation of (D.8), (D.9), (D.10) and (D.11).

D.2.1 Steady state

Let us first consider a steady state in which $\zeta_t = \zeta$, $A_t = A$, $\epsilon_t = \epsilon$, $q_t = q$, $G_t = G$, $m_t = m$, $s_t = s$, $\gamma_t^c = \gamma^c$, $\gamma_t^b = \gamma^b$, $\phi_t = \phi$, $\Pi_t = \Pi$.⁴⁶ The steady-state versions of equations (12), (15), (18), (22), and (26) imply:

$$w^{new} = \frac{\alpha}{1 + \gamma^b} \left(\frac{\epsilon - 1}{\epsilon} A \right)^{\frac{1}{\alpha}} \left(\frac{q}{1 - \alpha} \right)^{-\frac{1-\alpha}{\alpha}}, \quad (\text{D.12})$$

$$N + U = F, \quad (\text{D.13})$$

$$\tilde{U} = U + sN, \quad (\text{D.14})$$

$$w^{flex} = \frac{1}{m} \frac{\gamma^c}{\gamma^b} \theta^\eta, \quad (\text{D.15})$$

$$N = \frac{m}{s} \tilde{U} \theta^{1-\eta} \quad (\text{D.16})$$

Moreover

$$w^{new} = \max(w^{ex}, w^{flex}) \quad (\text{D.17})$$

with

$$w^{ex} = (w^{ex} \Pi^{-1} (\Pi)^\delta)^\lambda (w^{flex})^{1-\lambda} \phi. \quad (\text{D.18})$$

We consider a steady state in which $w^{flex} \leq w^{ex}$ and therefore $w^{new} = w^{ex} = \bar{w}$, with

$$\bar{w} = \frac{\alpha}{1 + \gamma^b} \left(\frac{\epsilon - 1}{\epsilon} A \right)^{\frac{1}{\alpha}} \left(\frac{q}{1 - \alpha} \right)^{-\frac{1-\alpha}{\alpha}}.$$

In this steady state $\bar{\theta} \leq \bar{\theta}^*$, which requires ϕ to satisfy the inequality $\phi \geq \Pi^{\lambda(1-\delta)}$. Note that $\bar{\theta}^*$ is defined when $w^{flex} = w^{new} = w^{ex} = \bar{w}$, therefore, in this case, $\phi = \Pi^{\lambda(1-\delta)}$ whereas $w^{flex} = w^{new}$ implies

$$\bar{\theta}^* = \left[\frac{\alpha m \gamma^b}{\gamma^c (1 + \gamma^b)} \left(\frac{\epsilon - 1}{\epsilon} A \right)^{\frac{1}{\alpha}} \left(\frac{q}{1 - \alpha} \right)^{-\frac{1-\alpha}{\alpha}} \right]^{\frac{1}{\eta}}.$$

In our approximation, we consider a steady state in which $\bar{\theta} < \bar{\theta}^*$, requiring, therefore, $\phi > \Pi^{\lambda(1-\delta)}$. For a given ϕ , satisfying the inequality, we can then use (D.15) into (D.18) noting that $w^{ex} = \bar{w}$, to determine $\bar{\theta}$. We can then combine (D.13), (D.14) and (D.16) to obtain the steady-state value of N , U and \tilde{U} .

⁴⁶We are generalizing the analysis by having z stochastic.

D.2.2 Derivation of equation (37)

In a log-linear approximation of equation (D.8), the AS equation is:

$$\pi_t - \pi = \frac{(\epsilon - 1)}{\varsigma} (\hat{\mu}_t + \alpha(\hat{w}_t^{new} + d_\gamma \hat{\gamma}_t^b) - \hat{A}_t + (1 - \alpha)\hat{q}_t) + \beta E_t(\pi_{t+1} - \pi), \quad (\text{D.19})$$

in which $d_\gamma \equiv \gamma^b / (1 + \gamma^b)$.

Consider first the case in which $\theta_t \geq \theta_t^*$ and $w_t^{new} = w^{ex}$, then it follows that $\hat{w}_t^{new} = -c_w + \hat{w}_t^{flex}$ where $c_w = \ln(\bar{w}/\bar{w}^{flex})$, with $c_w \geq 0$ and, in particular, $c_w = 0$ whenever the steady-state approximation is taken at the kink point, $\bar{\theta} = \bar{\theta}^*$. A log-linear approximation of equation (D.10) implies that:

$$\hat{w}_t^{flex} = \eta \hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t. \quad (\text{D.20})$$

Using (D.20) into the AS equation, we obtain:

$$\pi_t - \pi = -c + \kappa^{tight} \hat{\theta}_t + \kappa_v^{tight} (\hat{v}_t + \hat{\theta}_t^{tight}) + \beta E_t(\pi_{t+1} - \pi), \quad (\text{D.21})$$

given the following parameters

$$\begin{aligned} \kappa_c^{tight} &= \frac{(\epsilon - 1)\alpha}{\varsigma} \\ \kappa^{tight} &= \frac{(\epsilon - 1)\alpha\eta}{\varsigma}, \\ \kappa_v^{tight} &= \frac{(\epsilon - 1)}{\varsigma}, \\ c &= \kappa_c^{tight} c_w, \end{aligned}$$

having defined

$$\begin{aligned} \hat{v}_t &\equiv \hat{\mu}_t - \hat{A}_t + (1 - \alpha)\hat{q}_t, \\ \hat{\theta}_t^{tight} &= \alpha(\hat{\gamma}_t^c - (1 - d_\gamma)\hat{\gamma}_t^b - \hat{m}_t). \end{aligned}$$

Now consider the state in which $\theta_t < \theta_t^*$ and $w_t^{new} = w_t^{ex}$, then it follows that $\hat{w}_t^{new} = \hat{w}_t^{ex}$, and therefore using (D.11) that:

$$\begin{aligned} \hat{w}_t^{new} &= \lambda \hat{w}_{t-1} - \lambda(\pi_t - \pi) + \lambda \delta E_t(\pi_{t+1} - \pi) + (1 - \lambda)\hat{w}_t^{flex} + \hat{\phi}_t \\ &= \lambda \hat{w}_{t-1} - \lambda(\pi_t - \pi) + \lambda \delta E_t(\pi_{t+1} - \pi) + (1 - \lambda)(\eta \hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t) + \hat{\phi}_t, \end{aligned}$$

in which we have used \hat{w}_{t-1} in place of \hat{w}_{t-1}^{ex} and equation (D.10).

We can then substitute the wage norm into (D.19) to write it as

$$\begin{aligned} \pi_t - \pi = & \frac{(\epsilon - 1)}{\zeta} \left\{ \hat{\mu}_t + \alpha[\lambda \hat{w}_{t-1} - \lambda(\pi_t - \pi) + \lambda \delta E_t(\pi_{t+1} - \pi) + d_\gamma \hat{\gamma}_t^b] + \right. \\ & \left. + (1 - \lambda)(\eta \hat{\theta}_t + \hat{\gamma}_t^c - \hat{\gamma}_t^b - \hat{m}_t) + \hat{\phi}_t - \hat{A}_t + (1 - \alpha) \hat{q}_t \right\} + \beta E_t(\pi_{t+1} - \pi), \end{aligned}$$

which can be written more compactly as

$$\pi_t - \pi = \kappa_w \hat{w}_{t-1} + \kappa \hat{\theta}_t + \kappa_\nu (\hat{\nu}_t + \hat{\theta}_t) + \kappa_\beta E_t(\pi_{t+1} - \pi), \quad (\text{D.22})$$

given the following parameters

$$\begin{aligned} \kappa_w &= 1 - \psi, \\ \kappa &= (1 - \lambda) \psi \kappa^{tight}, \\ \kappa_\nu &= \psi \kappa_\nu^{tight}, \\ \kappa_\beta &= (1 - \psi) \delta + \psi \beta, \end{aligned}$$

with ψ being a positive parameter with $0 < \psi \leq 1$ defined as

$$\psi \equiv \frac{1}{1 + \frac{(\epsilon - 1)}{\zeta} \alpha \lambda}.$$

in which

$$\hat{\theta}_t = \alpha(1 - \lambda)(\hat{\gamma}_t^c - \hat{m}_t) + \alpha \hat{\phi}_t + \alpha(d_\gamma - (1 - \lambda)) \hat{\gamma}_t^b.$$

Note that $\kappa < \kappa^{tight}$ and $\kappa_\nu < \kappa_\nu^{tight}$, since $0 < \psi \leq 1$ and $0 < \lambda \leq 1$.

Note that

$$\pi_t - \pi = -c + \kappa^{tight} \hat{\theta}_t + \kappa_\nu^{tight} (\hat{\nu}_t + \hat{\theta}_t^{tight}) + \beta E_t(\pi_{t+1} - \pi),$$

applies when $\hat{\theta}_t \geq \hat{\theta}_t^*$ while

$$\pi_t - \pi = \kappa_w \hat{w}_{t-1} + \kappa \hat{\theta}_t + \kappa_\nu (\hat{\nu}_t + \hat{\theta}_t) + \kappa_\beta E_t(\pi_{t+1} - \pi),$$

whenever $\hat{\theta}_t < \hat{\theta}_t^*$. A requirement for this to be a log-linear approximation is that c must be of the same order as the norm of the shocks.

E Appendix: empirical results using Kalman-filter estimation

As an alternative benchmark to capture nonlinearities, we consider a specification that allows for time-varying coefficients, focusing on the period from 2008 Q3 to 2024 Q4. Generally, the results support our previous findings. We follow closely the existing literature, see Blanchard, Cerutti and Summers (2015). We consider the regression reported in Table 1, but the parameters are now allowed to vary over time by a random walk. The model is estimated by a Kalman filter using as initial conditions the OLS estimates generated by a regression up to 2008 Q2. Figure 14 shows how the estimated coefficients vary over time from 2008 Q3 to 2024 Q4 with red lines. The blue lines correspond to one-standard-deviation confidence bands.

The main conclusion is that the estimated coefficients shift sharply towards the end of the sample, once $\theta > 1$. The slope of the curve steepens significantly in the post-COVID period, ending with a value close to 2. This is consistent with the results in Table 1, although smaller in magnitude. The coefficient on the supply shock also increases from zero to over 0.15. This is of the same order, even if slightly smaller, than the estimated value of the supply shock in Table 4 when $\theta > 1$.

The inflation-persistence coefficient declines over time and hovers near zero at end of the sample. This, too, is one of our model's main predictions when $\theta > 1$, and is consistent with the benchmark regression.

Figure 15 highlights how poorly a forecaster would have done using either our benchmark regression or our regression with time varying coefficients if the forecaster fails to incorporate the non-linearities. The results reported in that figure give a natural explanation for why both policy makers and professional forecasters consistently failed to forecast the scope of the surge in inflation, as well as its persistence.

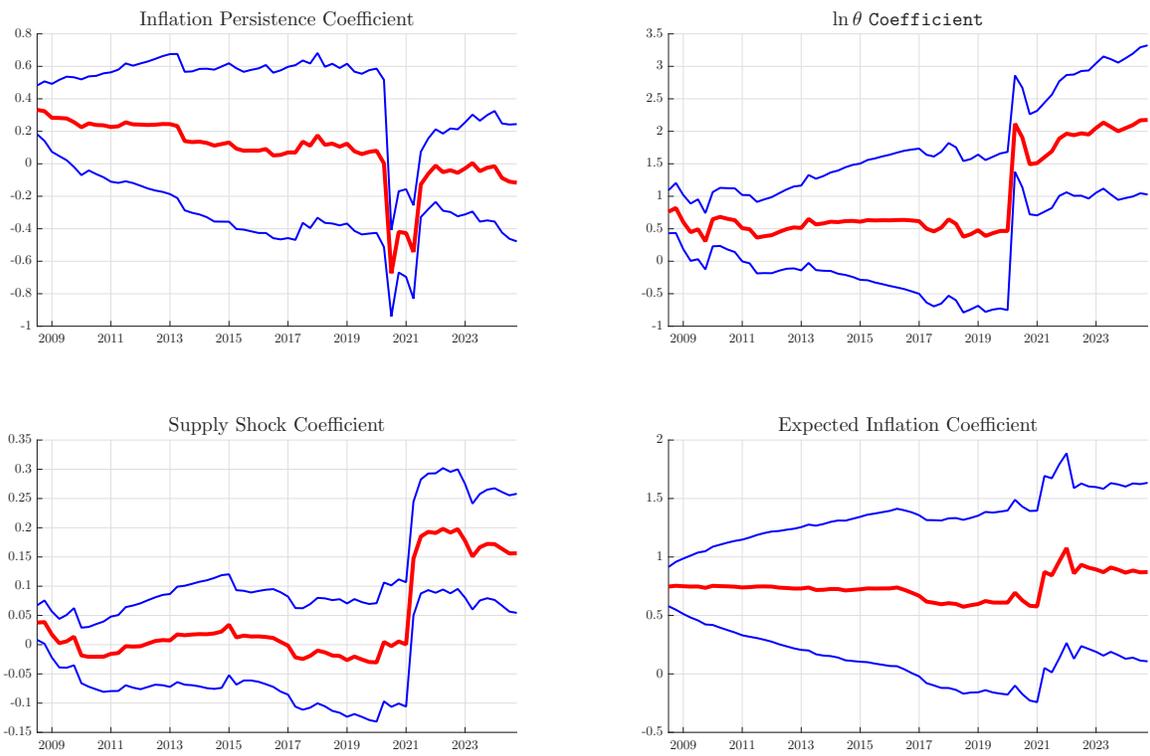


Figure 14: Estimates of the Kalman Filter with time-varying parameters on sample 2008 Q3 – 2024 Q4 with one-standard-deviation confidence bands.

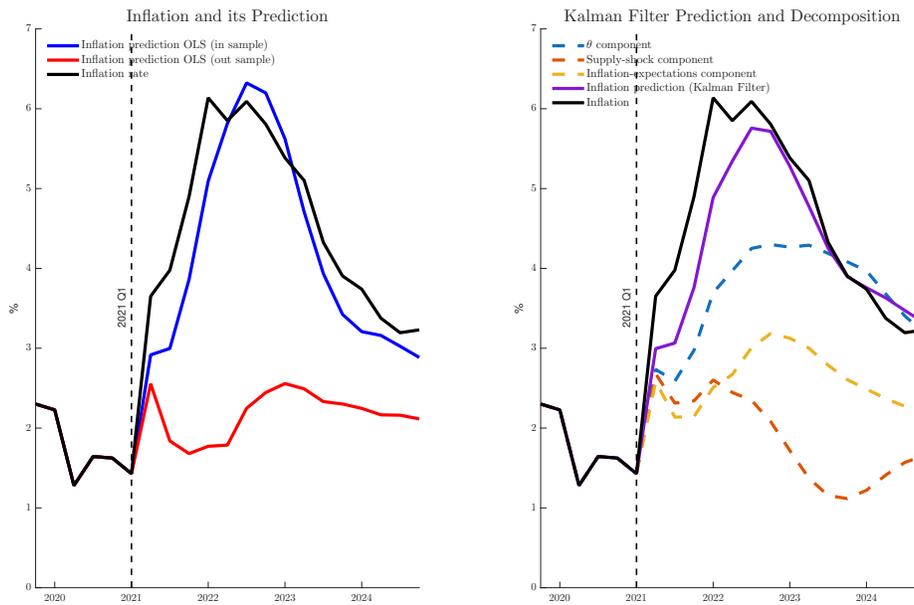


Figure 15: Left panel: CPI inflation rate at annual rates (black line); out-of-sample inflation prediction (red line) using OLS regression (41) without the dummy variable on the sample 2008 Q3 – 2021 Q1; in-sample inflation prediction (blue line) using OLS regression (41) on the sample 2008 Q3 – 2024 Q4. Right panel: CPI inflation rate at annual rates (black line); in-sample inflation prediction (purple line) using Kalman-Filter estimation with time-varying coefficients on the sample 2008 Q3 – 2024 Q4. The three dashed lines represent the inflation prediction using the Kalman-Filter estimates by restricting only to the variable θ , or the supply shock or the inflation expectations, respectively.

F Appendix: Data Description

Table 3

Table 3 reports a set of specification tests designed to assess the presence of a kink in the Phillips curve and the location of the Beveridge threshold. The first block presents tests of linearity. The global sup test is a sup-Wald (or sup-LR) statistic of the null $H_0 : \delta = \varrho_1 = \gamma_1 = 0$ over the full trimmed grid of thresholds. For each candidate θ^* , an F-test of the slope restrictions is computed and the supremum across the grid is taken; bootstrap critical values are obtained by moving-block resampling of residuals. The restricted sup test repeats the same procedure but confines the grid to the economically relevant range $\theta^* \in [0.75, 1.25]$, thereby increasing power around the unit-tightness benchmark. The fixed-break Wald test imposes $\theta^* = 1$ and tests the same null by conventional HAC-based Wald statistics.

The second block of the table reports tests of location. These are profile likelihood-ratio tests of $H_0 : \theta^* = 1$ against the alternative of a freely estimated threshold $\hat{\theta}^*$. For each sample, the LR statistic is computed as

$$LR(\theta^* = 1) = \frac{SSR(\theta^* = 1) - SSR(\hat{\theta}^*)}{\hat{\sigma}^2},$$

with $\hat{\sigma}^2 = SSR(\hat{\theta}^*) / (T - k_{\text{eff}})$. Bootstrap p -values are reported using both the pointwise distribution of $LR(\theta^*)$ and the more conservative sup distribution. Together, these procedures distinguish the question of whether a kink exists from the question of whether the threshold is located at unity.

Table 4

Table 4 presents estimates of

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) \ln \theta_t + (\beta_\varrho + \beta_{\varrho_d} D_t) \varrho_t + (\beta_{\pi^e} + \beta_{\pi_d^e} D_t) \pi_t^e + \varepsilon_t, \quad (\text{F.23})$$

where π_t is the annualized quarterly inflation rate (log change) in deviation from a 2% target. Inflation is measured using core CPI (excluding food and energy); π_{t-1} is its lag. CPI data are from FRED and are converted to quarterly frequency by averaging monthly observations within each quarter; $\ln \theta_t$ is the log vacancy–unemployment ratio. Vacancies are from the BLS JOLTS database from December 2000 onward; prior to that we backcast using Barnichon’s composite vacancy index (Barnichon, 2010b), which is available monthly from January 1951. In our sample we use the series from 1960Q1. All vacancy and unemployment data are monthly and are averaged within the quarter before taking logs. The indicator D_t equals one when $\theta_t \geq 1$ and zero otherwise.

ϱ_t is the four–quarter average of the first principal component of three “relative price gap” series: (i) headline CPI minus core CPI inflation, (ii) headline PCE minus core PCE inflation, and (iii) import–price inflation minus GDP–deflator inflation. All inflation rates are annualized log changes.

The underlying price series are taken from FRED and converted to quarterly frequency by averaging monthly observations. Let pr_t denote the (quarterly) principal component; then

$$q_t = \frac{pr_t + pr_{t-1} + pr_{t-2} + pr_{t-3}}{4}.$$

Inflation expectations, π_t^e , are proxied by the 1-year CPI inflation expectations from the U.S. Survey of Professional Forecasters (from FRED), available since 1981Q3. We extend the series back to 1960Q1 using the 12-month Livingston Survey, which has semi-annual frequency; missing quarters are filled by spline interpolation that preserves the levels. In all regressions, both π_t and π_t^e enter as deviations from a 2% annual target.

Table 5

Table 5 reports estimates of the kinked Phillips curve with an interaction on inflation expectations above the threshold,

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + \beta_\theta \ln \theta_t + \beta_{\theta_d} (\ln \theta_t - \ln \theta^*) D_t + (\beta_\varrho + \beta_{\varrho_d} D_t) \cdot q_t + (\beta_{\pi^e} + \beta_{\pi_d^e} D_t) \cdot \pi_t^e + \varepsilon_t, \quad (\text{F.24})$$

where $D_t = \mathbf{1}\{\ln \theta_t > \ln \theta^*\}$. The model is estimated on the same data as in Table 4, with the threshold fixed at the values $\theta^* = 0.5006$ for the full sample (1960Q1–2024Q4) and $\theta^* = 0.9305$ for the post-crisis subsample (2008Q3–2024Q4). These values correspond to the maximizers of the profile likelihood shown in Figure 13. Coefficient estimates are obtained by ordinary least squares conditional on θ^* , while inference relies on heteroskedasticity- and autocorrelation-consistent (HAC) standard errors (Newey–West with Bartlett kernel and fixed bandwidth), computed on the rank-reduced design matrix to avoid collinearity at the threshold. The HAC procedure ensures robustness to both heteroskedasticity and serial correlation in the regression residuals, and the effective degrees of freedom are adjusted accordingly. Reported t -statistics therefore account for the sampling variability induced by serially correlated inflation dynamics as well as the discrete threshold design.

Table 6

Table 6 presents the OLS estimates of regression (41) with the same variables as Table 4, except that the inflation lag and the log of the vacancy–unemployment ratio (θ) are instrumented with the fitted values from OLS regressions on their first lags. Specifically, the regressors π_{t-1} and $\ln \theta_t$ are replaced with the fitted values from the respective auxiliary regressions:

$$\begin{aligned} \pi_t &= \beta_0 + \beta_1 \pi_{t-1} + \varepsilon_t, \\ \ln \theta_t &= \gamma_0 + \gamma_1 \ln \theta_{t-1} + \varepsilon_t. \end{aligned}$$

Table 7

Table 7 presents the OLS estimates of regression (41) with the same variables as Table 4, except that the log of the vacancy–unemployment ratio (θ) is replaced with its lag.

Table 8

Table 8 uses as a measure of the supply shock the four-quarter average of the CPI headline shock, as described in Table 4.

Table 9

Table 9 uses as a measure of the supply shock the four-quarter average of the import-price shock, as described in Table 4.

Table 10

Table 10 uses as a proxy for inflation expectations the 2-year inflation expectations of the Federal Reserve Bank of Cleveland, available since 1982Q1 from FRED. The series is patched backward to 1960Q1 using interpolated 12-month Livingston Survey inflation expectations.

Table 11

Table 11 uses as a proxy for inflation expectations the 5-year inflation expectations of the Federal Reserve Bank of Cleveland, available since 1982Q1 from FRED. The series is patched backward with 1-year GDP-deflator inflation expectations from the U.S. Survey of Professional Forecasters, retrieved from Thomson Reuters Datastream (available since 1970Q2), and further back to 1960Q1 using interpolated 12-month Livingston Survey expectations.

Table 12

Table 12 uses as a proxy for inflation expectations the 5-year–5-year forward inflation expectations backcast by Groen and Middeldorp (2013) until 1971Q4. The series is patched back to 1960Q1 using interpolated 12-month Livingston Survey expectations.

Table 13

Table 13 uses as a proxy for inflation expectations the 1-year inflation expectations of the Federal Reserve Bank of Cleveland, available since 1982Q1 from FRED. The series is patched with the 1-year GDP-deflator inflation expectations from the U.S. Survey of Professional Forecasters (Datastream, available since 1970Q2), and further back to 1960Q1 using interpolated 12-month Livingston Survey expectations.

Table 14

Table 14 uses as a proxy for inflation expectations the interpolated 12-month Livingston Survey expectations.

Table 15

Table 15 uses as a proxy for inflation expectations the 12-month household expectations from the Michigan Survey, using the median until 1978Q1 and the mean thereafter back to 1960Q1. Missing values occur in the 1960s.

Table 16

Table 16 uses core PCE inflation at an annualized quarterly rate. The core PCE price index is obtained from FRED and aggregated quarterly as the average of monthly observations.

Table 17

Table 17 uses the level of θ rather than $\ln \theta$.

Table 18

Table 18 repeats the estimation of Table 4, replacing the lag of inflation with the detrended real wage. The real wage is constructed by deflating the BLS series *Nonfarm Business Sector: Unit Labor Costs for All Workers (ULCNFB)* with the GDP deflator from FRED. Since the series is non-stationary, it is detrended using the Hamilton (2018) procedure to extract the cyclical component.

Tables 19–21

Tables 19–21 use core CPI inflation (annualized quarterly) together with different measures of expectations: 1-year, 2-year, and 5-year inflation expectations from the Cleveland Fed; 1-year CPI expectations from the U.S. Survey of Professional Forecasters; 5-year–5-year forward expectations; 12-month Michigan Survey household expectations; and 12-month Livingston Survey expectations. Data are taken from FRED and Thomson Reuters Datastream and are converted to quarterly frequency.

Figure 1

Figure 1 presents the scatter plots of inflation and labor market tightness in the United States for the samples 1960 Q1–1969 Q4, 1970 Q1–1987 Q2, 1987 Q3–2008 Q2, 2008 Q3–2024 Q4. Inflation is annual inflation rate computed using the quarterly CPI. CPI quarterly observations are the average of the relevant monthly observations. $\ln \theta$ is the log of the ratio of vacancies to unemployed workers provided by Barnichon (2010b) and updated by the author. Data are monthly. Accordingly, the quarterly series is the average of the relevant monthly observations.

Figure 2

The top panel of Figure 2 plots the annual CPI inflation rate for 1913Q1–1959Q4, constructed from quarterly CPI data. Quarterly values are computed as the average of monthly observations. The panel also reports θ , the vacancy–unemployment ratio, which Petrosky-Nadeau and Zhang (2021) extend back to 1919. As the underlying data are monthly, the quarterly series is constructed as the average of monthly observations

The bottom panel reports the same series for 1960Q1–2024Q4. The vacancy–unemployment ratio is based on JOLTS data from December 2000 onward, with earlier values provided by Barnichon (2010b). As above, the underlying data are monthly, and quarterly series are constructed as the average of monthly observations.

Figure 3

Figure 3 plots the annual inflation rate computed using the quarterly CPI. CPI quarterly observations are the average of the relevant monthly observations. Inflation expectations are from the Livingston survey of the Federal Reserve Bank of Philadelphia for the 12-month horizon on CPI. The frequency of the graph is twice yearly, consistently with the Livingston Survey data. The inflation expectations of the Livingston Survey are also used in Table 14 in the robustness analysis of Table 4.

Figure 4

The Figure uses the estimates of Table 4, column 4, to draw inflation as a piecewise linear function of $\ln \theta$.

Figure 5

Figure 5 plots: (i) blue dashed— a dynamic projection starting in 2021Q1 based on a Phillips curve estimated over 1960Q1–2021Q1 with tightness-region dummies; (ii) red dotted— the corresponding projection from the no-dummy specification estimated on the same sample; (iii) in-sample fit (thin solid; see legend)— fitted values from the dummy specification; (iv) dash-dot (see legend)— a projection obtained using coefficients estimated on 1960Q1–1969Q4 with dummies. Projections are conditional on realized paths of labor-market tightness (V/U), the supply-shock proxy, and inflation expectations, and are anchored at 2021Q1 (vertical dashed line), so differences emerge from 2021Q2 onward. Axes: calendar years (ticks at Q1) and percent.

Figure 6

Figure 6 builds a decomposition among the different regressors of Table 4 column 4. Consider a situation in which inflation is on target, expectation on target, and $\ln \theta_t = \ln \bar{\theta}$ where $\ln \bar{\theta}$ correspond to $\ln \theta_t$ being neither inflationary or deflationary. In this case

$$0 = \beta_c + \beta_\theta \ln \bar{\theta}$$

Hence:

$$\ln \bar{\theta} = -\frac{\beta_c}{\beta_\theta}.$$

This implies that the fitted value for inflation can be written as follows:

$$\begin{aligned} \pi_t = & \underbrace{\beta_\pi \pi_{t-1}}_{\text{Lagged Inflation}} + \underbrace{\beta_\theta ((\min(\ln(\theta_t), 0) - \ln \bar{\theta}))}_{\text{Labor Market Tightness}} + \underbrace{D_t \left[\beta_{\theta_d} \ln \theta_t \right]}_{\text{Labor Market Tightness when } \theta \geq 1} \\ & + \underbrace{\beta_Q Q_t + \beta_{Q_d} D_t Q_t}_{\text{Cost Push Shock}} + \underbrace{\beta_\pi \pi_t^e}_{\text{Inflation Expectations}} \end{aligned}$$

Note that the contribution of the component $\ln \theta_t$ when $\theta_t \geq 1$ is given by the component $\beta_\theta ((\min(\ln(\theta_t), 0) - \ln \bar{\theta}))$ up to the unitary value and the component $\beta_{\theta_d} \ln \theta_t$. Therefore the overall contribution is $\beta_c + \beta_{\theta_d} \ln \theta_t$. Moreover, when $\theta_t \geq 1$ the contribution of the cost push shock is given by the sum of $\beta_Q Q_t + \beta_{Q_d} D_t Q_t$. Figure 6 shows this decomposition together with core inflation, both at annualized rates.

Figure 7

Figure 7 builds a decomposition among the different regressors of Table 4 column 3 for the sample 1960 Q1 – 2024 Q4. The procedure follows the description given under Figure 6.

Figure 8

Data for the PCE-index inflation and its forecasts of the Summary of Economic Projections (Panel a) are from the FRED database. Data on PCE-index inflation and its forecasts of the Survey of Professional Forecasters (Panel b) are from the FRED database.

Figure 9

Figure 9 plots the annual inflation rate computed using the quarterly CPI. CPI quarterly observations are the average of the relevant monthly observations. Inflation expectations are the 5-year inflation expectations of the Federal Reserve of Cleveland. Data are from the FRED database. The Figure also plots the 5-year 5-year forward inflation expectations of Groen and Middelcorp (2013) update from FRED database using the series T5YIFR with end-of-month data. The 5-year inflation expectations of the Federal Reserve of Cleveland are used in Table 11 in the robustness analysis of Table 4, while the 5-year 5-year forward inflation expectations of Groen and Middelcorp (2013) in Table 12.

Figure 10

Figure 10 presents the three different measures of the supply shock that we use to build the proxy for q_t , namely the four-quarter averages of the principal component of the two headline shocks (using CPI and PCE price index) and the import-price shock, as described under Table 4.

Figure 11

Figure 11 plots the inflation rate and inflation expectations used in Table 4, Table 10 and Table 15. Inflation rate is the annualized quarterly inflation rate computed using core CPI. Inflation expectations, used in Table 4, are the 2-year inflation expectations of the Federal Reserve Bank of Cleveland, collected from FRED quarterly and available since 1982 Q2. The series is patched backward to 1960 Q1 with the 12-month inflation expectations from the Livingston survey. Since the latter is twice yearly, missing observations are interpolated through a spline curve-preserving function. Inflation expectations in Table 10 are the 1-year CPI inflation expectations of the Survey of Professional Forecasters, retrieved from Thomson Reuters Datastream, which starts in 1981 Q3. We patch this series backward to 1960 Q1 again using interpolated 12-month Livingston inflation expectations. Inflation expectations in Table 15 represent the 12-month consumer inflation expectations from the University of Michigan survey, retrieved from the FRED database at a quarterly frequency. This data starts in

1979 Q1 and corresponds to the median of the survey. The series is extended backward to 1960 Q1 using the mean of the same survey.

Figure 12

Figure 12 plots wage growth, in its decomposition between the category job switchers and job stayers, and core CPI inflation. Data on wages are monthly and collected from the website of the Atlanta Fed at Wage Growth Tracker - Federal Reserve Bank of Atlanta (atlantafed.org).

Figure 13

Figure 13 depicts the profile likelihood–ratio statistic for the kinked Phillips curve regression

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + \beta_\theta \ln \theta_t + \beta_{\theta_d} (\ln \theta_t - \ln \theta^*) D_t + (\beta_\rho + \beta_{\rho_d} D_t) \cdot \rho_t + \beta_{\pi^e} \cdot \pi_t^e + \varepsilon_t, \quad (\text{F.25})$$

where $D_t = \mathbf{1}\{\ln \theta_t > \ln \theta^*\}$. For each candidate threshold θ^* on a grid trimmed to the interior of the support of θ_t (excluding the bottom and top 2% of observations), we run OLS of (1), compute the residual sum of squares $SSR(\theta^*)$, and form the profile likelihood–ratio curve

$$LR(\theta^*) = \frac{SSR(\theta^*) - SSR(\hat{\theta}^*)}{\hat{\sigma}^2}, \quad \hat{\sigma}^2 = \frac{SSR(\hat{\theta}^*)}{T - k_{\text{eff}}},$$

with $\hat{\theta}^* = \arg \min_{\theta^*} SSR(\theta^*)$ and k_{eff} the effective number of regressors after rank reduction. The figure plots $LR(\theta^*)$ as a function of the level threshold θ^* . The top panel corresponds to the post-crisis subsample 2008Q3–2024Q4, while the bottom panel covers the full sample 1960Q1–2024Q4. The vertical dashed line marks $\theta = 1$, the unit tightness benchmark. Confidence intervals for $\hat{\theta}^*$ are obtained by comparing $LR(\theta^*)$ to bootstrap critical values derived from moving-block resampling of residuals (999 replications, fixed seed, parallel execution).

Figure 14

Figure 14 presents the estimates through Kalman Filter of the measurement equation

$$\pi_t = \beta_{c,t} + \beta_{\pi,t} \pi_{t-1} + \beta_{\theta,t} \ln \theta_t + \beta_{\rho,t} \rho_t + \beta_{\pi^e,t} \pi_t^e + \varepsilon_t,$$

in which ε_t is distributed as $N(0, \sigma_\varepsilon^2)$ with the state equations given by

$$\begin{aligned} \beta_{c,t} &= \beta_{c,t-1} + \epsilon_{c,t} \\ \beta_{\pi,t} &= \beta_{\pi,t-1} + \epsilon_{\pi,t} \\ \beta_{\theta,t} &= \beta_{\theta,t-1} + \epsilon_{\theta,t} \\ \beta_{\rho,t} &= \beta_{\rho,t-1} + \epsilon_{\rho,t} \\ \beta_{\pi^e,t} &= \beta_{\pi^e,t-1} + \epsilon_{\pi^e,t} \end{aligned}$$

in which $\epsilon_{c,t} \sim N(0, \sigma_{\epsilon_c}^2)$, $\epsilon_{\pi,t} \sim N(0, \sigma_{\epsilon_\pi}^2)$, $\epsilon_{\theta,t} \sim N(0, \sigma_{\epsilon_\theta}^2)$, $\epsilon_{\rho,t} \sim N(0, \sigma_{\epsilon_\rho}^2)$, $\epsilon_{\pi^e,t} \sim N(0, \sigma_{\epsilon_{\pi^e}}^2)$. The Kalman Filter is initialized by running an OLS regression of the measurement equation with constant coefficients on the sample period 1960 Q1 – 2008 Q2. Then, the Kalman Filter estimation runs from 2008 Q3 to 2024 Q4. σ_{ϵ}^2 is initialized as the variance of the residuals of the OLS regression on the pre-sample; β_c , β_π , β_θ , β_ρ and β_{π^e} are initialized with OLS estimates of the respective coefficients on the pre-sample; $\sigma_{\epsilon_c}^2$, $\sigma_{\epsilon_\pi}^2$, $\sigma_{\epsilon_\theta}^2$, $\sigma_{\epsilon_\rho}^2$, $\sigma_{\epsilon_{\pi^e}}^2$ are initialized with the variance of the respective coefficients of the OLS regression on the pre-sample. Figure 14 plots the estimated time-varying coefficients $\beta_{\pi,t}$, $\beta_{\theta,t}$, $\beta_{\rho,t}$ and $\beta_{\pi^e,t}$ using the Kalman Filter and their one-standard-deviation confidence bands.

Figure 15

The black line of the left panel of Figure 15 is the annual CPI inflation rate excluding food and energy sectors. The red line represents the out-of-sample prediction of equation

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + \beta_\theta \ln \theta_t + \beta_\rho \rho_t + \beta_{\pi^e} \pi_t^e + \epsilon_t,$$

estimated for the sample 2008 Q3 – 2021 Q1 for the period 2021 Q2 – 2024 Q4. The model produces forecasts for the quarterly inflation rate in deviations of a 2% target. Accordingly, we build the corresponding predicted inflation at annual rates.

The blue line represents the in-sample prediction of equation

$$\pi_t = \beta_c + \beta_\pi \pi_{t-1} + (\beta_\theta + \beta_{\theta_d} D_t) \cdot \ln \theta_t + (\beta_\rho + \beta_{\rho_d} D_t) \rho_t + \beta_{\pi^e} \pi_t^e + \epsilon_t,$$

estimated for the sample 2008 Q3 – 2024 Q4 for the period 2021 Q2 – 2024 Q4. The model produces predictions for the quarterly inflation rate in deviation of a 2% target. Accordingly, we build the corresponding predicted inflation at annual rates.

The black line in the right panel of Figure 15 is the annual CPI inflation rate excluding food and energy. The purple line is the in-sample prediction using the non-linear Kalman Filter estimates, while ‘ θ component’, ‘Supply Shock component’, ‘Inflation Expectations component’ correspond, respectively, to the in-sample prediction derived from the following three equations using the Kalman Filter estimates.

$$\begin{aligned} \pi_t^\theta &= \beta_{\pi,t} \pi_{t-1}^\theta + \beta_{\theta,t} (\ln \theta_t - \ln \bar{\theta}_t), \\ \pi_t^\rho &= \beta_{\pi,t} \pi_{t-1}^\rho + \beta_{\rho,t} \rho_t, \\ \pi_t^* &= \beta_{\pi,t} \pi_{t-1}^* + \beta_{\pi^e,t} \pi_t^e. \end{aligned}$$

in which

$$\ln \bar{\theta}_t = -\beta_{c,t}^{-1} \beta_{\theta,t},$$

and initial conditions are given by the inflation rate in 2021 Q1. The model produces inflation predictions at quarterly frequency in deviations of a 2% target, so we build the corresponding annual inflation predictions plotted in the Figure.